Circle

Conic : $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

If $\Delta \neq 0$, $a = b$ & $h = 0$ $\Rightarrow$ Circle

Def^n: Locus of a • which moves in a plane, so that its dist. from a fixed pt. in plane is always const.

Here, fix. pt. is known as Centre and const. dist. is known as Radius

Forms of Circle

1) Centre radius form – $\boxed{(x-\alpha)^2 + (y-\beta)^2 = a^2}$

Centre $\equiv (\alpha, \beta)$; Radius $= a$

2) Standard form – $\boxed{x^2 + y^2 = a^2}$

Centre $\equiv (0,0)$; Radius $= a$

3) General form – $\boxed{x^2+y^2+2gx+2fy+c=0}$

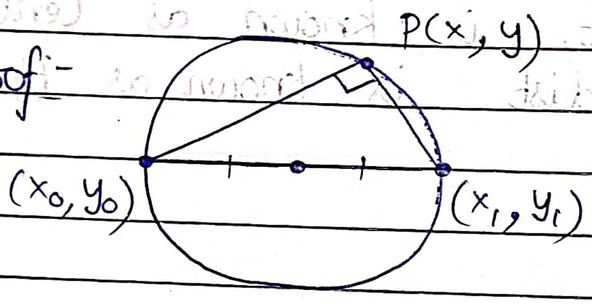
Centre $\equiv (-g,-f)$; Radius $= \sqrt{g^2+f^2-c}$

If $g^2+f^2-c > 0 \Rightarrow$ Real Circle
" " $< 0 \Rightarrow$ Imag. Circle
" " $= 0 \Rightarrow$ • Circle

4) Diameter form – $\boxed{(x-x_0)(x-x_1)+(y-y_0)(y-y_1)=0}$

Endpts. of diameter : (x_0, y_0) ; (x_1, y_1)

Proof :-



$$\left(\frac{y-y_1}{x-x_1}\right)\left(\frac{y-y_0}{x-x_0}\right) = (-1)$$

Centre $\equiv \left(\frac{x_0+x_1}{2}, \frac{y_0+y_1}{2}\right)$

5) Parametric form – $\boxed{\begin{array}{l} x = \alpha + a\cos(\theta) \\ y = \beta + a\sin(\theta) \end{array}}$

This represents $(x-\alpha)^2+(y-\beta)^2 = a^2$

Here, $\theta \in [0, 2\pi)$

Q) Find centre ~~of~~ and radius of $\odot$,

$$2x^2 + 2y^2 = 3x - 5y + 7$$

Q) P.t. the radii of $\odot$s,

$x^2 + y^2 = 1$, $x^2 + y^2 - 2x - 6y = 6$ and $x^2 + y^2 - 4x - 12y - 9 = 0$ are in A.P.

Q) Find eq^n of $\odot$ whose centre is the $\cap$ of $2x - 3y + 4 = 0$ & $3x + 4y - 5 = 0$ and passes thru origin.

Q) Find eq^n of $\odot$ concentric with, $x^2 + y^2 - 8x + 6y - 5 = 0$ and passing thru $(-2, -7)$.

Q) A $\odot$ has radius 3 and its centre lies on $y = x - 1$. Find its eq^n if it passes thru $(7, 3)$.

Q) If para form of $\odot$s ~~is~~ are $x = (-4) + 5c_\theta$, $y = (-3) + 5s_\theta$

and $x = a c_\alpha + b s_\alpha$, $y = a s_\alpha - b c_\alpha$, find their cartesian form.

A) $x^2 + y^2 - \frac{3}{2}x + \frac{5}{2}y - \frac{7}{2} = 0 \Rightarrow$ Centre $\equiv (3/4, -5/4)$

Radius $= \left(\frac{3\sqrt{5}}{2\sqrt{2}}\right)$

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	Eqⁿ	Radius
A)	$x^2+y^2-1=0$	1
	$x^2+y^2-2x-6y-6=0$	4
	$x^2+y^2-4x-12y-9=0$	7

A) Centre $\equiv \left(-1/17, \cancel{41/31}\right)$ 22/17 ; Eqⁿ : $(17x+1)^2+(17y-22)^2=485$

A) Centre $\equiv (4,-3) \Rightarrow$ Eqⁿ : $(x-4)^2+(y+3)^2=52$

A) Let centre $(t, t-1)$

$\Rightarrow$ Eqⁿ : $(x-t)^2+(y-(t-1))^2=9$

Passes thru $(7,3)$ $\Rightarrow (t-7)^2+(t-4)^2=9$

$\Rightarrow t^2-11t+\cancel{56}=0 \Rightarrow t=4,7$
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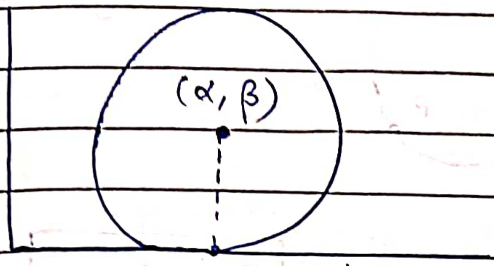
Eqⁿ : $(x-4)^2+(y-3)^2=9$, $(x-7)^2+(y-6)^2=9$

A) $x=(-4)+5c_\theta$
$y=(-3)+5s_\theta$ $\rightarrow$ $(x+4)^2+(y+3)^2=25$

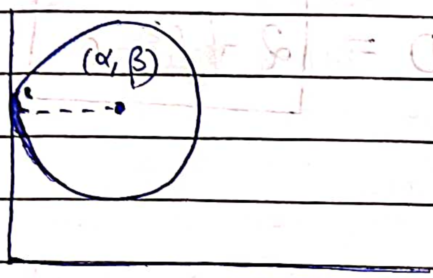
$x=ac_\alpha+bs_\alpha$
$y=as_\alpha-bc_\alpha$ $\rightarrow$ $x^2+y^2=a^2+b^2$

Special forms –

1) 

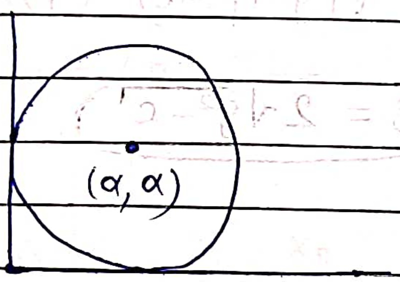
$$\boxed{(x-\alpha)^2+(y-\beta)^2=\beta^2}$$

(Touching X axis.)

2) 

$$\boxed{(x-\alpha)^2+(y-\beta)^2=\alpha^2}$$

(Touching Y axis)

3) 

$$\boxed{(x-\alpha)^2+(y-\alpha)^2=\alpha^2}$$

(Touching X & Y axes)

4) If ⊙ thru $(0,0) \Rightarrow$ $\boxed{x^2+y^2+2gx+2fy=0}$

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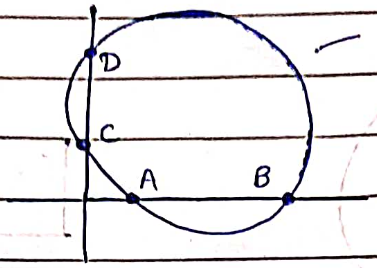
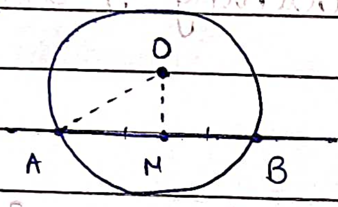
Intercepts made by Circle



Intercept on X axis = $AB = \boxed{2\sqrt{g^2-c}}$

Intercept on Y axis = $CD = \boxed{2\sqrt{f^2-c}}$

Proof:



$AM = \sqrt{OA^2 - OM^2}$

$= \sqrt{(g^2+f^2-c)-(-f)^2}$

$\Rightarrow$ $AB = 2\sqrt{g^2-c}$

Q) Find eq^n of $\odot$ end pts. of whose diameter are centres of $\odot$s $x^2+y^2+6x-14y=1$ & $x^2+y^2-4x+10y=2$

Q) The abscissa of 2 ·s A and B are roots of eq^n $x^2+2ax-b^2=0$ and their ordinates are roots of eq^n $x^2+2px-q^2=0$. Find eq^n of circle with

Q) Find eq^n of ⊙ which touches Y axis at dist. of 4 from origin, and cuts intercept of 6 from X axis.

Q) Find eq^n of ⊙, which passes thru origin and makes intercepts of length a & b on X & Y axis resp.

Q) A ⊙ of radius 5 touches coordinate axes in 1st quad. If ⊙ makes 1 complete roll on X axis (+ve) dirx^n, find its eq^n in new posit.

A) Centre$_1$ $\equiv (-3, 7)$ Eq^n: $\boxed{(x-2)(x+3)+(y+5)(y-7)=0}$

Centre$_2$ $\equiv (2, -5)$

A) Let roots x_1, x_2 & y_1, y_2 of eq^{n}s resp.

