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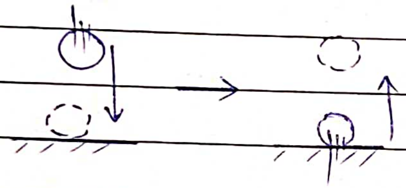
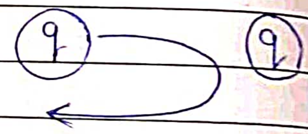
Collision

m_1 m_2

If 2 objs interact, & due to this their momentum changes in very short time, then interaction is called Collision.

Eg: 

Collision ✓

Eg: 

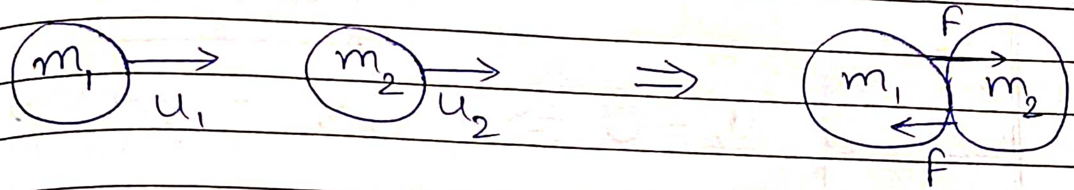
Collision ✗

In collision, force is Impulsive.

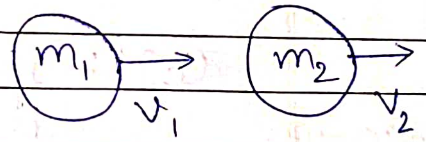
Collision
- Elastic
  - p Consrv.
  - K Consrv.
- Inelastic
  - p Consrv
  - K NOT consrv.

(If nothing given, assume inelastic)

1) Elastic Collision –



During interaction, some K converted to U.
During later interaction, U reconverted to K.



At end, $K_{\text{Total just before collision}} = K_{\text{Total just after collision}}$

Momentum Consrv., $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$$\Rightarrow m_1(v_1 - u_1) = m_2(u_2 - v_2) \quad \text{— (1)}$$

K Consrv., $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$$\Rightarrow m_1(v_1 - u_1)(v_1 + u_1) = m_2(u_2 - v_2)(u_2 + v_2)$$

Using (1),

$$v_1 + u_1 = v_2 + u_2 \quad \text{— (2)}$$

Solving (1) & (2) gives, ☆

$$\boxed{\begin{aligned} v_1 &= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2 \\ v_2 &= \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 \end{aligned}}$$

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$$\left[\begin{pmatrix}\%\text{ Momentum}\\ \text{Tranferred by } m_1\end{pmatrix} = \left|\frac{P_f - P_i}{P_i}\right| \times 100\%\right]$$

Date:

C1: If $m_1 = m_2 \Rightarrow$ $\boxed{\begin{aligned} v_1 &= u_2 \\ v_2 &= u_1 \end{aligned}}$

C2: If $u_2 = 0 \Rightarrow$ $\boxed{\begin{aligned} v_1 &= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 \\ v_2 &= \left(\frac{2m_1 u_1}{m_1 + m_2}\right) \end{aligned}}$

Cc 2.1: If $m_1 = m_2 \Rightarrow$ $\boxed{\begin{aligned} v_1 &= 0 \\ v_2 &= u_1 \end{aligned}}$

(Momentum tranfer 100%
KE tranfer 100%)

(max. in any case) → KE tranfer 100%

Cc 2.2: If $m_1 >> m_2 \Rightarrow$ $\boxed{\begin{aligned} v_1 &= u_1 \\ v_2 &= 2u_1 \end{aligned}}$

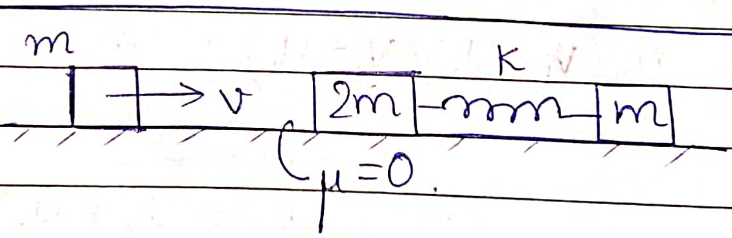
(Momentum tranfer 0%
KE tranfer 0%)

