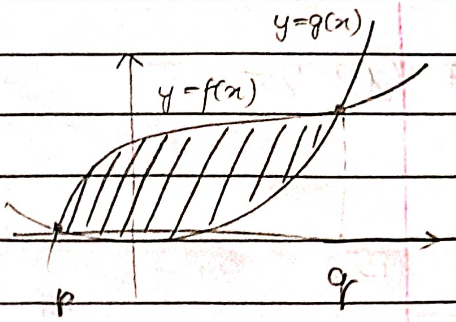
AREA

02/08/2023

- Area b/w 2 curves :-



Solve $f(x) = g(x)$

Let $x = l, m$ be the soln's.

$$\Rightarrow \boxed{\text{Area} = \int_p^q |f(x) - g(x)|\, dx}$$

NOTE: If $f'(x) = f(x) + \lambda$ & $g'(x) = g(x) + \lambda$,

$$\boxed{\begin{pmatrix}\text{area b/w} \\ f(x) \ \& \ g(x)\end{pmatrix} = \begin{pmatrix}\text{area b/w} \\ f'(x) \ \& \ g'(x)\end{pmatrix}}$$

- Area under known curves

Q Find area bounded by

(i) $\frac{x^2}{4} + y^2 = 1$

(ii) $y = 1+x^2$, x-axis & $x = \pm 1$

(iii) $y = \frac{3x^2}{4}$ & $3x-2y+12=0$

(iv) $y = 2-x^2$ & $x+y=0$

(v) $y = |x| - 1$ & $y = -|x|+1$

(vi) $y = (x-1)(x-2)(x-3)$, x-axis & lying b/w $x=0$ & 3.

(vii) $x = -2y^2$ & $x = 1-3y^2$

☆(viii) $x^2+y^2=4$, $x^2 = -\sqrt{2}y$ & $x=y$

A (i) 2π (ii) $\Delta = \int_{-1}^{1} 1+x^2\,dx = \left[x+\frac{x^3}{3}\right]_{-1}^{1} = \left(\frac{8}{3}\right)$

(iii) $3x - 2\left(\frac{3x^2}{4}\right) + 12 = 0 \Rightarrow 6x - 3x^2 + 24 = 0$

$\Rightarrow x^2 - 2x - 8 = 0 \Rightarrow \underline{x = 4, -2}$

$\Rightarrow \left|\int_{-2}^{4} \frac{3x^2}{4} - \frac{3x}{2} - 6\,dx\right| = \left|\left[\frac{x^3}{4} - \frac{3x^2}{4} - 6x\right]_{-2}^{4}\right|$

$= |(16-12-24) + (2+3-12)| = |-20-7| = \underline{27}$

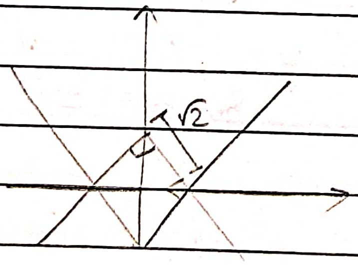
(iv) $2 - x^2 = -x \Rightarrow x^2 - x - 2 = 0 \Rightarrow (x-2)(x+1) = 0$

$\Rightarrow \underline{x = -1, 2}$

$\Delta = \left|\int_{-1}^{2} x^2 - x - 2\,dx\right| = \left|\left[\frac{x^3}{3} - \frac{x^2}{2} - 2x\right]_{-1}^{2}\right|$

$= \left|\left(\frac{8}{3} - 2 - 4\right) + \left(\frac{1}{3} + \frac{1}{2} - 2\right)\right| = \left|\frac{-10}{3} - \frac{27}{6}\right| = \underline{\left(\frac{9}{2}\right)}$

(v)



$\Delta = 2$

(vi) $\Delta = \int_{0}^{1} -(x^3 - 6x^2 + 11x - 6)\,dx$

$+ \int_{1}^{2} (x^3 - 6x^2 + 11x - 6)\,dx$

$+ \int_{2}^{3} -(x^3 - 6x^2 + 11x - 6)\,dx$

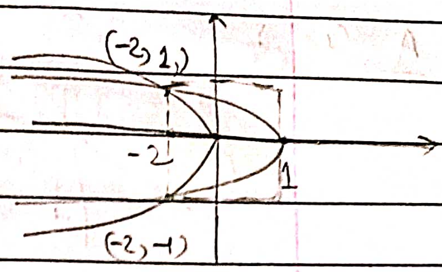
$= \left[\frac{x^4}{4} - 2x^3 + \frac{11x^2}{2} - 6x\right]_{1,1,3}^{0,2,2}$

$= 2\left(4 - 16 + 22 - 12\right) - 2\left(\frac{1}{4} - 2 + \frac{11}{2} - 6\right)$

$- \left(\frac{81}{4} - 54 + \frac{99}{2} - 18\right)$

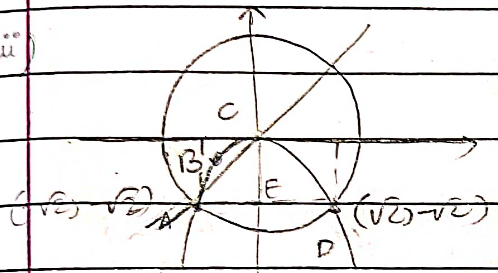
$= \left(\frac{11}{4}\right)$

(vii) $-2y^2 = 1 - 3y^2 \Rightarrow \quad y = \pm 1$



$$\Delta = \frac{2}{3}\left[(2\times3) - (2\times2)\right] = \left(\frac{4}{3}\right)$$