$x^2+2ax-b^2=0 \ \langle x_1, x_2$ $\qquad$ $y^2+2py-q^2=0 \ \langle y_1, y_2$

$\Rightarrow \quad x_1+x_2=-2a \qquad y_1+y_2=-2p$

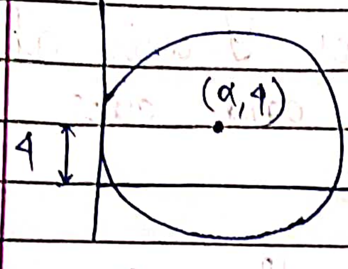
$x_1x_2=-b^2 \qquad y_1y_2=-q^2$

Eq^n: $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

$\Rightarrow \quad x^2+y^2-(x_1+x_2)x-(y_1+y_2)y+(x_1x_2+y_1y_2)=0$

$\Rightarrow \quad \boxed{x^2+y^2+2ax+2py-(b^2+q^2)=0}$

A)

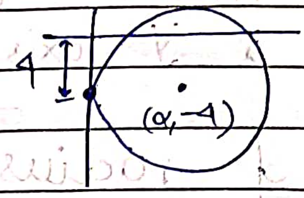


$(x-\alpha)^2+(y-4)^2=\alpha^2$

$\Rightarrow x^2+y^2-2\alpha x-8y+16=0$

Also, $2\sqrt{\alpha^2-16}=6 \Rightarrow \alpha=\pm 5$

Circle can also be like:



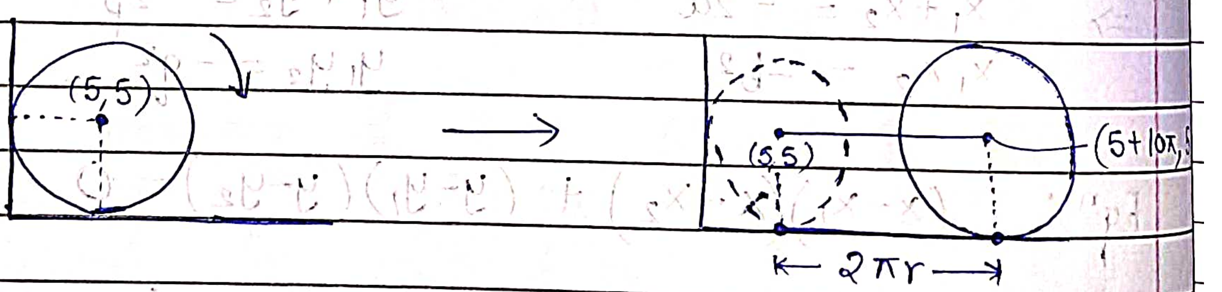
Eqns : $\boxed{\begin{matrix}(x-5)^2+(y-4)^2=25 \\ (x+5)^2+(y-4)^2=25\end{matrix}}$, $\boxed{\begin{matrix}(x-5)^2+(y+4)^2=25 \\ (x+5)^2+(y+4)^2=25\end{matrix}}$

A) $2\sqrt{g^2-c}=a$, $2\sqrt{f^2-c}=b$

Thru origin $\Rightarrow c=0 \Rightarrow \begin{matrix} g=\pm(a/2) \\ f=\pm(b/2)\end{matrix}$

Eqns: $\boxed{\begin{matrix}x^2+y^2+ax+by=0 \\ x^2+y^2-ax+by=0\end{matrix}}$, $\boxed{\begin{matrix}x^2+y^2+ax-by=0 \\ x^2+y^2-ax-by=0\end{matrix}}$

A)

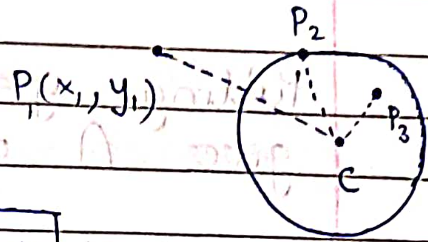


Eqn : $\boxed{(x-(5+10\pi))^2+(y-5)^2=25}$

Position of a • wrt ⊙

$S: x^2 + y^2 + 2gx + 2fy + c = 0$

Pt. (x_1, y_1)



Pt. outside ⊙ $\Rightarrow$ $\boxed{S(x_1, y_1) > 0}$

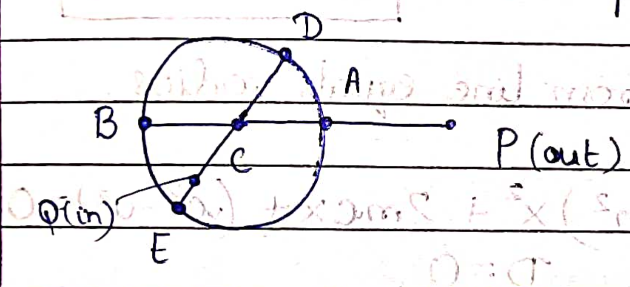
Pt. inside ⊙ $\Rightarrow$ $\boxed{S(x_1, y_1) < 0}$

Pt. on ⊙ $\Rightarrow$ $\boxed{S(x_1, y_1) = 0}$

Proof: If $P(x_1, y_1)$ outside ⊙ $\Rightarrow$ $PC > r$ (radius)

$\Rightarrow PC^2 > r^2 \Rightarrow (x_1 + g)^2 + (y_1 + f)^2 > (g^2 + f^2 - c)$

Max. & min. dist of • from ⊙



Dist. max. $= PB = |PC + r|$

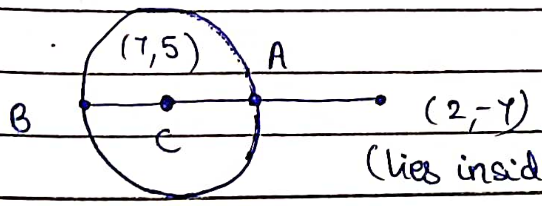
Dist. min. $= PA = |PC - r|$

Q) Find max. & min. dist. of $(2, -7)$ from $x^2 + y^2 - 14x - 10y - 151 = 0$

A) Centre $\equiv (7, 5)$

Dist. max. $= 13 + 15 = 28$

Dist. min. $= 15 - 13 = 2$



(lies inside ⊙)

Radius $= 15$

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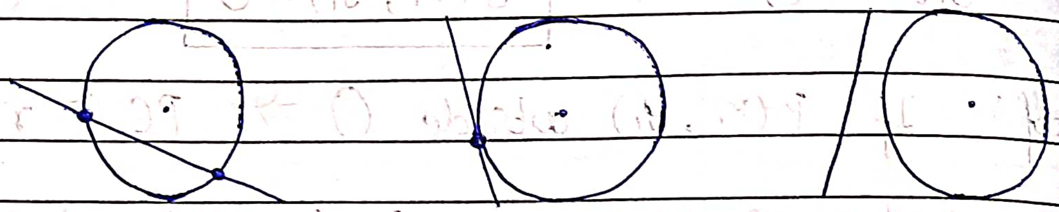
Intersection of a Line & Circle

$\odot$: $x^2+y^2=a^2$ $/$: $y=mx+c$

- Putting $y=mx+c$ in eq^n of circle gives $\cap$ of $\odot$ & $/$.

We will have a quad. in 'x'. If D is discriminant of that eq^n, then



$D>0$ $D=0$ $D<0$

1) Line touches the Circle, $\boxed{c^2=a^2(1+m^2)}$

Proof: • $\perp$ dist of centre from line equals radius.

- Req. quad. is $(1+m^2)x^2+2mcx+(c^2-a^2)=0$
for tangency, set $D=0$.

2) Lines $\boxed{y=mx\pm a\sqrt{1+m^2}}$ touch the give $\odot$ $\forall m\in\mathbb{R}$ at pts. $\boxed{\left(\frac{\pm am}{\sqrt{1+m^2}}, \frac{\mp a}{\sqrt{1+m^2}}\right)}$

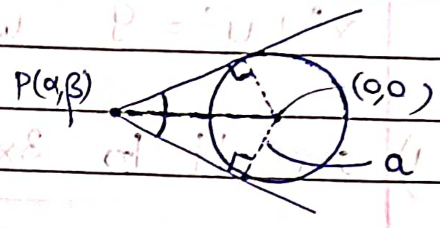
i.e. $\boxed{\left(\frac{\pm a^2m}{c}, \frac{\mp a^2}{c}\right)}$

3) ∠ b/w Pair of Tangents –

If tangents drawn from external pt. (α, β) on ⊙, then

[Don't remember formula. Remember procedure.]

Proof: • By geometry.



• Since tangents.

$\Rightarrow$ Eqⁿ: $y = mx \pm a\sqrt{1+m^2}$

Since this pass thru P $\Rightarrow$ $(\beta - m\alpha) = (\pm a)\sqrt{1+m^2}$

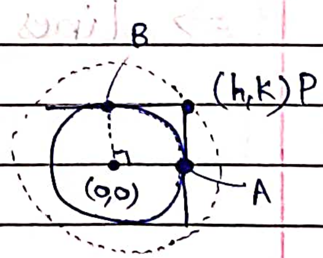
$\Rightarrow (\alpha^2 - a^2)m^2 - (2\alpha\beta)m + (\beta^2 - a^2) = 0$ $\left.\begin{matrix} m_1 \\ m_2 \end{matrix}\right\}$ Slopes of Tangents.

Now find angle b/w lines.

4) Director ⊙ –

Locus of • s.t. pair of tangents drawn from an external pt. to the ⊙ are ⊥ to each other.

Proof: • Obviously PAOB is sq.



$\Rightarrow h^2 + K^2 = (a\sqrt{2})^2$

$\Rightarrow$ Locus: $x^2 + y^2 = 2a^2$

- Earlier we found, $m_1+m_2=\left(\frac{2\alpha\beta}{\alpha^2-a^2}\right)$ $m_1m_2=\left(\frac{\beta^2-a^2}{\alpha^2-a^2}\right)$

  For $\perp$ $\Rightarrow$ $1+m_1m_2=0$ $\Rightarrow$ $\alpha^2+\beta^2=2a^2$

Q) Find eqⁿs of tangents to circle $x^2+y^2=9$ which —

1) are $\parallel$ to $3x+4y-5=0$
2) are $\perp$ to $2x+3y+7=0$
3) make an angle $60°$ with X axis.

Q) P.t. $lx+my+n=0$ touches the ⊙ $(x-a)^2+(y-b)^2=r^2$ if $(al+bm+n)^2=r^2(l^2+m^2)$

A) 1) $m=\left(\frac{-3}{4}\right)$, $a=3$ $\Rightarrow$ $y=\left(\frac{-3x}{4}\right)\pm 3\sqrt{1+\frac{9}{16}}$

$\Rightarrow$ Lines: $4y+3x+15=0$
$4y+3x-15=0$

2) $m = \left(\frac{3}{2}\right)$, $a = 3 \Rightarrow y = \left(\frac{3x}{2}\right) \pm 3\sqrt{1+\frac{9}{4}}$

$\Rightarrow$ Lines: $2y = 3x + 3\sqrt{13}$
$2y = 3x - 3\sqrt{13}$

3) $\angle 60°$ with X axis $\Rightarrow m = \sqrt{3}$ or $-\sqrt{3}$

$\Rightarrow y = \sqrt{3}x \pm 3\sqrt{1+3}$ or $y = -\sqrt{3}x \pm 3\sqrt{1+3}$

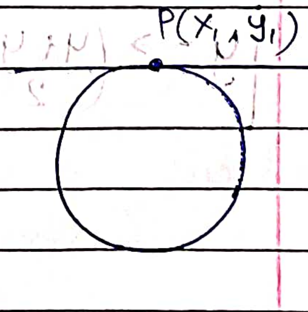
$\Rightarrow y = x\sqrt{3} + 6$; $y = -x\sqrt{3} + 6$
$y = x\sqrt{3} - 6$; $y = -x\sqrt{3} - 6$

A) Line touches ⊙ $\Rightarrow$ ($\perp$ dist. of centre from line equal to radius)

$$\Rightarrow \left|\frac{al+bm+n}{\sqrt{l^2+m^2}}\right| = r \Rightarrow \boxed{(al+bm+n)^2 = r^2(l^2+m^2)}$$

☆ Eqn of tangent at a • $P(x_1, y_1)$ on ⊙ $x^2+y^2+2gx+2fy+c=0$

On diff., $2x + 2yy' + 2g + 2fy' = 0$



$\Rightarrow$ $y' = -\left(\frac{g+x_1}{f+y_1}\right)$ — Slope of Tangent

Since tangent thru $P(x_1, y_1)$,

Tangent : $xx_1 + yy_1 + 2g\left(\frac{x+x_1}{2}\right) + 2f\left(\frac{y+y_1}{2}\right) + c = 0$

Imp. Results —

$T = 0$, $T^2 = SS_1$, $T = 0$, $T = S_1$

(Eq^n of Tangent) (Pair of Tangents) (Chord of Contact) (Chord with given Midpt.)