Cc 2.3: If $m_2 >> m_1 \Rightarrow$ $\boxed{\begin{aligned} v_1 &= (-u_1) \\ v_2 &= 0 \end{aligned}}$

(max. in any case) → Momentum tranfer 200%

(Momentum tranfer 200%
KE tranfer 0%)

Q)

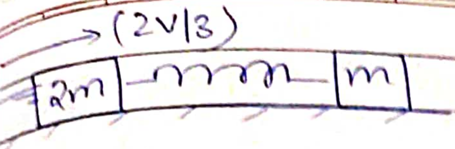


Find max. compression in spring.

A) $m \rightarrow v$ $\quad$ $2m \rightarrow 0$

$$v_1 = \left(\frac{m - 2m}{m + 2m}\right)v = \left(\frac{-v}{3}\right), \quad v_2 = \left(\frac{2m}{m + 2m}\right)v = \frac{2v}{3}$$

$$\left[\left(\begin{array}{c}\% \text{ KE} \\ \text{retained by } m_1\end{array}\right) = \left(\frac{v_1}{u_1}\right)^2 \times 100\%\right]$$



Wrt CoM,

$$\left(\frac{1}{2}\right)\left(\frac{2m \cdot m}{m+2m}\right)\left(\frac{2v}{3}\right)^2 = \frac{1}{2}Kx^2 \Rightarrow \boxed{x = \sqrt{\frac{8m}{27K}}\, v}$$

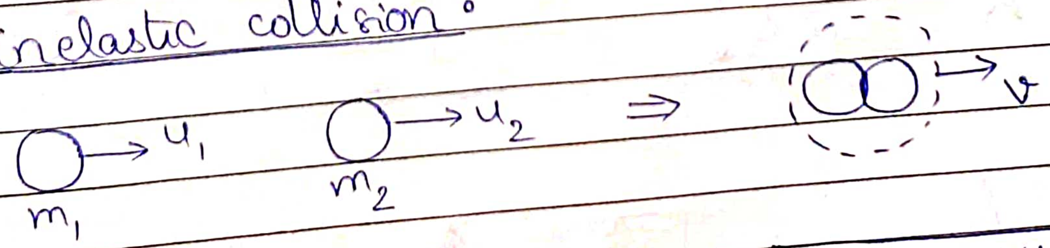
2) Inelastic Collision —

$$\boxed{(\text{Coeff. of Restitution}) = \left(\frac{\text{Rel. vel. of sep.}}{\text{Rel. vel. of approach}}\right)}$$

$$\Rightarrow \quad \boxed{e = \left(\frac{v_2 - v_1}{u_1 - u_2}\right)} \quad \{0 < e < 1 \text{ always}\}$$

for elastic collision, " $e = 1$.
for perfectly inelastic collision, $e = 0$.
(obj. stick together)

- Perfectly inelastic collision :



$$P_i = P_f \Rightarrow m_1u_1 + m_2u_2 = (m_1+m_2)v \Rightarrow \boxed{v = \left(\frac{m_1u_1 + m_2u_2}{m_1+m_2}\right)}$$

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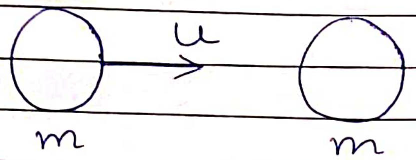
Loss in KE $= \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \frac{1}{2}(m_1+m_2)v^2$

$= \boxed{\frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)(u_1-u_2)^2}$

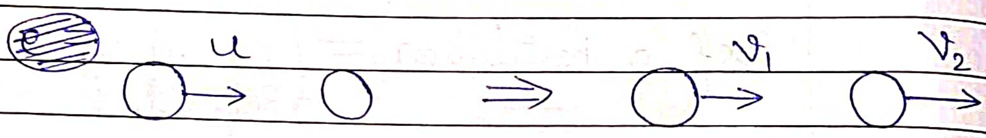
Q)



Coeff. of Restt. = e

Find vel. of A & B just after collision.