(viii)



$$\Delta = [ACE] + [CED] + [AED]$$

$$= \frac{1}{2}(\sqrt{2})(\sqrt{2}) + \left(\frac{2}{3}\right)(\sqrt{2})(\sqrt{2})$$

$$+ \left(\frac{\pi(2)^2}{4} - \frac{1}{2}(2\sqrt{2})(\sqrt{2})\right)$$

$$= \left(\pi + \frac{1}{3}\right)$$

$x^2 + y^2 = 4$

& $x^2 = -\sqrt{2}y$

$\Rightarrow y^2 - \sqrt{2}y = 4$

$\Rightarrow y^2 - \sqrt{2}y - 4 = 0$

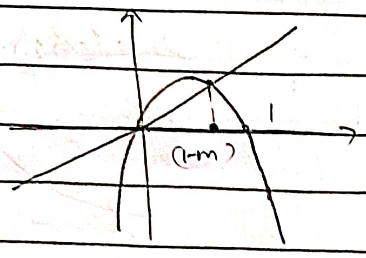
$\Rightarrow y = \frac{\sqrt{2} \pm \sqrt{18}}{2} = -\sqrt{2}, \underbrace{2\sqrt{2}}_{\times}$

Q (ix) For what value of 'm' the area of the region bounded by $y = x - x2$ & the line $y = mx$ equals $9/2$.

(x) Find the area of the region bounded by the graphs $y = x^2 + 2$, $y = x$, $x = 0$ & $x = 1$.

Sketch the region

A. (ix)



$mx = x - x^2 \Rightarrow x = 1-m$

& $x = 0$

$A = 9/2$

$$\Delta = \int_0^{(1-m)} |x - x^2 - mx|\, dx = \left|\left[\frac{x^2}{2} - \frac{x^3}{3} - \frac{mx^2}{2}\right]_0^{(1-m)}\right|$$

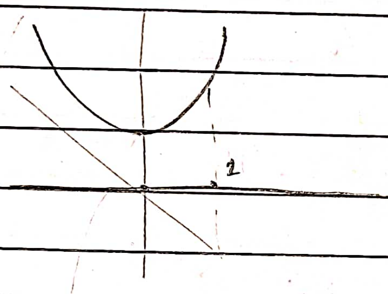
$$= \left|\frac{(1-m)^2 - m(1-m)^2}{2} - \frac{(1-m)^3}{3}\right|$$

$\Rightarrow \left|\frac{(1-m)^3}{6}\right| = \frac{9}{2}$

$\Rightarrow 1 - m = \pm 3$

$\Rightarrow m = -2, 4$

(x)



$$\Delta = \int_0^1 (x^2+2) - (-x)\, dx = \left[\frac{x^3}{3} + \frac{x^2}{2} + 2x\right]_0^1$$

$$= \left(\frac{17}{6}\right)$$

Q. Find the area bounded by

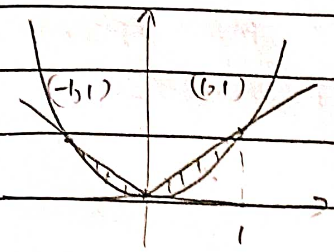
(xi) $y \geq x^2$, $y \leq |x|$

(xii) $f(x) = \max\{sx, cx\}$, $x = 0$, $x = 2\pi$ & x-axis

(xiii) $y = tx$, $y = t^2x$ & $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

(xiv) $y = 2x^4 - x^2$, x-axis & ordinate of 2 minima of the curve.

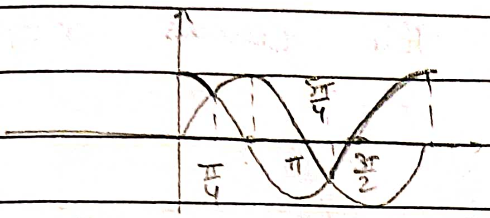
A (xi)



$$\Delta = 2\left[\left(\frac{2}{3}\right)(1)(1) - \left(\frac{1}{2}\right)(1)(1)\right]$$

$$= \left(\frac{1}{3}\right)$$

(xii)



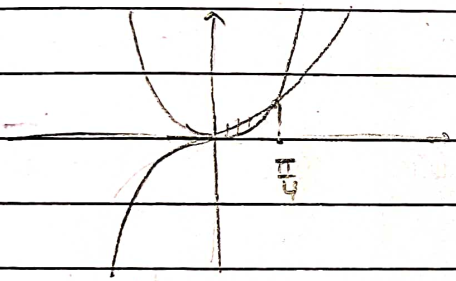
$$\Delta = \int_0^{\pi/4} c_x\,dx + \int_{\pi/4}^{\pi} s_x\,dx + \int_{\frac{3\pi}{2}}^{2\pi} c_x\,dx$$

$$= \int_{\pi}^{5\pi/4} s_x\,dx - \int_{\frac{5\pi}{4}}^{3\pi/2} c_x\,dx$$

$$= \frac{1}{\sqrt{2}} + 1 + 1 + \frac{1}{\sqrt{2}} + 1 + \frac{1}{\sqrt{2}} + 1 + \frac{1}{\sqrt{2}}$$

$$= \underline{4+\sqrt{2}}$$

(xiii)