Meaning of T & S, S_1 :

Consider $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$
and a pt. $P(x_1, y_1)$.

Here, $S \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c$

$S_1 \equiv ax_1^2 + 2hx_1y_1 + by_1^2 + 2gx_1 + 2fy_1 + c$

T is obtained by following transformations in S,

$\boxed{x^2 \to xx_1}$, $\boxed{y^2 \to yy_1}$, $\boxed{x \to \left(\frac{x+x_1}{2}\right)}$,

$\boxed{y \to \left(\frac{y+y_1}{2}\right)}$, $\boxed{xy \to \left(\frac{xy_1 + yx_1}{2}\right)}$

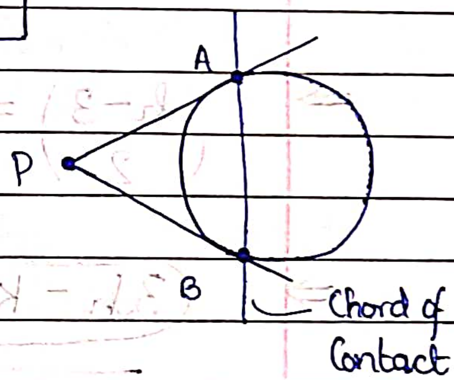
In particular, for a $\odot$ $x^2 + y^2 + 2gx + 2fy + c = 0$

1) (Eqⁿ of Tangent on any $P(x_1, y_1)$ on $\odot$) $\equiv$ $\boxed{T = 0}$

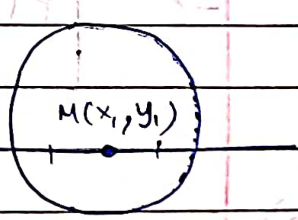
2) (Joint Eqⁿ of Pair of Tangents from Ext. $\cdot$) $\equiv$ $\boxed{T^2 = SS_1}$ [Last Resort]

i.e. $(xx_1 + yy_1 + g(x+x_1) + f(y+y_1) + c)^2 = (x^2+y^2+2gx+2fy+c)(x_1^2+y_1^2+2gx_1+2fy_1+c)$

3) (Chord of Contact) $\equiv$ $\boxed{T = 0}$



4) (Eqⁿ of Chord with a given Mid pt.) $\equiv$ $\boxed{T = S_1}$



Q) Tangents are draw to $\odot$ $x^2 + y^2 - 6x - 4y + 3 = 0$ from a pt. P s.t. the midpt. of chord of contact is (1,1). Find P.

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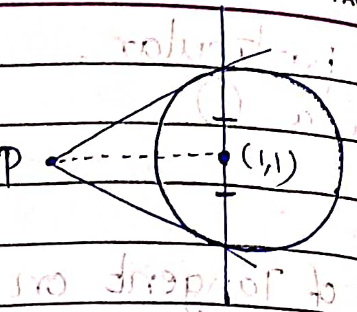
A) $\begin{pmatrix}\text{Chord of}\\ \text{Contact}\\ \text{wrt P}\end{pmatrix} \equiv \begin{pmatrix}\text{Chord with}\\ \text{given Midpt.}\end{pmatrix}$



If we let $P(h,k)$, then

$$\left(hx + ky - 3(x+h) - 2(y+k) + 3 = 0\right)$$

$$\equiv \left(x + y - 3(x+1) - 2(y+1) + 3 = 1^2+1^2-6-4+3\right)$$

$$\Rightarrow \left(x(h-3) + y(k-2) + (3-3h-2k) = 0\right)$$

$$\equiv \left(-2x - y + 3 = 0\right)$$

$$\Rightarrow \left(\frac{h-3}{2}\right) = \left(\frac{k-2}{1}\right) = \left(\frac{3h+2k-3}{3}\right)$$

$$\Rightarrow (3h - k + 3 = 0) \quad \& \quad (h - 2k + 1 = 0)$$

$$\Rightarrow \boxed{P(h,k) \equiv (-1, 0)}$$

Q) Find eq^n of tangent to ⊙ $x^2+y^2=16$ from pt. (1, 4)

Q) The angle b/w a pair of tangents from a pt. P to ⊙ $x^2+y^2+4x-6y + 9\sin^2(\alpha) + 13\cos^2(\alpha)$ is 2α. Find locus of P.

A) Let m be the slope of tangent. Then,

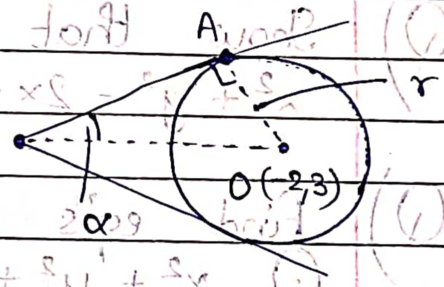
Tangent: $y = mx \pm 4\sqrt{1+m^2}$. Since thru $(1,4)$.

$\Rightarrow \quad 4 = m \pm 4\sqrt{1+m^2} \quad \Rightarrow \quad m^2 - 8m + 16 = 16 + 16m^2$

$\Rightarrow \quad 15m^2 + 8m = 0 \quad \Rightarrow \quad m = 0, -8/15$

Eq^n: $y = 4$, $15y + 8x = 68$

A) Let $P(h,k)$



$r = \sqrt{2^2 + 3^2 - 9s^2_\alpha - 13c^2_\alpha}$

$\Rightarrow \quad r = 2s_\alpha$

Now, $\quad OA = PO\, s_\alpha \quad \Rightarrow \quad 2 = \sqrt{(h+2)^2 + (k-3)^2}$

$\Rightarrow \quad \boxed{\text{Locus: } (x+2)^2 + (y-3)^2 = 4}$

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Length of Tangent

$\boxed{\text{Length of Tangent} = \sqrt{S_1}}$

If PA, PB be tangents drawn from $P(x_1, y_1)$ on given ⊙ $x^2 + y^2 + 2gx + 2fy + c = 0$, then

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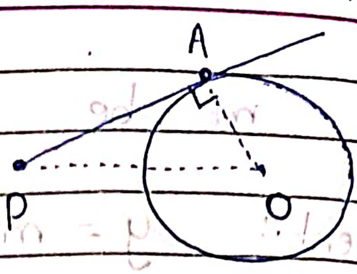
$PA = \sqrt{PO^2 - OA^2}$



$\Rightarrow \boxed{PA = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}}$

Q) Show that length of tangent from any pt. on ⊙ $x^2 + y^2 + 2gx + 2fy + c = 0$ to the ⊙ $x^2 + y^2 + 2gx + 2fy + c_1 = 0$ is $\sqrt{c_1 - c}$.

Q) Show that the / $3x - 4y = 1$ touches ⊙ $x^2 + y^2 - 2x + 4y + 1 = 0$. Find pt. of contact

Q) Find eq^ns of tangents from pt. A(3,2) to ⊙ $x^2 + y^2 + 4x + 6y + 8 = 0$.

A) Let (x_0, y_0) be any pt. of $x^2 + y^2 + 2gx + 2fy + c = 0$

$\Rightarrow x_0^2 + y_0^2 + 2gx_0 + 2fy_0 + c = 0$

Now, length of tangent $= \sqrt{S_1}$

$= \sqrt{x_0^2 + y_0^2 + 2gx_0 + 2fy_0 + c_1} = \sqrt{(x_0^2 + y_0^2 + 2gx_0 + 2fy_0 + c) + (c_1 - c)}$

$\Rightarrow \boxed{\text{Length of Tangent} = \sqrt{c_1 - c}}$

A) Let (x_0, y_0) be pt. of contact. By $T=0$, eq^n of tangent,

$$xx_0 + yy_0 - (x+x_0) + 2(y+y_0) + 1 = 0$$

$$\Rightarrow x(x_0-1) + y(y_0+2) + (2y_0+1-x_0) = 0$$

$$\equiv 3x - 4y - 1 = 0$$

$$\Rightarrow \left(\frac{x_0-1}{3}\right) = \left(\frac{y_0+2}{-4}\right) = \left(\frac{2y_0+1-x_0}{-1}\right)$$

$$\Rightarrow 3y_0 + 4x_0 + 2 = 0, \quad 7y_0 - 4x_0 + 2 = 0$$

$$\Rightarrow \boxed{(x_0, y_0) \equiv (-1/5, -2/5)}$$

A) Let slope of tangent be 'm'

Tangent: $(y+3) = m(x+2) \pm \sqrt{5}\sqrt{1+m^2}$ Thru $(3,2)$

$$\Rightarrow 5 = 5m \pm \sqrt{5}\sqrt{1+m^2} \Rightarrow 5m^2 - 10m + 5 = m^2 + 1$$

$$\Rightarrow 4m^2 - 10m + 4 = 0 \Rightarrow (2m-1)(m-2) = 0$$

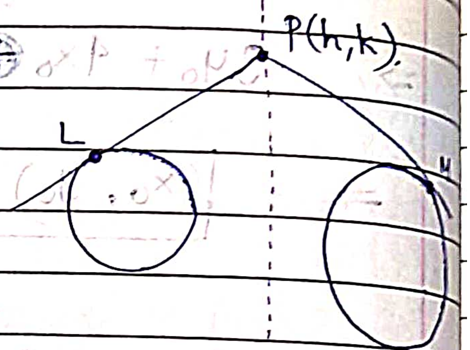
$$\Rightarrow m = 1/2, 2 \Rightarrow \text{Eq}^n\text{s}: \boxed{y - 2x + 4 = 0}; \boxed{2y - x = 1}$$

Radical Axis

The Radical Axis of 2 $\odot$s is a locus of a $\cdot$ which moves s.t. the length of tangents drawn from it to the 2 $\odot$s are $=$.