A)

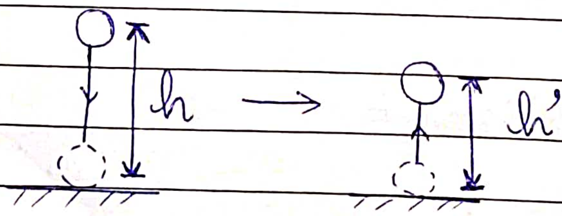


✓ $\left(\frac{v_2-v_1}{u}\right) = e \Rightarrow v_2 - v_1 = eu$

✓ $mu = mv_1 + mv_2 \Rightarrow v_2 + v_1 = u$

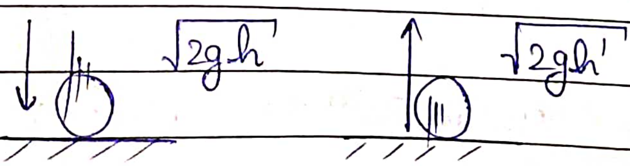
$\Rightarrow \boxed{v_2 = \left(\frac{1+e}{2}\right)u}, \boxed{v_1 = \left(\frac{1-e}{2}\right)u}$

☆ Q)



Find e.

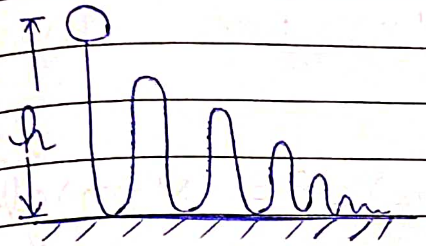
A)



$e = \frac{\sqrt{2gh'}}{\sqrt{2gh}}$

$\Rightarrow \boxed{e = \sqrt{\frac{h'}{h}}}$

Q) If ball dropped from height 'h' and coeff. of restt. of collision with ground is 'e', find total dist. travelled by ball before stopping.

A) 

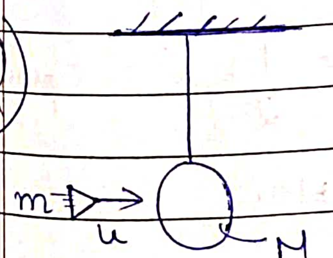
$$\text{Dist.} = h + 2he^2 + 2he^4 + \dots$$

$$= \boxed{h\left(\frac{1+e^2}{1-e^2}\right)}$$

Q) Find time taken by ball to stop.

A) $$t = \sqrt{\frac{2h}{g}} + 2\sqrt{\frac{2he^2}{g}} + 2\sqrt{\frac{2he^4}{g}} + \dots$$

$$= \left(2\sqrt{\frac{2h}{g}}\right)\left(1 + e + e^2 + \dots\right) - \sqrt{\frac{2h}{g}} = \boxed{\sqrt{\frac{2h}{g}}\left(\frac{1+e}{1-e}\right)}$$

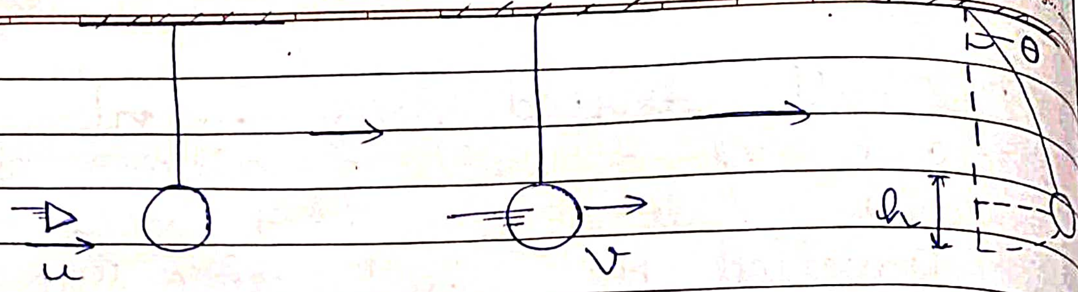
☆ Q) 

Bullet strikes bob and gets embedded into it.