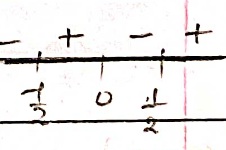


$$\int_0^{\pi/4} t - t^2\,dx$$

$$\Rightarrow \left[\ell|sec| - t + x\right]_0^{\pi/4}$$

$$= \frac{1}{2}\ell(2) - 1 + \frac{\pi}{4}$$

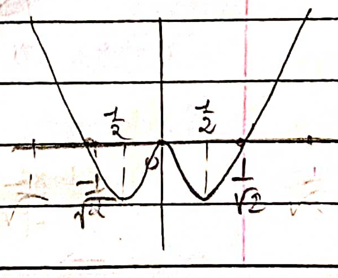
(xiv) $y' = 8x^3 - 2x = 0 \Rightarrow x = \pm\frac{1}{2}, 0$



$y = 0 \Rightarrow x^2(2x^2-1) = 0 \Rightarrow x = 0, \pm\frac{1}{\sqrt{2}}$

$$\Delta = 2\left[\int_0^{1/2} x^2 - 2x^4\,dx\right]$$

$$= 2\left[\frac{x^3}{3} - \frac{2x^5}{5}\right]^{1/2} = 2\left[\frac{1}{24} - \frac{1}{80}\right]$$



$= \underline{7/120}$ ^{2} units

Q. (xv) Sketch the region bounded by the curves $|x+y| \leq 1$, $|y-x| \leq 1$ and $3x^2+3y^2=1$. Find its area.

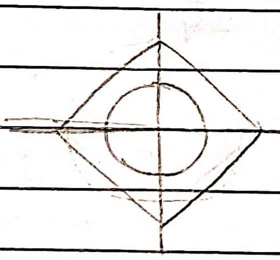
(xvi) Find the area of the region in the I Quadrant bounded by the curves $x^2+y^2=25$, $4y=|4-x^2|$ & y-axis

(xvii) Define a region $S = \{P(x,y) : [x]+[y]=5\}$, s.t $P(x,y)$ lies in I Quad. Find the area of region S.

(xviii) Draw the graph of the fn $y = ex\,l(x)$ & $y = \frac{l(x)}{ex}$.

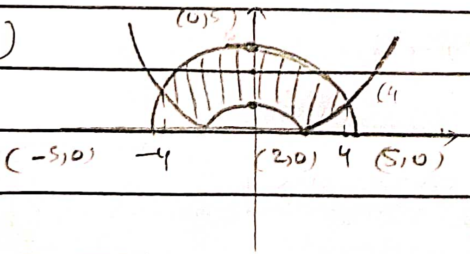
Find the area bounded b/w the 2 curves.

A. (xv)



$$\Delta = (\sqrt{2})^2 - \pi\left(\frac{1}{\sqrt{3}}\right)^2 = 2 - \frac{\pi}{3}$$

(xvi)



$$\Delta = 2\left[\int_0^4 \sqrt{25-x^2}\,dx - \int_0^2 1-\frac{x^2}{4}\,dx + \int_2^4 \frac{x^2}{4}-1\,dx\right]$$

$$= 2\left[\left\{\frac{x\sqrt{25-x^2}}{2} + \frac{25}{2}\sin^{-1}\left(\frac{x}{5}\right)\right\}_0^4 - \left(x-\frac{x^3}{12}\right)_0^2\right]$$

$$\left(1-\frac{x^2}{4}\right)^2 + x^2 = 25$$

$$\Rightarrow \frac{x^4}{16} + \frac{x^2}{2} = 24$$

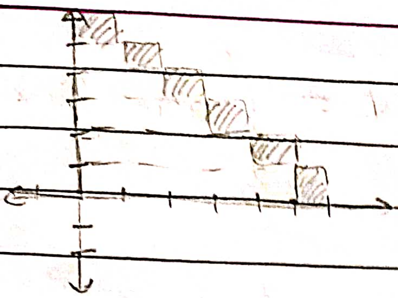
$$= 2\left[6 + \frac{25}{2}\sin^{-1}\left(\frac{4}{5}\right) - \frac{16}{3} + 4\right] - \left(\frac{x^3}{12} - x\right)_2^4\Big]$$

$$\Rightarrow x^4 + 8x^2 - (16)(24) = 0 \Rightarrow x^2 = 16, \cancel{-24}$$

$$\underline{x = \pm 4}$$

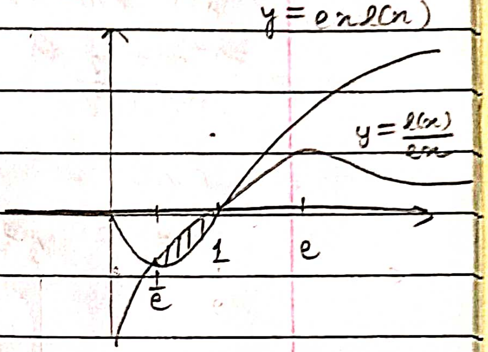
$$= \underline{4 + 25\sin^{-1}\left(\frac{4}{5}\right)}$$

(xvii)



6

(xviii) $\frac{l(x)}{ex} = ex\,l(x) \Rightarrow \underline{x = 1/e, 1}$



$$\Delta = \left| \int_{1/e}^{1} ex\,l(x) - \frac{l(x)}{ex}\, dx \right|$$

$$= \left| \left[\frac{ex^2 l(x)}{2} - \frac{ex^2}{4} - \frac{l^2(x)}{2e} \right]_{1/e}^{1} \right| = \underline{\frac{e}{4} - \frac{5}{4e}}$$

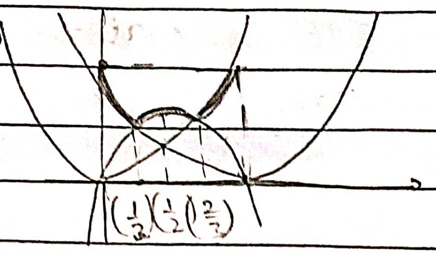
Graph – ① $y' = el(x) + e \Rightarrow x = \frac{1}{e}$ (min) 1/e

② $y' = \frac{(1 - l(x))}{ex^2} \Rightarrow x = e$ (max) + − e

Q. (xix) Let $f(x) = \max\{x^2, (1-x)^2, 2x(1-x)\}$, $x \in [0,1]$. Determine the area of the region bounded by $y = f(x)$, x-axis, $x = 0$, $x = 1$.