Consider, $C_1 : x^2 + y^2 + 2gx + 2fy + c = 0$
$C_2 : x^2 + y^2 + 2g_1 x + 2f_1 y + c_1 = 0$

$PL = PM \Rightarrow PL^2 = PM^2$



Radical Axis : $\boxed{S_1 - S_2 = 0}$

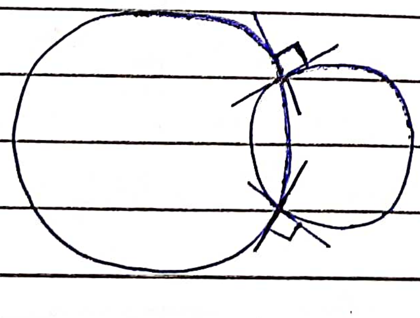
$S_1 : h^2 + k^2 + 2gh + 2fk + c = 0$
$S_2 : h^2 + k^2 + 2g_1 h + 2f_1 k + c_1 = 0$

☆ Radical Axis is ALWAYS a line.

Orthogonal $\cap$ of 2 $\odot$s

$C_1 : x^2 + y^2 + 2gx + 2fy + c = 0$
$C_2 : x^2 + y^2 + 2g_1 x + 2f_1 y + c_1 = 0$

$\cap$ orthogonally if $\boxed{2gg_1 + 2ff_1 = c + c_1}$



Angles b/w tangents at $\cap \cdot$s $= 90°$

Proof: Let (α, β) be of $\cap$

$\Rightarrow \alpha^2+\beta^2+2g\alpha+2f\beta+c=0$; $\Rightarrow \alpha^2+\beta^2+2g_1\alpha+2f_1\beta+c_1=0$

$\Rightarrow$ Tangent 1: $x(\alpha+g)+y(\beta+f)+(g\alpha+f\beta+c)=0$

Tangent 2: $x(\alpha+g_1)+y(\beta+f_1)+(g_1\alpha+f_1\beta+c_1)=0$.

These are $\perp$ $\Rightarrow$ $\left(\frac{\beta+f_1}{\alpha+g_1}\right)\left(\frac{\beta+f}{\alpha+g}\right)=(-1)$

$\Rightarrow \beta^2+(f+f_1)\beta+ff_1=-\alpha^2-(g+g_1)\alpha-gg_1$

$\Rightarrow 2ff_1+2gg_1=(-\alpha^2-\beta^2-2g\alpha-2f\beta-c)+(-\alpha^2-\beta^2-2g_1\alpha-2f_1\beta-c_1)+(c+c_1)$

$\Rightarrow 2ff_1+2gg_1=c+c_1$

Prop^{ts} of Radical Axis

1) Radical Axis is $\perp$ to line joining centres of the 2 $\odot$s.

2) Radical Axis bisects the Common tangents of the 2 $\odot$s.

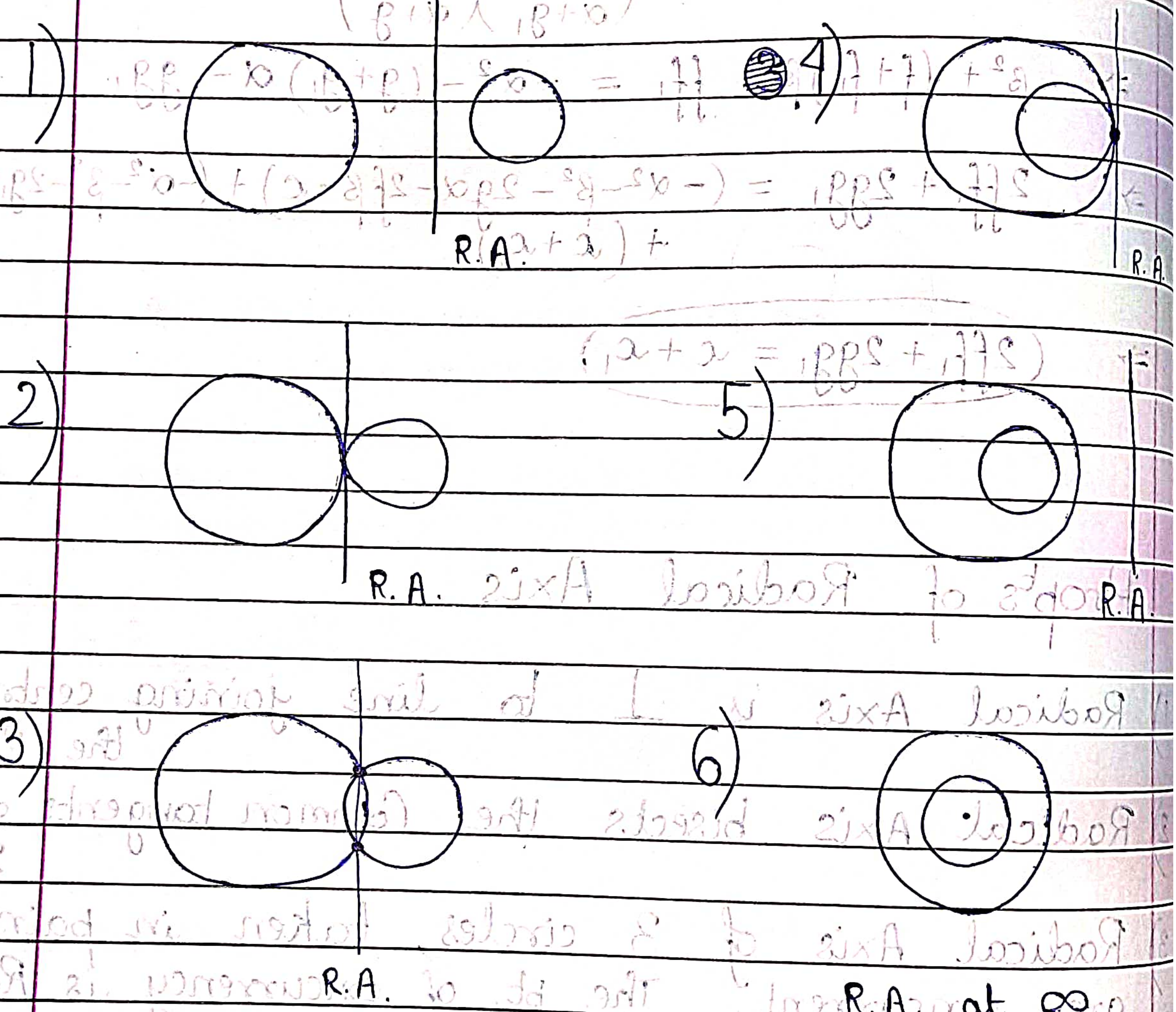
3) Radical Axis of 3 circles, taken in pairs, are concurrent. The pt. of concurrency is Radical Centre

4) If 2 $\odot$s cut a 3rd $\odot$ orthogonally, the radical axis of the 2 circles will pass thru the centre of 3rd $\odot$.

~~Common Tangents~~

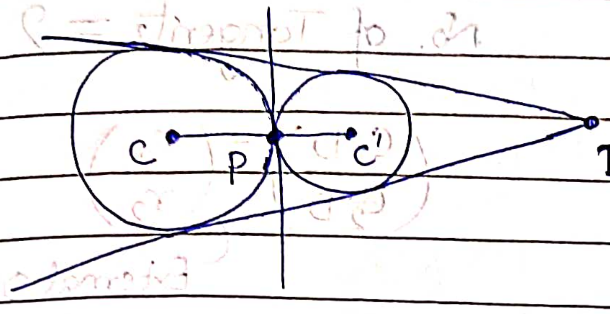
~~1)~~

Location of Radical Axis



Common Tangents

1)



$|CC'| = r_1 + r_2$

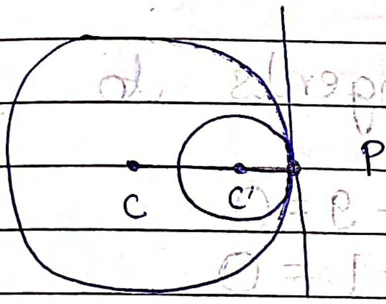
No. of Tangents = 3

$\left(\frac{C_1P}{C_2P}\right) = \left(\frac{r_1}{r_2}\right)$ Internal Division

Let $C_1(x_1, y_1)$ & $C_2(x_2, y_2)$.