Find max. height attained by system

A) ☆ Can't directly Consrv. Energy as inelastic collision.

A)

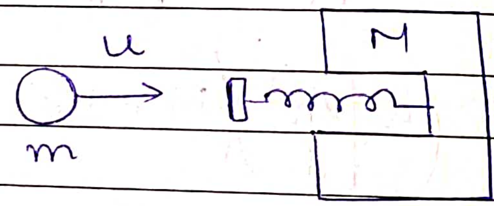


$$v = \left(\frac{mu}{M+m}\right)$$ by Momentum Conserv.

Energy Conserv., $\frac{1}{2}(M+m)v^2 = (M+m)gh$

$$\Rightarrow \boxed{h = \frac{m^2u^2}{2g(M+m)^2}}$$

Q) Find max. frac. of energy of m stored in spring. Everything lies on a flat table.

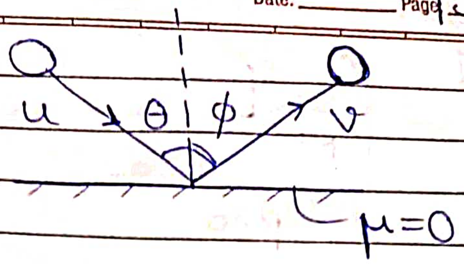


A) Wrt. CoM, $\left(\frac{1}{2}\right)\left(\frac{Mm}{M+m}\right)u^2 = \frac{1}{2}kx^2 = E_{spring}$

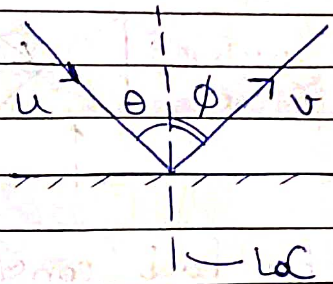
$$\Rightarrow \left(\frac{E_{spring}}{E_{m\,init}}\right) = \frac{(1/2\, mu^2)(M)/(M+m)}{(1/2\, mu^2)}$$

$$\Rightarrow \boxed{Req. = \left(\frac{M}{M+m}\right)}$$

Q1) Find e.



A) e calced along Line of Collision



Along LoC : $e = \dfrac{v c_\phi}{u c_\theta}$

Since no friction, $v s_\phi = u s_\theta$

$\Rightarrow \quad e = \left(\dfrac{v}{u}\right)\left(\dfrac{c_\phi}{c_\theta}\right) \Rightarrow \boxed{e = \dfrac{t_\theta}{t_\phi}}$

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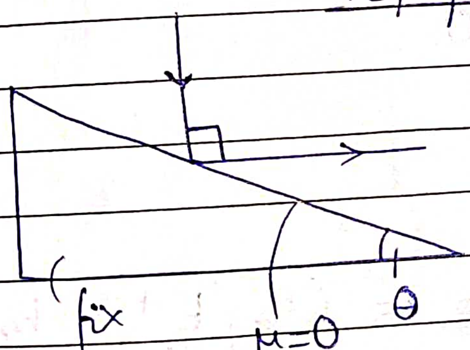
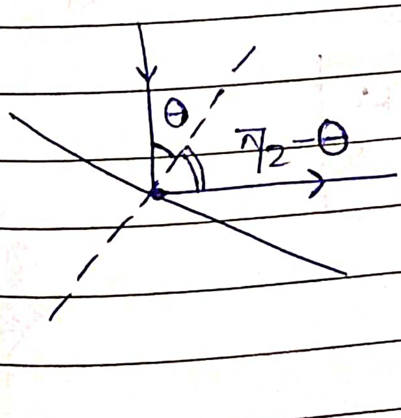
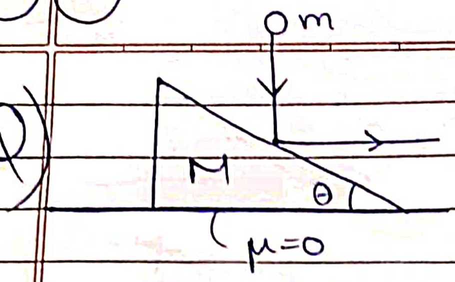
Q1) Find e.