(xx) Find the area of the region bounded by $y = l(x)$, $y = s^4 \pi x$ & $x = 0$

A (xix)



$$\Delta = \int_0^{1/3} (x-1)^2\, dx + \int_{1/3}^{2/3} 2x-2x^2\, dx$$

$$+ \int_{2/3}^{1} x^2\, dx$$

$$x^2 = 2x - 2x^2$$

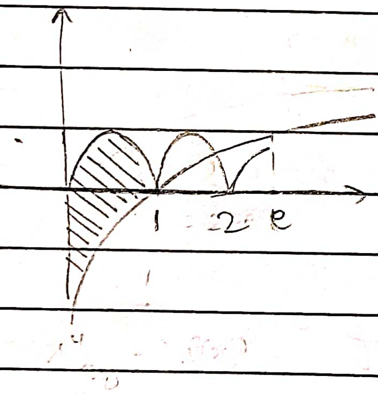
$$\Rightarrow x = \frac{2}{3}$$

$$= \left[\frac{(x-1)^3}{3}\right]_0^{1/3} + \left[x^2 - \frac{2}{3}x^3\right]_{1/3}^{2/3} + \left[\frac{x^3}{3}\right]_{2/3}^{1}$$

$$\Rightarrow \quad \Delta = \frac{8}{81} + \frac{4}{9} - \frac{16}{81} - \frac{1}{9} + \frac{8}{81} + \frac{1}{3} - \frac{8}{81} + \frac{1}{3}$$

$$= 1 - \frac{80}{81} = \boxed{\frac{17}{27}}$$

(xx)



$$\Delta = \int_0^1 s^4_{\pi x} - l(x)\, dx$$

$$= \int_0^1 \frac{3}{8} + \frac{c_{4\pi x}}{8} - \frac{c_{2\pi x}}{2} - l(x)\, dx$$

$$= \left[\frac{3x}{8} + \frac{s_{4\pi x}}{32\pi} - \frac{s_{2\pi x}}{4\pi} + x - x\,l(x)\right]_0^1$$

$$= \frac{13}{8} + 1 = \boxed{\frac{11}{8}}$$

$$s^4_{\pi x} = \frac{(1-c_{2\pi x})^2}{4}$$

$$= \frac{1}{4} - \frac{c_{2\pi x}}{2} + \frac{1 + c_{4\pi x}}{8}$$

Q (xxi) Find the area of the smaller region bounded by the curves
$$\frac{(x-1)^2}{4} + (y-1)^2 = 1 \quad \& \quad (y-1)^2 = \frac{3}{4}(x-1).$$

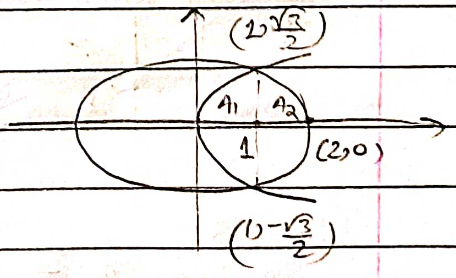
(xxii) If 'f' is a real valued fnⁿ satisfying $f\left(\frac{x}{y}\right) = f(x) - f(y)$. $\forall\ x \in \mathbb{R}^+$. and $\lim_{x\to 0} \frac{f(1+x)}{x} = 3$, then find area of the region bounded by the curves. $y = f(x)$; y-axis & $y = 3$.

(xxiii) Find the area of the region containing the $P(x, y)$ satisfying $|y| + \frac{1}{2} \geq e^{-|x|}$ and $\max\{|x|, |y|\} \leq 2$

A. (xxi) Area isn't affected by shifting of origin.

so $$\Delta\left(\frac{(x-1)^2}{4} + (y-1)^2 = 1,\ (y-1)^2 = \frac{3}{4}(x-1)\right)$$
$$= \Delta\left(\frac{x^2}{4} + y^2 = 1,\ y^2 = \frac{3x}{4}\right)$$

$$\frac{x^2}{4} + \frac{3x}{4} = 1 \Rightarrow x^2 + 3x - 4 = 0$$
$$\Rightarrow x = 1, \cancel{-4}$$



$$A_1 = \left(\frac{2}{3}\right)(\sqrt{3}) = \left(\frac{2\sqrt{3}}{3}\right)$$

$$A_2 = 2\int_1^2 \sqrt{1 - \frac{x^2}{4}}\, dx = \int_1^2 \sqrt{4 - x^2}\, dx$$
$$= \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_1^2 = \pi - \frac{\sqrt{3}}{2} - \frac{\pi}{3} = \frac{2\pi}{3} - \frac{\sqrt{3}}{2}$$

$$\Delta = \frac{2\sqrt{3}}{3} + \frac{2\pi}{3} + \frac{\sqrt{3}}{2} = \frac{2\pi}{3} + \frac{1}{2\sqrt{3}}$$