$\Rightarrow \quad P \equiv \left(\frac{r_2x_1 + r_1x_2}{r_1 + r_2}, \frac{r_1y_2 + r_2y_1}{r_1 + r_2}\right)$

2)

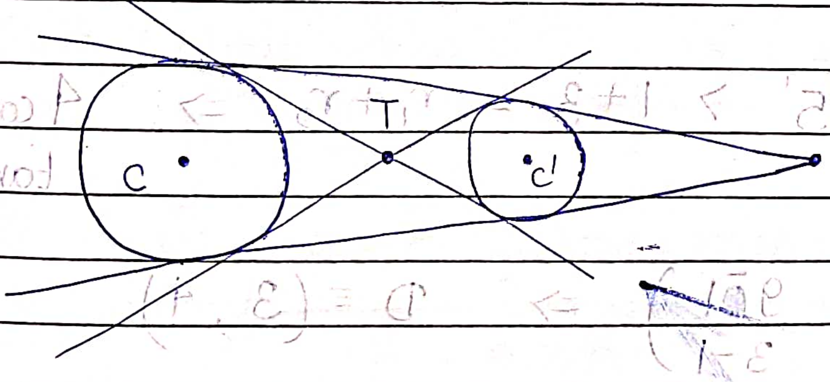


$|CC'| = |r_1 - r_2|$

No. of Tangents = 1

$\left(\frac{C_1P}{C_2P}\right) = \left(\frac{r_1}{r_2}\right)$ External Division

3)



$|CC'| > r_1 + r_2$

No. of Tangents = 4

$D \equiv \left(\frac{r_1x_2 - r_2x_1}{r_1 - r_2}, \frac{r_1y_2 - r_2y_1}{r_1 - r_2}\right)$; $\quad T \equiv \left(\frac{r_1x_2 + r_2x_1}{r_1 + r_2}, \frac{r_1y_2 + r_2y_1}{r_1 + r_2}\right)$

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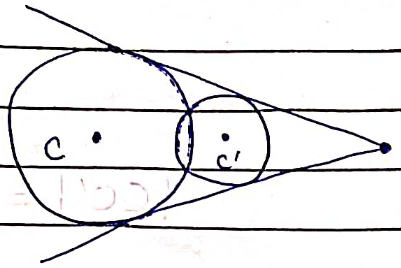
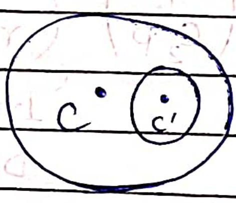
4) 

$|r_1 - r_2| < |CC'| < r_1 + r_2$

No. of Tangents = 2

$\left(\frac{C_1D}{C_2D}\right) = \left(\frac{r_1}{r_2}\right)$ External Division

5) 

$|CC'| < |r_1 - r_2| \Rightarrow$ No Common Tangent

☆ Q) Find all common tangents to circles,

$$x^2 + y^2 - 2x - 6y + 9 = 0$$
$$x^2 + y^2 + 6x - 2y + 1 = 0$$

A) $C_1 \equiv (1, 3)$ $\quad$ $C_2 \equiv (-3, 1)$

$r_1 = 1$ $\quad$ $r_2 = 3$

$|C_1C_2| = 2\sqrt{5} > 1 + 3 = r_1 + r_2 \Rightarrow$ 4 common tangents

$D \equiv \left(\frac{3+3}{3-1}, \frac{9-1}{3-1}\right) \Rightarrow D \equiv (3, 4)$

$T \equiv \left(\frac{3-3}{3+1}, \frac{9+1}{3+1}\right) \Rightarrow T \equiv (0, 5/2)$

Let D.C.T. be $(y-4) = m(x-3)$.

☆ (Dist. of Centre from Line) = (Radius of ⊙)

$$\Rightarrow \quad 1 = \left|\frac{-2m+1}{\sqrt{m^2+1}}\right| \Rightarrow m^2+1 = 4m^2-4m+1$$

$$\Rightarrow m = 0, 4/3$$

$$\Rightarrow \quad \boxed{\begin{array}{ll} DCT_1: & y = 4 \\ DCT_2: & 3y = 4x \end{array}}$$

Let TCT be $(y - 5/2) = mx$.

☆ (Dist. ~~from~~ of Centre from Line) = (Radius of ⊙)

$$\Rightarrow \quad 1 = \left|\frac{m - 1/2}{\sqrt{m^2+1}}\right| \Rightarrow 4m^2+4 = 4m^2-4m+1$$

$$\Rightarrow m = -3/4, \infty$$

$$\Rightarrow \quad \boxed{\begin{array}{ll} TCT_1: & x = 0 \\ TCT_2: & 4y + 3x = 10 \end{array}}$$

☆ (Q) Find angle b/w ⊙s $x^2+y^2-4x+6y+11=0$ and $x^2+y^2-2x+8y+13=0$.

(Q) Find ⊙ which cuts each of the 3 given ⊙s orthogonally $x^2+y^2-2x+3y-7=0$, $x^2+y^2+5x-5y+9=0$, $x^2+y^2+7x-9y+29=0$

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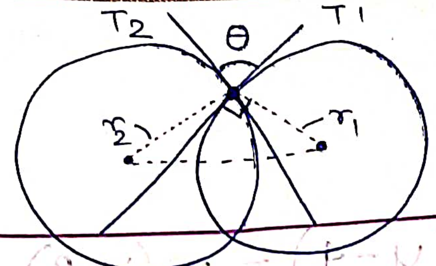

$T_1 \wedge T_2 = \theta$

$r_1 \wedge r_2 = \pi - \theta$

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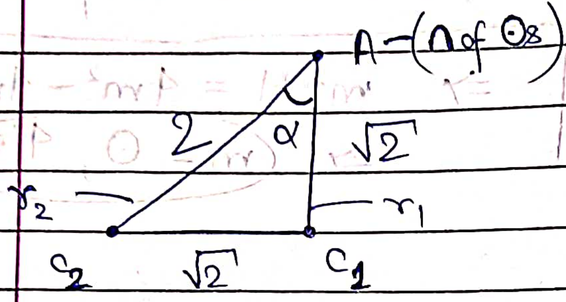
A) $C_1 \equiv (2, -3)$ $\quad$ $C_2 \equiv (1, -4)$ $\Rightarrow$ $|C_1C_2| = \sqrt{2}$

$r_1 = \sqrt{2}$ $\quad$ $r_2 = 2$



$\boxed{\alpha = 45^\circ}$ $\Rightarrow$ $\boxed{\theta = 135^\circ}$

A) Let eq^n of ⊙ be $x^2 + y^2 + 2gx + 2fy + c = 0$

By condition of orthogonality $2gg_1 + 2ff_1 = c + c_1$

$-2g + 3f = c - 7$ — (1)

$5g - 5f = c + 9$ — (2)

$7g - 9f = c + 29$ — (3)

(1) + (3) − (2) $\Rightarrow$ $-f = c + 13$

Substituting in (1) & (2), $\quad g - 2f = 10$

$5g - 4f = (-4)$

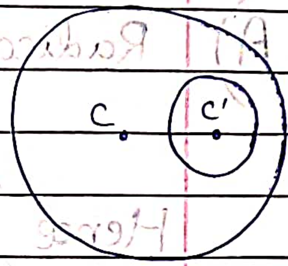
$\Rightarrow$ $\boxed{g = (-8)}$, $\boxed{f = (-9)}$, $\boxed{c = (-4)}$

$\Rightarrow$ $\boxed{⊙ : x^2 + y^2 - 16x - 18y - 4 = 0}$

Q) If ⊙ $x^2 + y^2 + 2ax + b = 0$ lies completely inside $x^2 + y^2 + 2a_2x + b = 0$, then pt.

1) $a_1 a_2 > 0$ 2) $b > 0$

A) Centres : $(-a_1, 0)$; $(-a_2, 0)$
Radii : $\sqrt{a_1^2 - b}$, $\sqrt{a_2^2 - b}$



$|CC'| < |r_1 - r_2| \Rightarrow |a_1 - a_2| < |r_1 - r_2|$

$\Rightarrow |a_1 - a_2| < \left|\sqrt{a_1^2 - b} - \sqrt{a_2^2 - b}\right| = \sqrt{a_2^2 - b} - \sqrt{a_1^2 - b}$

$\Rightarrow a_1^2 + a_2^2 - 2a_1a_2 < a_2^2 - b + a_1^2 - b - 2\sqrt{(a_1^2 - b)(a_2^2 - b)}$

$\Rightarrow b^2 - 2a_1a_2b + \cancel{4}a_1^2a_2^2 > \cancel{4}(a_1^2a_2^2 - ba_1^2 - ba_2^2 + b^2)$

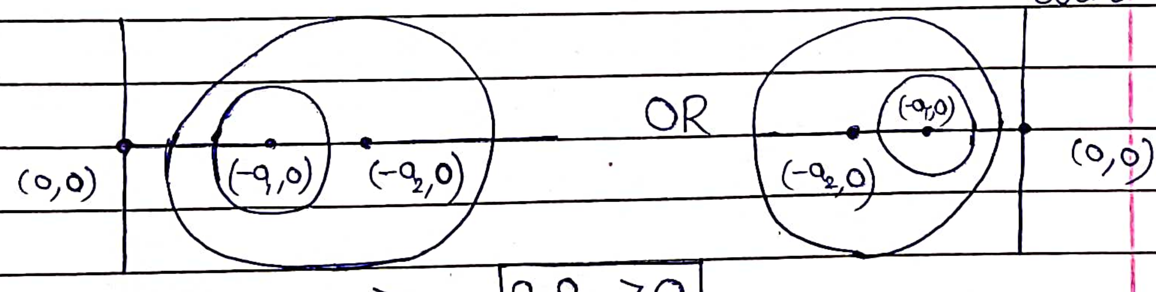
$\Rightarrow 0 > 3b^2 - 4ba_1^2 - 4ba_2^2 + 4a_1a_2b$

$\Rightarrow (b)(4a_1^2 + 4a_2^2 - (4a_1a_2 - 3b)) > 0$

$(b)(a_1 - a_2)^2 > 0 \Rightarrow \boxed{b > 0}$

Now, $S_1(0,0) = S_2(0,0) = b > 0 \Rightarrow (0,0)$ outside both circles.

$\Rightarrow$



$\Rightarrow \boxed{a_1 a_2 > 0}$

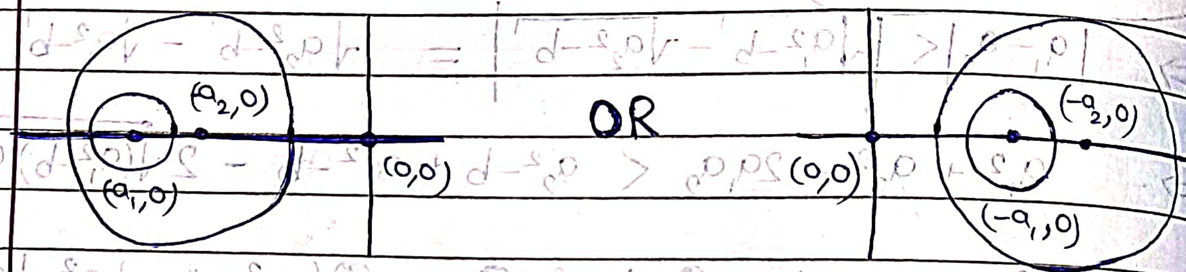
Q) If $\odot$ $x^2+y^2+2a_1x+b=0$ lies completely inside $x^2+y^2+2a_2x+b=0$, the pt. —