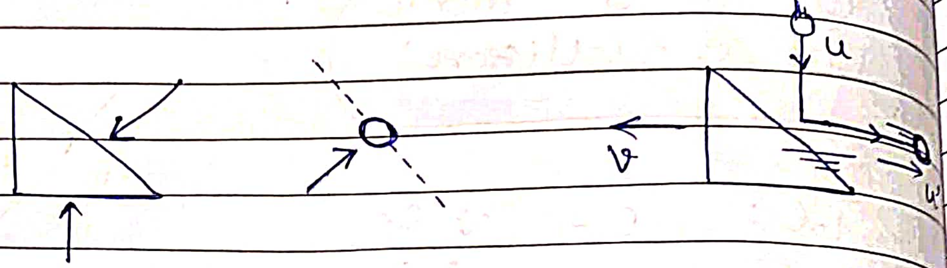


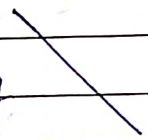
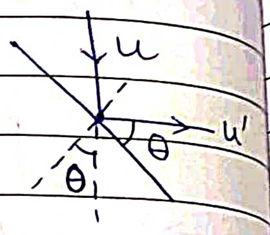
A)



$e = \dfrac{t_\theta}{t_{\pi/2-\theta}} \Rightarrow \boxed{e = t_\theta^2}$

Q)  find e.

A) Impulsive forces: 

for ball consrv p along  

$\Rightarrow \quad mu\, s_\theta = mu'\, c_\theta$

$\Rightarrow \quad u'/u = t_\theta$

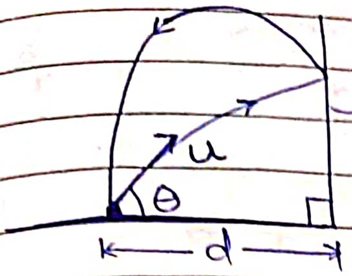
for system consrv. p along ——.

$\Rightarrow \quad Mv = mu'$

Now, $\quad e = \dfrac{u' s_\theta + v s_\theta}{u c_\theta} = (t_\theta)\left(t_\theta + \dfrac{m}{M} t_\theta\right)$

$\Rightarrow \quad \boxed{e = \left(1 + \dfrac{m}{M}\right) t_\theta^2}$

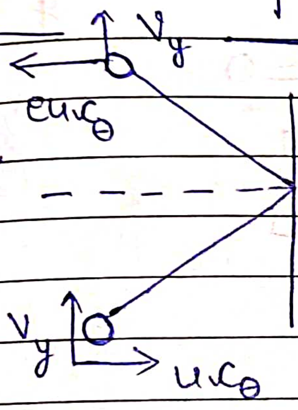
Q)



Find e.