(xxii) $f(x) = f(x) - f(1) \Rightarrow \underline{f(1)=0}$

$$\underset{x\to 0}{\text{lt}} \frac{f(1+x)}{x} \overset{\left(\frac{0}{0}\right)\to LH}{=} \frac{f'(1+x)}{1} = f'(1) = 3.$$

$$f'(x) = \underset{h\to 0}{\text{lt}} \frac{f(x+h)-f(x)}{h} = \underset{h\to 0}{\text{lt}} \frac{f\left(1+\frac{h}{x}\right)}{\left(\frac{h}{x}\right)} \left(\frac{1}{x}\right)$$

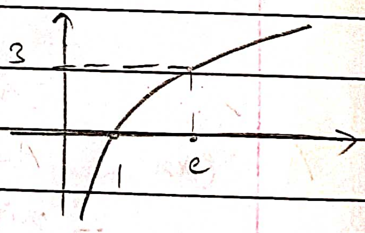
$$= \left(\frac{3}{x}\right)$$

$f(x) = 3\ell(x) + C$ $\quad\because f(1)=0 \Rightarrow C=0$

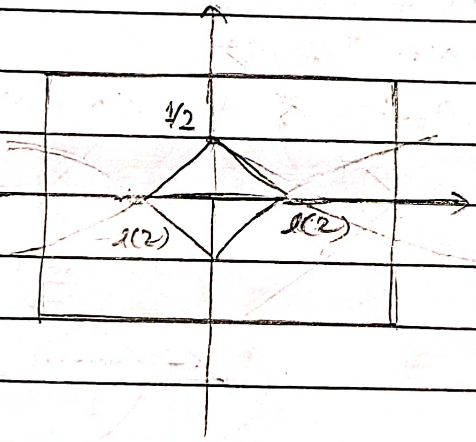
$\Rightarrow \underline{f(x) = 3\ell(x)}$

$$\Delta = \int_{-\infty}^{3} e^{y/3}\, dy = 3\left[e^{y/3}\right]_{-\infty}^{3}$$

$$= \underline{3e}$$



(xxiii)



$A_1 = (4)(4) = 16$

$$A_2 = 4\left\{\int_0^{\ell(2)} e^{-x} - \frac{1}{2}\, dx\right\}$$

$$= 4\left[-e^{-x} + \frac{x}{2}\right]_0^{\ell(2)}$$

$$= 4\left[\frac{1}{2} + \frac{\ell(2)}{2} - 1\right]$$

$\Delta = A_1 - A_2 = \underline{2\left[9 - \ell(2)\right]}$

Q. (xxiv) Let $f(x) = \min\{e^x, 3/2, 1+e^{-x} : x\in[0,1]\}$. Find the area of the region bounded by $y=f(x)$, x-axis, y-axis & the line $x=1$.

(xxv) Find the area of the region which contains all the pts. satisfying $|x-2y| + |x+2y| \leq 8$ & $xy \geq 2$.

(xxvi) A curve has parametric eqn $x=\sqrt{3}t^2+4t$, $y=2t^3-8t$, where $t\geq 0$. Find the values of 't' for which the curve lies below the x-axis. Also find the magn. of area enclosed by this part of the curve and the x-axis.

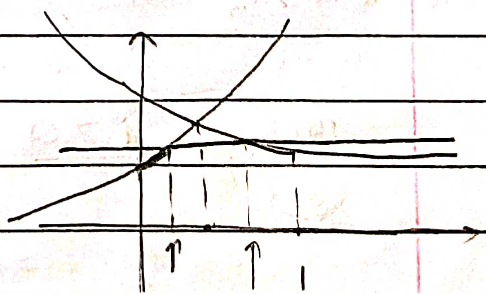
(xxvii) Consider a square with vertices at $(1,1)$, $(-1,1)$, $(1,-1)$ & $(-1,1)$.
Let S be the region consisting of all pts. inside the square which are nearer to origin than to any edges.
Sketch the region S and find its area.

A. (xxiv)

$$e^x = 1+\frac{1}{e^x}$$

$$\Rightarrow e^{2x}-e^x-1=0$$

$$\Rightarrow e^x = \frac{1+\sqrt{5}}{2}$$

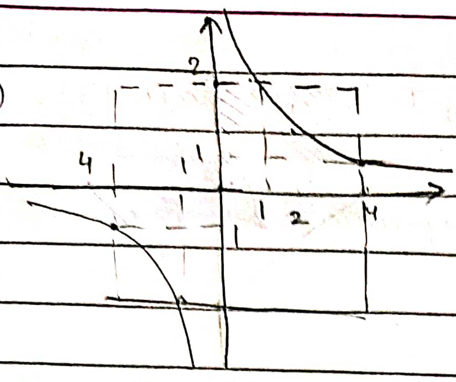


$$\Delta = \int_0^{l(3/2)} e^x\,dx + \int_{l(3/2)}^{l(2)} \frac{3}{2}\,dx + \int_{l(2)}^{1} 1+e^{-x}\,dx$$

$$= \left(\frac{3}{2}-1\right) + \frac{3}{2}\,l\left(\frac{4}{3}\right) + 1-l(2) + \frac{1}{2}-\frac{1}{e}$$

$$= 2 + l\left(\frac{4}{3\sqrt{3}}\right) - \frac{1}{e}$$

(xxv)