1) $a_1a_2>0$ 2) $b>0$

A) Radical Axis : $(x^2+y^2+2a_1x+b)-(x^2+y^2+2a_2x+b)=0$

$\Rightarrow$ $x=0$

Hence, we have the following configuration



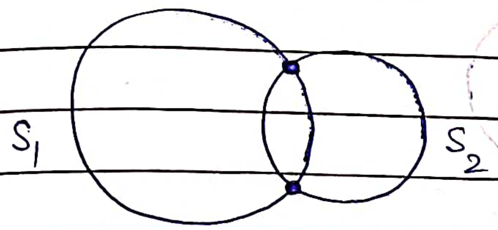
1) In both cases either $a_1, a_2>0$ or $a_1, a_2<0$ $\Rightarrow$ $\boxed{a_1a_2>0}$

2) In both cases $(0,0)$ lies outside both $\odot$s

$\Rightarrow$ $\boxed{b>0}$

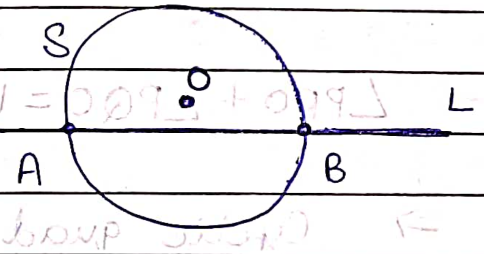
Family of Circles :—

1) Let $S_1 \equiv x^2+y^2+2g_1x+2f_1y+c_1=0$ & $S_2 \equiv x^2+y^2+2g_2x+2f_2y+c_2=0$ are 2 intersecting ⊙s, then family of ⊙s passing thru' their pts of ∩ is



$$\boxed{S_1+\lambda S_2=0}$$

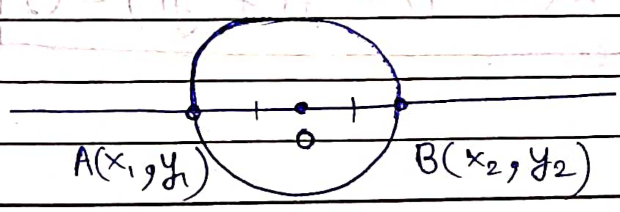
2) Let ⊙ $S=0$ & / $L=0$ intersect at 2 real and distinct pts. A & B, then family of ⊙s passing thru A & B is



$$\boxed{S+\lambda L=0}$$

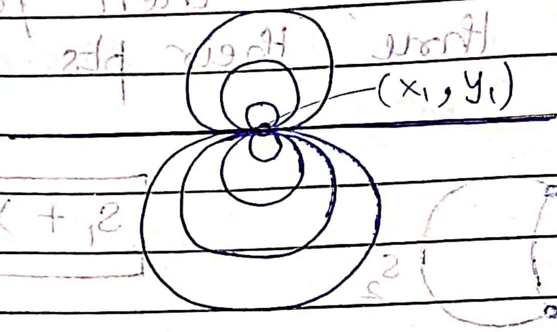
3) The eqn of family of ⊙s passing thru 2 given pts. $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)+\lambda\left[(y-y_1)-\left(\frac{y_2-y_1}{x_2-x_1}\right)(x-x_1)\right]=0$$



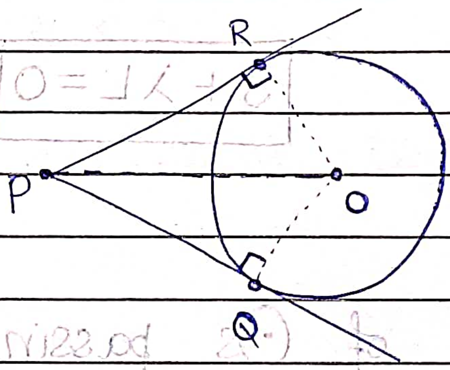
4) The eq^n of family of $\odot$s which touch the line $(y-y_1) = m(x-x_1)$ at (x_1, y_1) for $m \in \mathbb{R}$ is

$$(x-x_1)^2+(y-y_1)^2+\lambda\left[(y-y_1)-m(x-x_1)\right]=0$$



Q) Tangents PQ & PR are drawn to $x^2+y^2=a^2$ from $P(x_1, y_1)$. P.t. eq^n of circum$\odot$ of ΔPQR is $x^2+y^2-xx_1-yy_1=0$

A)



$\angle PRO + \angle PQO = 180°$

$\Rightarrow$ Cyclic quad.

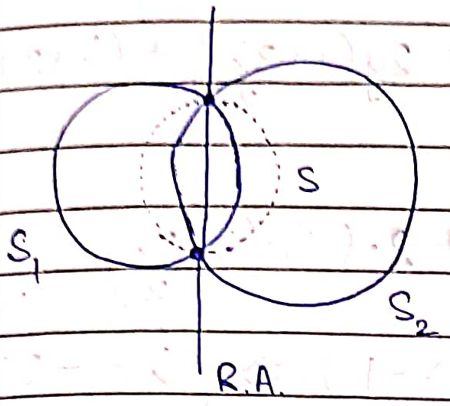
$\Rightarrow$ P, R, Q, O concyclic

Now, $\angle PRO = \angle PQO = 90° \Rightarrow$ PO diameter of circum circle.

$$\Rightarrow \boxed{\text{Circum}\odot : \quad x(x-x_1)+y(y-y_1)=0}$$

Q) Find eqⁿ of ⊙ described on common chord of $x^2+y^2-4x-5=0$ & $x^2+y^2+8y+7=0$ as diameter.

A)



$$S = S_1 + \lambda S_2$$

$$\Rightarrow S \equiv (1+\lambda)x^2 + (1+\lambda)y^2 - 4x + 8\lambda y + (7\lambda - 5) = 0$$

$$\Rightarrow \text{Centre of } S \equiv \left(\frac{2}{1+\lambda}, \frac{-4\lambda}{1+\lambda}\right)$$

$\left(\begin{array}{l}\text{Centre of } S \\ \text{lies on Common Chord}\end{array}\right) \wedge \left(\begin{array}{l}\text{Common Chord} \\ \equiv \text{Radical Axis}\end{array}\right) \Rightarrow \left(\begin{array}{l}\text{Centre of } S \\ \text{on R.A.}\end{array}\right)$

Now, R.A. $\equiv 4x + 8y + 12 = 0$

$$\Rightarrow \left(\frac{2}{1+\lambda}\right) + 2\left(\frac{-4\lambda}{1+\lambda}\right) + 3 = 0$$

$$\Rightarrow 2 - 8\lambda + 3 + 3\lambda = 0 \Rightarrow \boxed{\lambda = 1}$$

$$\Rightarrow \boxed{\odot : \ x^2 + y^2 - 2x + 4y + 1 = 0}$$

Q) Find eqⁿ of ⊙ touching $2x+3y+1=0$ at $(1,-1)$; and cutting orthogonally circle having line segment joining $(0,3)$ and $(-2,-1)$ as diameter

A) $C_2 : x(x+2) + (y-3)(y+1) = 0$

$C_1 : (x-1)^2 + (y+1)^2 + \lambda(2x+3y+1) = 0$

$$\Rightarrow x^2 + y^2 + 2x(\lambda-1) + 2y\left(\frac{1+3\lambda}{2}\right) + (\lambda+2) = 0$$

For orthogonal, $2\cdot 1\cdot(\lambda-1) + 2\cdot(-1)\cdot\left(\frac{1+3\lambda}{2}\right) = (\lambda+2) + (-3)$

$$\Rightarrow (2\lambda-2) - (2+3\lambda) = (\lambda-1) \Rightarrow \lambda = \left(-\frac{3}{2}\right)$$

$$\Rightarrow \boxed{C : x^2 + y^2 - 5x - \frac{5y}{2} + \frac{1}{2} = 0}$$

Q) Consider a family of $\odot$s passing thru 2 fix. pts. $A(3,7)$ and $B(6,5)$. Show that chords in which $\odot$ $x^2+y^2-4x-6y-3=0$ cuts members of family are concurrent at a pt. Find this pt.

A) $C_1 : x^2 + y^2 - 4x - 6y - 3 = 0$

$C_2 : (x-3)(x-6) + (y-7)(y-5) + \lambda(3y+2x-27) = 0$

$$\Rightarrow x^2 + y^2 + (2\lambda-9)x + (3\lambda-12)y + (53-27\lambda) = 0$$

We use the fact that,

Common Chord $\equiv$ Radical Axis

if the $\odot$s intersect.

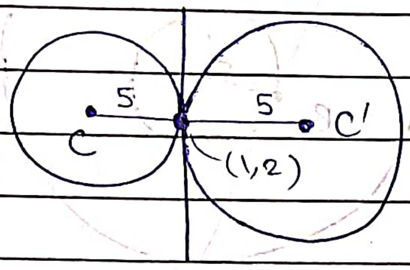
$\Rightarrow$ Chord : $(2\lambda-5)x+(3\lambda-6)y+(56-27\lambda)=0$

$\Rightarrow \lambda(2x+3y-27)-(5x+6y-56)=0$

which is a Family of Lines thru the pt. $\boxed{(2, 23/3)}$

Q) 2 circles each of radius 5 touch each other at (1,2). If eqn of their common tangent is $4x+3y=10$. Find eqn of ⊙s.

A)



① Let $C \equiv (\alpha, \beta)$. Condt 1 : Dist. b/w $(1,2)$ & $(\alpha,\beta)=5$.
Condt 2: Line join $(1,2)$ & $(\alpha,\beta) \perp$ $4x+3y=10$

② Use Family of ⊙s. Find λ by using Radius $=5$.

③ Using (1,2) & parametric form, of line $\perp$ to $4x+3y=10$, find C & C' taking $r=\pm 5$.

$CC' \equiv 4y - 3x = 5$

$\Rightarrow \left(\frac{x-1}{-3/5}\right) = \left(\frac{y-2}{+4/5}\right) = \pm 5 \Rightarrow C \equiv (-2,6)$, $C' \equiv (4,-2)$

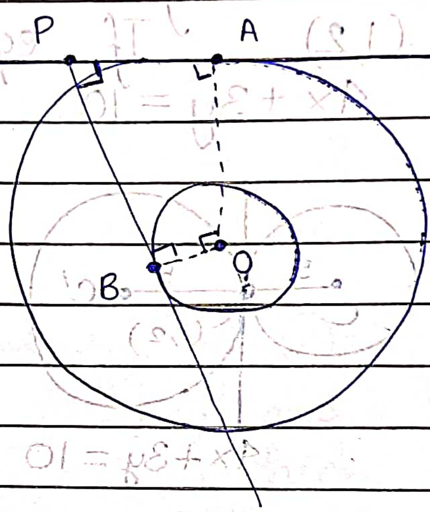
$\Rightarrow \odot : (x+2)^2 + (y-6)^2 = 25$

$\odot : (x-4)^2 + (y+2)^2 = 25$

★ Q) Find locus of • s.t. tangents drawn from it to $x^2+y^2=9$ & $x^2+y^2=16$ are $\perp$.