A) Since no impulse in y axis

$\Rightarrow$ Time of flight $= \dfrac{2u\sin\theta}{g}$



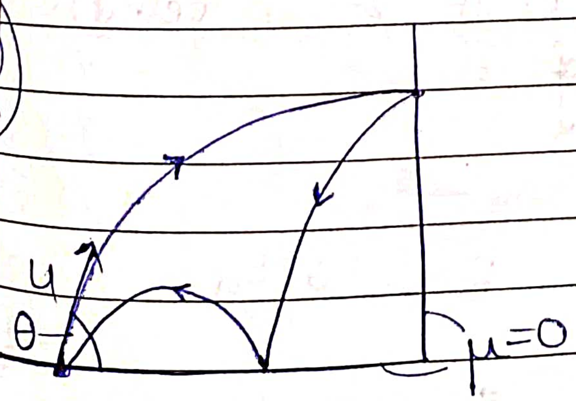
Now,

$$T = \frac{2u\sin\theta}{g} = \frac{d}{u\cos\theta} + \frac{d}{eu\cos\theta}$$

$$\Rightarrow \frac{u^2\sin 2\theta}{g} = d\left(1 + \frac{1}{e}\right)$$

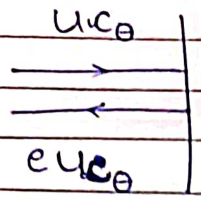
$$\Rightarrow \boxed{e = \left(\frac{gd}{u^2\sin 2\theta - gd}\right)}$$

Q)

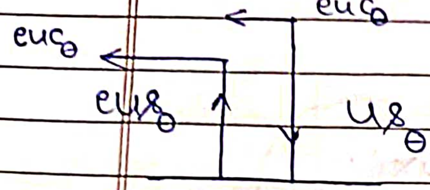


Obj. collide with wall at max. height. If e for both collisions is same, find e.

A)

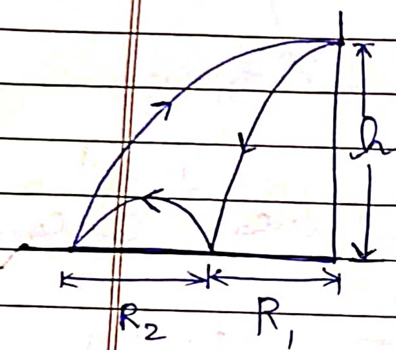


$$h = \frac{u^2 s_\theta^2}{2g} \Rightarrow t = \frac{u s_\theta}{g}$$



$$R = R_1 + R_2$$

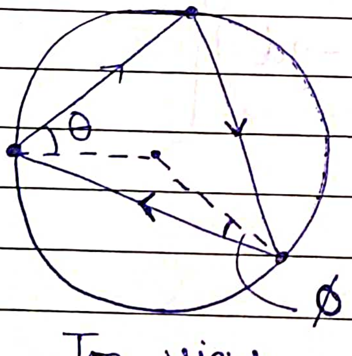
$$\Rightarrow \frac{2u^2 s_\theta c_\theta}{2g} = (euc_\theta)\left(\frac{us_\theta}{g}\right) + \frac{2(euc_\theta)(eus_\theta)}{g}$$



$$\Rightarrow 2e^2 + e - 1 = 0$$

$$\Rightarrow \boxed{e = 1/2}$$

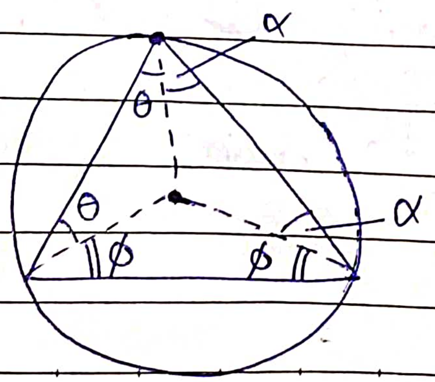
Q)



Top view

If e same for both collisions, find rel^n b/w θ & ϕ in terms of e.

A)