$\underline{\text{I Quad}}$ & $x \geq 2y \Rightarrow x - 2y + x + 2y \leq 8$

$\Rightarrow \underline{x \leq 4}$

& $x < 2y \Rightarrow 2y - x + x + 2y \leq 8$

$\Rightarrow \underline{y \leq 2}$

$y = \left(\frac{2}{x}\right)$

$$\Delta = 2\left[(3)(2) - \int^{4} \frac{2}{x}\, dx\right]$$

$$= 2\left[6 - 2l(4)\right]$$

$$= \underline{12 - 8l(2)}$$

(xxvi) $y < 0 \Rightarrow 2t^3 - 8t < 0 \Rightarrow t(t+2)(t-2) < 0$

$\Rightarrow \underline{t \in (-\infty, -2) \cup (0, 2)}$

$$\Delta = \int y\, dx = \int (2t^3 - 8t)\, d(\sqrt{3}t^2 + 4t)$$

$$= \int_{-2}^{0} (2t^3 - 8t)(2\sqrt{3}t + 4)\, dt - \int_{0}^{2} (2t^3 - 8t)(2\sqrt{3}t + 4)\, dt$$

$$= \left[\frac{4\sqrt{3}}{5}t^5 + 2t^4 - \frac{16\sqrt{3}}{3}t^3 - 16t^2\right]_{-2\;\;2}^{0\;\;0}$$

$(2t^3 - 8t)(2\sqrt{3}t + 4)$

$= 4\sqrt{3}t^4 + 8t^3$
$- 16\sqrt{3}t^2 - 32t$

$$= (-2)\left[32 - 64\right] = \underline{64}$$

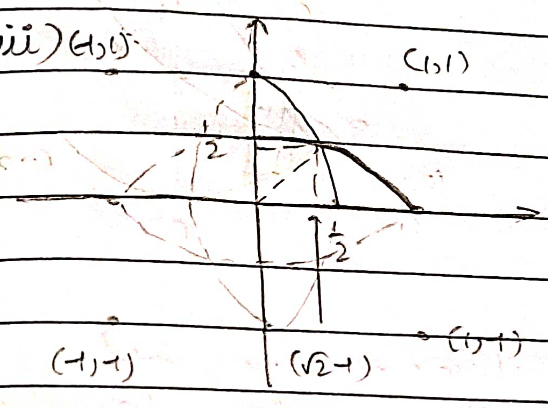
But $t \geq 0$ given.

so)

$$\Delta = -\int_{0}^{2} (2t^3 - 8t)(2\sqrt{3}t + 4)\, dt = \left[\frac{4\sqrt{3}}{5}t^5 + 2t^4 - \frac{16\sqrt{3}}{3}t^3 - 16t^2\right]_{2}^{0}$$

$$= -\frac{128\sqrt{3}}{5} - 32 + \frac{128\sqrt{3}}{3} + 64 = \underline{32\left(1 + \frac{8\sqrt{3}}{15}\right)}$$

(xxvii)



<u>I Quad</u> : $\sqrt{x^2+y^2} \leq \min\{1-x, 1-y\}$

① $x > y \Rightarrow \sqrt{x^2+y^2} \leq 1-x$

$\Rightarrow x^2+y^2 \leq x^2-2x+1$

$\Rightarrow y^2 \leq -2\left(x-\frac{1}{2}\right)$

② $x \leq y \Rightarrow \sqrt{x^2+y^2} \leq 1-y$

$\Rightarrow x^2+y^2 \leq y^2-2y+1$

$\Rightarrow x^2 \leq -2\left(y-\frac{1}{2}\right)$

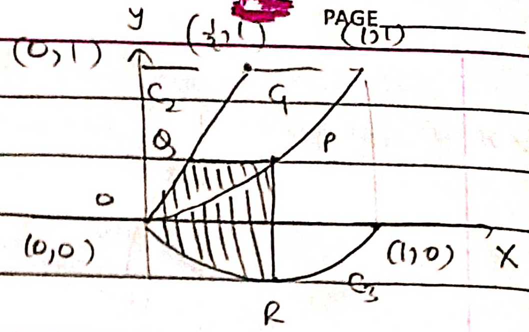
$$\Delta = 4\left[(\sqrt{2}-1)^2 + (2)\left(\frac{2}{3}\right)(\sqrt{2}-1)\left(\frac{1}{2}-\sqrt{2}+1\right)\right]$$

$$= 4\left[(\sqrt{2}-1)^2 + \frac{2}{3}(\sqrt{2}-1)^3\right] = 4(\sqrt{2}-1)^2\left[1+\frac{2\sqrt{2}}{3}-\frac{2}{3}\right]$$

$$= \underline{\frac{4}{3}(\sqrt{2}-1)^2(2\sqrt{2}+1)}$$

Q (xxviii) Let $O(0,0)$, $A(2,0)$ & $B(1,\frac{1}{\sqrt{3}})$ be the vertices of a Δ. Let R be the region consisting of all those pts. P inside ΔOAB which satisfy $d(P,OA) \leq \min\{d(P,OB), d(P,AB)\}$, where 'd' denotes the dist. from the pt. to the corresponding line.
Sketch the region R & find its area

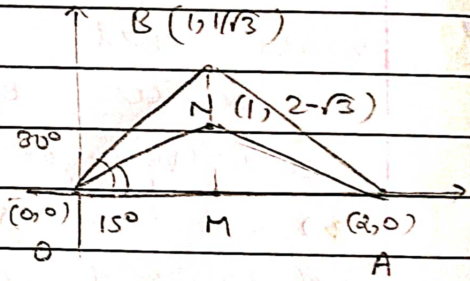
(xxix) Let C_1, C_2 be the graphs of the fn $y=x^2$ & $y=2x$ respectively, where $x \in [0,1]$.
Let C_3 be the graph of a fn $y=f(x)$, $x \in [0,1]$, $f(0)=0$.
For a pt. P on C_1, let the lines through P, parallel to the axis, meet C_2 & C_3 at Q & R respectively. If for every position P (on C_1) the area of the shaded region OPQ & ORP are equal, determine f(x).