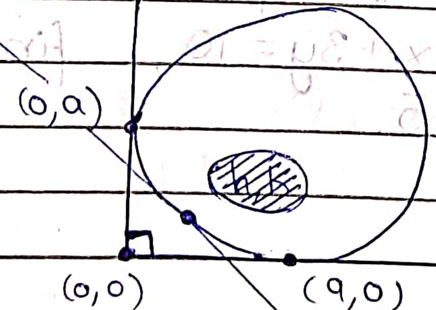
A) PAOB is rectangle.

$\Rightarrow \boxed{PO = 5}$



Q) A variable Δ has 2 of its sides along coordinate axes. Its 3rd side touches $x^2+y^2-2ax-2ay+a^2=0$. P.t. locus of circumcentre of Δ is $a^2 - 2a(x+y) + 2xy = 0$.

A)



$\odot \equiv (x-a)^2 + (y-a)^2 = a^2$

Let (h, k) be midpt. of tangent.
$90°$ at origin $\Rightarrow$ Circumcentre at (h, k).

$\Rightarrow$ Tangent $\equiv (x/2h) + (y/2k) = 1$

$\Rightarrow a = \left| \dfrac{a/2h + a/2k - 1}{\sqrt{1/4h^2 + 1/4k^2}} \right| = \left| \dfrac{ak + ah - 2hk}{\sqrt{h^2 + k^2}} \right|$

$\Rightarrow$ Locus $\equiv a^2(h^2+k^2) = (ak + ah - 2hk)^2$

Q) Let $\odot$ be $2x(x-a) + y(2y-b) = 0$
$a, b \neq 0$. Find condition on a, b; if 2 chords, each bisected by X axis, can be drawn from the pt. $(a, b/2)$

A) Let $(\alpha, 0)$ be the midpt. of chord.

Chord with midpt. $(\alpha, 0)$: $T = S_1 \Rightarrow 2\alpha x - a(x+\alpha) - \left(\frac{b}{2}\right)(y+0)$
$= 2\alpha(\alpha - a)$

$\Rightarrow$ $(2\alpha - a)x - \left(\frac{b}{2}\right)y = 2\alpha^2 - a\alpha$

This passes thru $(a, b/2)$.

$\Rightarrow (2\alpha - a)a - \left(\frac{b}{2}\right)^2 = 2\alpha^2 - a\alpha$

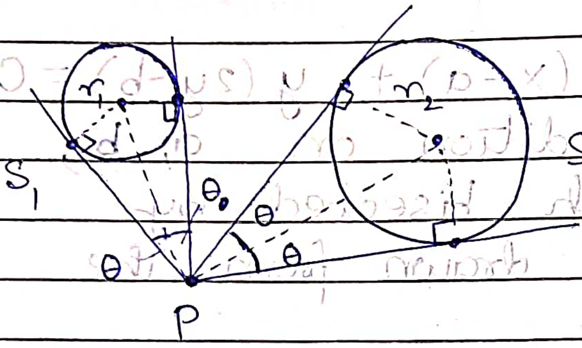
$\Rightarrow 2\alpha^2 - 3a\alpha + \left(\frac{b^2}{4}\right) + a^2 = 0$

This should give 2 distinct values for α.

$D > 0 \Rightarrow 9a^2 - 8\left(\frac{b^2}{4} + a^2\right) > 0 \Rightarrow \boxed{a^2 - 2b^2 > 0}$

Q) If $S_1 \equiv x^2+y^2+2g_1x+2f_1y+c_1=0$ and
$S_2 \equiv x^2+y^2+2g_2x+2f_2y+c_2=0$ are
2⊙s with radius r_1 & r_2 resp.
Show that the pts at which the
⊙s subtend equal angle lie of the
⊙ is $\left(\frac{S_1}{r_1^2}\right)=\left(\frac{S_2}{r_2^2}\right)$

A)



$\tan(\theta) = \frac{r_1}{\sqrt{S_1}} = \frac{r_2}{\sqrt{S_2}}$

$\Rightarrow \left(\frac{S_1}{r_1^2}\right)=\left(\frac{S_2}{r_2^2}\right)$

Q) P.t. tangents to $x^2+y^2=5$ at $(1,-2)$
also touches $x^2+y^2-8x+6y+20=0$
& find pts of contact.

A) $C_1 \equiv (0,0)$ & $C_2 \equiv (4,-3)$
$r_1 = \sqrt{5}$ & $r_2 = \sqrt{5}$

Tangent: $x - 2y = 5$

(Dist b/w C_2 & Tangent) $= \left|\frac{4+6-5}{\sqrt{5}}\right| = \sqrt{5} =$ (Radius $= r_2$)

$\Rightarrow$ Touches 2nd ⊙.

Let pt. of contact (h, k).

Tangent : $hx + ky - 4(x+h) + 3(y+k) + 20 = 0$

$\Rightarrow x(h-4) + y(k+3) + (20+3k-4h) = 0$

$\equiv x - 2y - 5 = 0$

$\Rightarrow \left(\frac{h-4}{1}\right) = -\left(\frac{k+3}{2}\right) = -\left(\frac{20+3k-4h}{5}\right)$

$\Rightarrow 2h + k = 5$ & $9h - 3k = 24$

$\Rightarrow$ Pt. of Contact $\equiv \left(13/5, -1/5\right)$

(See Pg 169)

Q) Find locus of midpt. of chords of $x^2 + y^2 + 2gx + 2fy + c = 0$, which subtend right angle at origin.

A) Let midpt. be (h, k)

Chord with midpt. (h,k) : $T = S_1 \Rightarrow hx + ky + g(x+h) + f(y+k) + c = h^2 + k^2 + 2gh + 2fk + c$

$\Rightarrow (h+g)x + (k+f)y = (h^2+k^2+gh+fk)$

Homogenize this with $\odot$.

$$1 = \frac{(h+g)x + (k+f)y}{(h^2+k^2+gh+fk)}$$

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(Worst Sol^n)

$$\Rightarrow x^2+y^2+2gx\left[\frac{(h+g)x+(k+f)y}{h^2+k^2+gh+fk}\right]+2fy\left[\frac{(h+g)x+(k+f)y}{h^2+k^2+gh+fk}\right]$$
$$+c\left[\frac{(h+g)x+(k+f)y}{h^2+k^2+gh+fk}\right]^2=0$$

$$\Rightarrow Ax^2+2Hxy+By^2+2Gx+2Fy+C=0.$$

where $A = 1+\dfrac{2g(h+g)}{h^2+k^2+gh+fk}+\dfrac{c(h+g)^2}{(h^2+k^2+gh+fk)^2}$

and $B = 1+\dfrac{2f(k+f)}{h^2+k^2+gh+fk}+\dfrac{c(f+k)^2}{(h^2+k^2+gh+fk)^2}$

for 90° at origin we need $A+B=0$.

$$\Rightarrow 2+\left(\frac{2gh+2fk+2g^2+2f^2}{h^2+k^2+gh+fk}\right)+(c)\left(\frac{(h+g)^2+(f+k)^2}{(h^2+k^2+gh+fk)^2}\right)=0$$

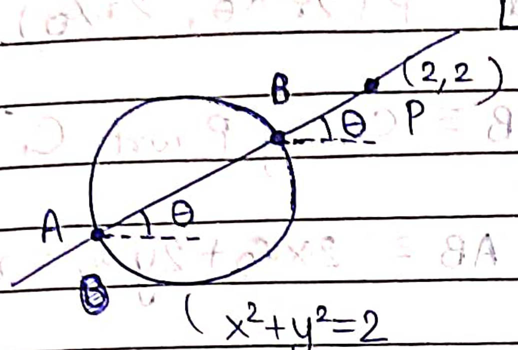
$$\Rightarrow 2(h^2+k^2+gh+fk)^2+2(h^2+k^2+gh+fk)(gh+fk+g^2+f^2)$$
$$+c\left((h+g)^2+(f+k)^2\right)=0$$

$$\Rightarrow 2(h^2+k^2+gh+fk)\left[h^2+2gh+g^2+k^2+2fk+f^2\right]$$
$$+c\left((h+g)^2+(f+k)^2\right)=0$$

$$\Rightarrow \left[(h+g)^2+(f+k)^2\right]\left[2h^2+2k^2+2gh+2fk+c\right]=0$$

Locus: $\left[(x+g)^2+(y+f)^2\right]\left[2x^2+2y^2+2gx+2fy+c\right]=0$

Q) Find eqn of chord AB of $x^2+y^2=2$ thru P(2,2) s.t. PA/PB = 3.

A) 

Let chord L be,

$(y-2) = t_\theta(x-2)$.

Let $B(2-rc_\theta,\ 2-rs_\theta)$

Both A and B lie on ⊙.

and $A(2-3rc_\theta,\ 2-3rs_\theta)$ where $r>0$.

$\Rightarrow (2-rc_\theta)^2+(2-rs_\theta)^2 = 2 \Rightarrow 6-4r(c_\theta+s_\theta)+r^2=0$

$\Rightarrow (2-3rc_\theta)^2+(2-3rs_\theta)^2 = 2 \Rightarrow 6-12r(c_\theta+s_\theta)+9r^2=0$

$\Rightarrow 4r(c_\theta+s_\theta) = 6+r^2 = 3r^2+2 \Rightarrow r = \sqrt{2}$

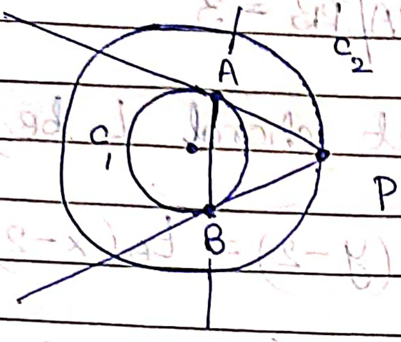
Substituting in eqn, $(4\sqrt{2})(c_\theta+s_\theta) = 8$

$\Rightarrow c_\theta+s_\theta = \sqrt{2} \Rightarrow \theta = \pi/4$

$\Rightarrow$ $\boxed{AB:\ x=y}$

Q) C_1 & C_2 are concentric ⊙s, radius of C_2 being twice of C_1. From a pt. P on C_2, tangents PA & PB are drawn to C_1. P.t. centroid of ΔPAB lies on C_1.

A) Let WLOG $C_1 : x^2+y^2=r^2$ & $C_2 : x^2+y^2=4r^2$



Let $P(2r c_\theta, 2r s_\theta)$

$AB \equiv$ C.C. of P wrt C_1

$\Rightarrow AB \equiv 2x c_\theta + 2y s_\theta = r$

$AB \cap C_1 \Rightarrow x^2 + \left(\dfrac{r-2x c_\theta}{2 s_\theta}\right)^2 = r^2$

$\Rightarrow 4x^2 - 4r c_\theta x + r^2(1-4s_\theta^2) = 0 \begin{array}{l} x_A \\ x_B \end{array}$

$\Rightarrow \boxed{x_A + x_B = r c_\theta}$

$AB \cap C_1 \Rightarrow y^2 + \left(\dfrac{r-2y s_\theta}{2 c_\theta}\right)^2 = r^2$

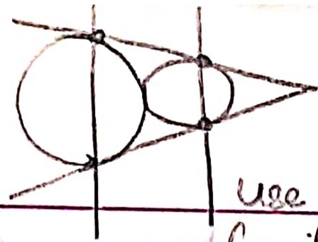
$\Rightarrow 4y^2 - 4r s_\theta y + r^2(1-4c_\theta^2) = 0 \begin{array}{l} y_A \\ y_B \end{array}$

$\Rightarrow \boxed{y_A + y_B = r s_\theta}$

G of $\Delta PAB \equiv \left(\dfrac{r c_\theta + 2r c_\theta}{3}, \dfrac{r s_\theta + 2r s_\theta}{3}\right)$

$\Rightarrow \boxed{G \equiv (r c_\theta, r s_\theta)} \Rightarrow$ $\boxed{\text{Lies on } C_1}$

(Worst Sol^n)

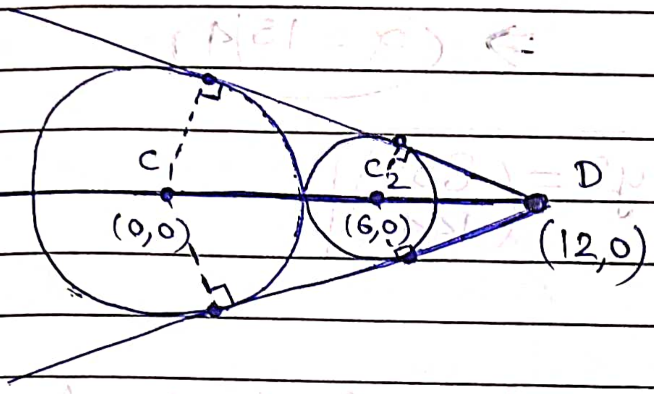


Use family ⊙

(See 169)

Q) Find eq^n of ⊙ passing thru pts. of contact of D.C.T. of $x^2+y^2=16$ and $x^2+y^2-12x+32=0$.

A) $C_1 \equiv x^2+y^2=16$ $\quad C_2 \equiv (x-6)^2+y^2=4$



$$\left(\frac{C_1D}{C_2D}\right)=\left(\frac{r_1}{r_2}\right)=2$$

$\Rightarrow D(12,0)$

Let ~~slope~~ eq^n of common tangent be

$$y=m(x-12)$$

(Dist. of Centre from Tangent) = (Radius)

$$\Rightarrow \left|\frac{6m}{\sqrt{1+m^2}}\right|=2 \Rightarrow m=\pm 1/2\sqrt{2}$$

(Pt. of Contact on C_2) : $\left(\frac{x-6}{\pm 1/2\sqrt{2}}\right)=\left(\frac{y}{-1}\right)=-\left(\frac{\mp 3/\sqrt{2}}{9/8}\right)$

$$\Rightarrow \left(\frac{20}{3},\frac{4\sqrt{2}}{3}\right);\left(\frac{20}{3},\frac{-4\sqrt{2}}{3}\right)$$

(Pt. of Contact on C_1) : $\left(\frac{x}{\pm 1/2\sqrt{2}}\right)=\left(\frac{y}{-1}\right)=-\left(\frac{\mp 6/\sqrt{2}}{9/8}\right)$

$$\Rightarrow (4/3, 8\sqrt{2}/3);\ (4/3, -8\sqrt{2}/3)$$

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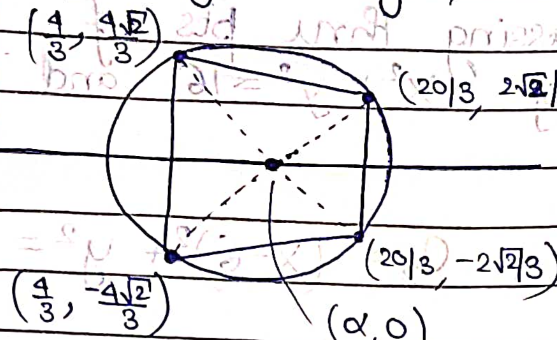
By symmetry, centre of req. ⊙ on X axis



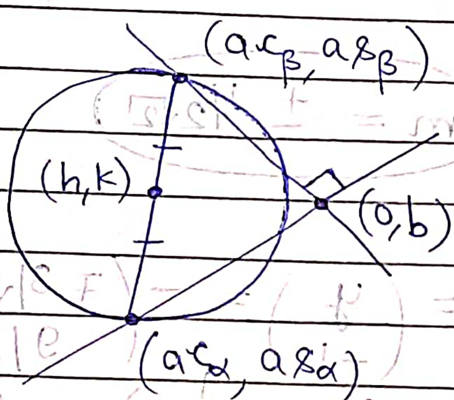
$$\left(\alpha-\frac{4}{3}\right)^2+\left(\frac{4\sqrt{2}}{3}\right)^2 = \left(\alpha-\frac{20}{3}\right)^2+\left(\frac{2\sqrt{2}}{3}\right)^2$$

$\Rightarrow \alpha = 15/4$

$\Rightarrow$ Eq^n: $\boxed{\left(x-\frac{15}{4}\right)^2+y^2=\left(\frac{853}{144}\right)}$

☆ Q) Find locus of midpt. of chord of $x^2+y^2=a^2$ that subtends $90°$ at $(0,b)$.

A)



$$\left(\frac{as_\alpha-b}{ac_\alpha}\right)\left(\frac{as_\beta-b}{ac_\beta}\right) = -1$$

$$\Rightarrow a^2c_\alpha c_\beta + a^2 s_\alpha s_\beta - ab(s_\alpha+s_\beta)+b^2=0$$

Now, $2h = a(c_\alpha+c_\beta)$, $2k = a(s_\alpha+s_\beta)$

$$\Rightarrow 4(h^2+k^2) = a^2(2+2c_\alpha c_\beta+2s_\alpha s_\beta)$$

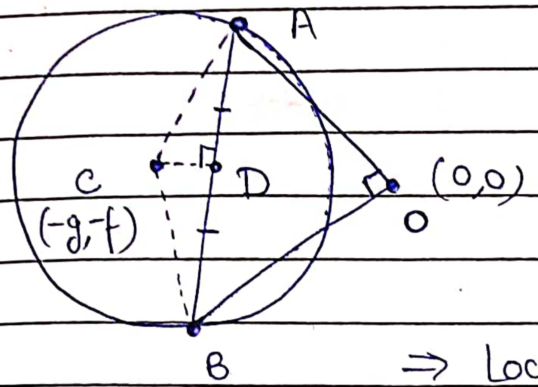
$$\Rightarrow (c_\alpha c_\beta + s_\alpha s_\beta) = \left(\frac{2}{a^2}\right)(h^2+k^2)-1$$

$\Rightarrow$ Locus: $2(x^2+y^2) - 2by + (b^2-a^2) = 0$

(See Pg 163)

☆Q) Find locus of midpt. of chord of $x^2+y^2+2gx+2fy+c=0$ which subtends right angle at origin.

A)



Observe, $OD = DA$

$\Rightarrow CD^2 + DO^2 = CA^2$

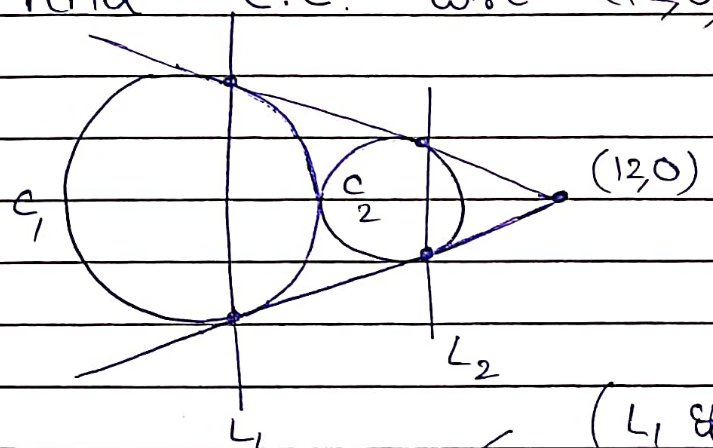
$\Rightarrow$ Locus of D: $(x+g)^2+(y+f)^2+x^2+y^2=(g^2+f^2-c)$

$\Rightarrow \boxed{2x^2+2y^2+2gx+2f+c=0}$

(See 167)

☆Q) (mn).

A) Find C.C. wrt (12,0) for both circles.



$L_1 \equiv x = 4/3$

$L_2 \equiv x = 20/3$

$(L_1 \ \& \ C_1) \Rightarrow x^2+y^2-16+\lambda(3x-4)=0$

Represent same ⊙

$(L_2 \ \& \ C_2) \Rightarrow x^2+y^2-12x+32+\mu(3x-20)=0$

$\Rightarrow \left(\frac{3\lambda}{3\mu-12}\right)=\left(\frac{-4\lambda-16}{32-20\mu}\right)=\left(\frac{\lambda+4}{5\mu-8}\right)=1 \Rightarrow \boxed{\begin{matrix}\lambda=(-2)\\ \mu=2\end{matrix}}$