(xxx) Let $b \neq 0$ for $j = 0, 1, 2 \ldots, n$. Let S_j be the area of the region bounded by the y-axis & the curve $xe^{ay} = \sin by$, $\frac{j\pi}{b} \leq y \leq \frac{(j+1)\pi}{b}$. Show that $S_0, S_1 \ldots S_n$ are in GP.

Also, find their sum for $a = 1$ & $b = \pi$

A. (xxviii) All pts in ONA are closer to OA & OB & AB



$\Rightarrow \frac{1}{2}(2)(2-\sqrt{3}) = 2-\sqrt{3}$

(xxix) Let $P(x, x^2)$

A.T.Q

$$\int_0^{x^2} \sqrt{y} - \frac{y}{2}\, dy = \int_0^x x^2 - f(x)\, dx$$

$$\Rightarrow \frac{2x^3}{3} - \frac{x^4}{4} = \frac{x^3}{3} - \int_0^x f(x)\, dx$$

$$\Rightarrow \underline{f(x) = x^3 - x^2}$$

(xxxx) $S_k = \int_{\frac{k\pi}{b}}^{\frac{(k+1)\pi}{b}} e^{-ay} s_{by}\, dy$

D	I
e^{-ay}	s_{by}
$-ae^{-ay}$	$-\frac{c_{by}}{b}$
a^2e^{-ay}	$-\frac{s_{by}}{b^2}$

$$S_k = \left[-\frac{1}{b} e^{-ay} c_{by}\right]_{\frac{k\pi}{b}}^{\frac{(k+1)\pi}{b}} - \left[\frac{a}{b^2} e^{-ay} s_{by}\right]_{\frac{k\pi}{b}}^{\frac{(k+1)\pi}{b}} - \frac{a^2 S_k}{b^2}$$

$$\Rightarrow \frac{(a^2+b^2)}{b^2} S_k = \frac{e^{-\frac{ak\pi}{b}}}{b} c_{k\pi} - \frac{e^{-\frac{a(k+1)\pi}{b}}}{b} c_{(k+1)\pi} - \frac{a\, e^{-\frac{a(k+1)\pi}{b}}}{b^2} s_{(k+1)\pi} + \frac{a\, e^{-\frac{k\pi}{b}}}{b^2} s_{k\pi}$$

$$\Rightarrow (a^2+b^2) S_k = \underline{e^{-\frac{ak\pi}{b}} (b c_{k\pi} + a s_{k\pi}) - e^{-\frac{a(k+1)\pi}{b}} (b c_{(k+1)\pi} + a s_{(k+1)\pi})}$$

$$= e^{-\frac{ak\pi}{b}} (1 - e^{\frac{a\pi}{b}}) (b c_{k\pi} + a s_{k\pi})$$

$$(a^2+b^2)^2 S_{(k-1)} S_{(k+1)} = e^{-\frac{a}{b}(k-1+k+1)\pi} (1-e^{\frac{a\pi}{b}})^2 (b c_{(k-1)\pi} + a s_{(k-1)\pi}) (b c_{(k+1)\pi} + a s_{(k+1)\pi})$$

$$= e^{-\frac{2ak\pi}{b}} (1-e^{\frac{a\pi}{b}})^2 (b c_{k\pi} + a s_{k\pi})^2$$

$$= (a^2+b^2)^2 S_k^2$$

$$\Rightarrow \underline{S_{(k-1)} S_{(k+1)} = S_k^2}$$

□

$$\underline{E_k = e^{-\frac{ak\pi}{b}} (b c_{k\pi} + a s_{k\pi})}$$

$(a^2+b^2) S_0 = E_0 - E_1$

$(a^2+b^2) S_1 = E_1 - E_2$

$\vdots$

$(a^2+b^2) S_n = E_n - E_{(n+1)}$

$$\Rightarrow (a^2+b^2)(\Sigma S_i) = E_0 - E_{(n+1)}$$

$$= b - e^{-\frac{a(n+1)\pi}{b}} (b c_{(n+1)\pi} + a s_{(n+1)\pi})$$

$$= \underline{b + e^{-\frac{a}{b}\pi(n+1)} (b c_{n\pi} + a s_{n\pi})}$$

Q (xxxi) Let the line $x=b$ divide the area enclosed by $y=(1-x)^2$, $y=0$ & $x=0$ into 2 parts R_1 ($x \in [0, b)$) & R_2 ($x \in [b, 1]$) s.t. $R_1 - R_2 = 1/4$. Find b

(xxxii) Determine the area enclosed by the curves $y = sx + cx$ & $y = |c-s|$ over the interval $[0, \pi/2]$

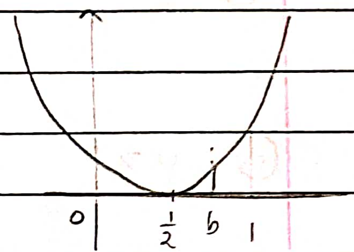
(xxxiii) For a pt. P in the plane, let $d_1(P)$ and $d_2(P)$ be the distances of the pt. P from the lines $x-y=0$ & $x+y=0$ respectively. The area of the region R consisting of all pts. P lying in the I Quad of the plane satisfying $2 \leq d_1(P) + d_2(P) \leq 4$ is S. Find S.

(xxxiv) Let S be the area of the region enclosed by $y = e^{-x^2}$, $y=0$, $x=0$ & $x=1$
Then P.T.

(I) $S \geq 1/e$ (II) $S \geq 1 - 1/e$ (III) $S \leq \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}\left(1 - \frac{1}{\sqrt{2}}\right)$

A. (xxxi) $R_1 = \int_0^b (x-1)^2 \, dx = \left[\frac{(x-1)^3}{3}\right]_0^b$

$= \frac{(b-1)^3+1}{3}$

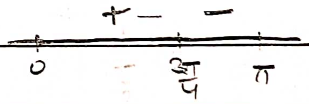


$R_1 + R_2 = \left[\frac{(x-1)^3}{3}\right]_0^1 = \frac{1}{3}$ $\Rightarrow$ $R_1 = \frac{7}{24}$ $\Rightarrow$ $8(b-1)^3+8 = 7$

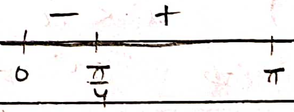
$R_1 - R_2 = \frac{1}{4}$ $\Rightarrow$ $b-1 = \frac{-1}{2}$

$\Rightarrow$ $b = 1/2$

(xxxii) $y_1 = s+c$



$y_2 = |s-c|$

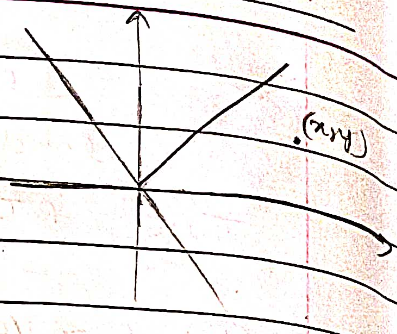


$\Delta = \int_0^{\pi/4} s+c+c-s \, dx + \int_{\pi/4}^{3\pi/4} s+c+s-c \, dx + \int_{3\pi/4}^{\pi} -s-c+s-c \, dx$

$= 2\boxed{\int_0^{\pi/4} c \, dx + \int_{\pi/4}^{3\pi/4} s \, dx - \int_{3\pi/4}^{\pi} c \, dx}$

$= 2\left[\left(\frac{1}{\sqrt{2}} - 0\right) + \left(\frac{1}{\sqrt{2}} - \left(\frac{-1}{\sqrt{2}}\right)\right) + \left(\frac{-1}{\sqrt{2}} + 1\right)\right]$

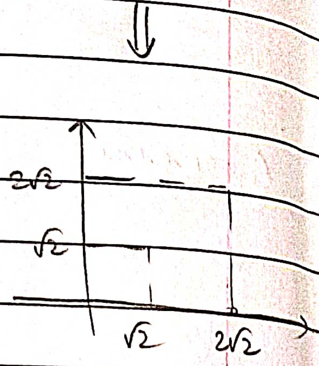
$= 2\sqrt{2} + 2$



(xxxiii) $d_1(P) = \left|\frac{x-y}{\sqrt{2}}\right|$

$d_2(P) = \left|\frac{x+y}{\sqrt{2}}\right|$

$\Rightarrow \quad 2\sqrt{2} \leq |x-y| + |x+y| \leq 4\sqrt{2}$



① $y<x \quad \Rightarrow \quad 2\sqrt{2} \leq x-y+x+y \leq 4\sqrt{2}$

$\Rightarrow \quad \underline{x \in [\sqrt{2}, 2\sqrt{2}]}$

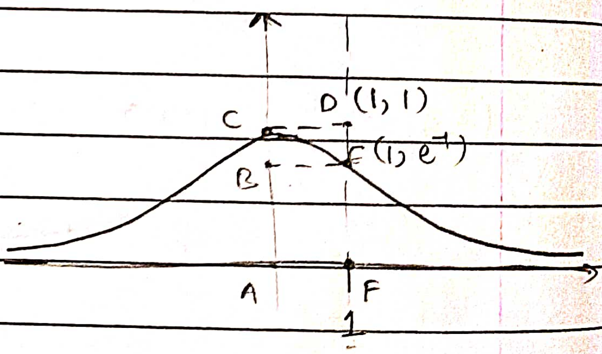
② $y \geq x \quad \Rightarrow \quad 2\sqrt{2} \leq y-x+x+y \leq 4\sqrt{2}$

$\Rightarrow \quad \underline{y \in [\sqrt{2}, 2\sqrt{2}]}$

$S = (2\sqrt{2})^2 - (\sqrt{2})^2 = \underline{6}$

(xxxiv) (I) $S > [ABEF]$

$> e^{-1}$



(II) $x^2 < x$

$\Rightarrow \quad e^{-x^2} > e^{-x}$

$\Rightarrow \quad \int_0^1 e^{-x^2} dx > \int_0^1 e^{x} dx$

$> \underline{\left(1 - \frac{1}{e}\right)}$

(III) $S < \left(\frac{1}{\sqrt{2}} - 0\right)1 + \frac{1}{\sqrt{e}}\left(1 - \frac{1}{\sqrt{2}}\right)$

$< \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}\left(1 - \frac{1}{\sqrt{2}}\right)$