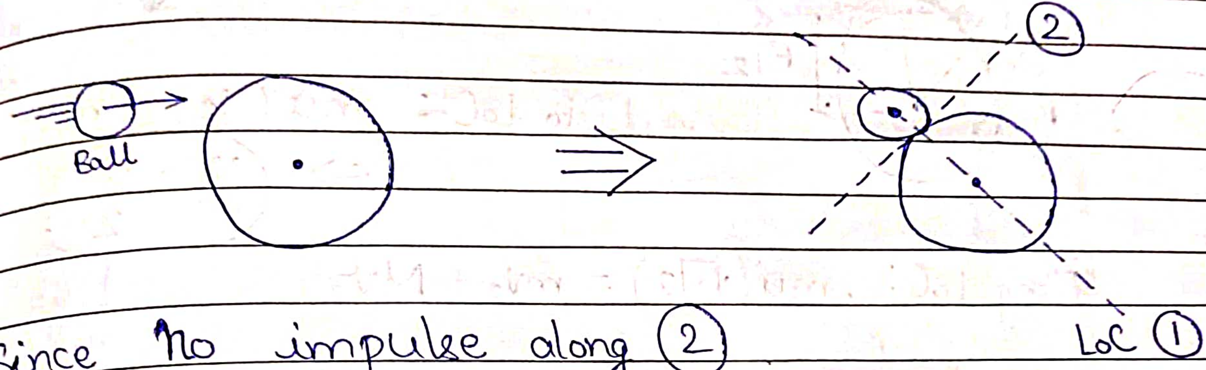
$$e = t_\theta / t_\alpha$$

$$e = t_\alpha / t_\phi$$

$$\Rightarrow \boxed{e = \sqrt{t_\theta / t_\phi}}$$

3) Oblique Collision –

When obj's vel. NOT pass thru other obj's centre.



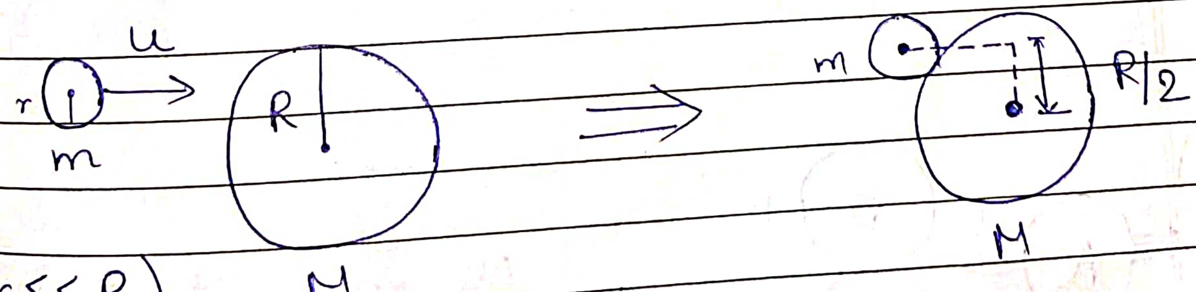
Since no impulse along ②

⇒ $\boxed{\text{p of Ball consrv. along ②}}$

Also, $\boxed{\text{p of system consrv. along ①}}$ as no fext.

---

①



(r << R)

Find vel. of bodies after collision.

Assume perfectly elastic collision.

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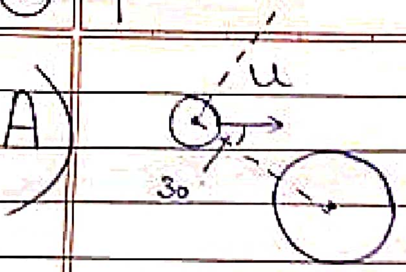
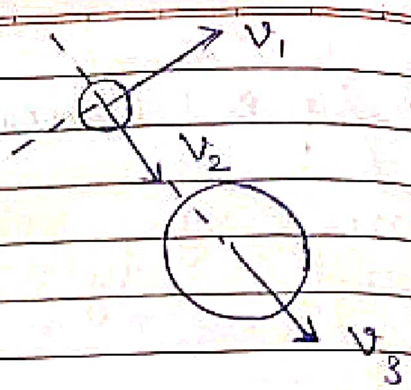
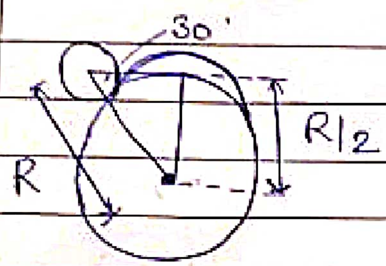
A)



$\Rightarrow$





$\perp$ to LoC : $mu\left(\frac{1}{2}\right) = mv_1$

$\Rightarrow \boxed{v_1 = \frac{u}{2}}$

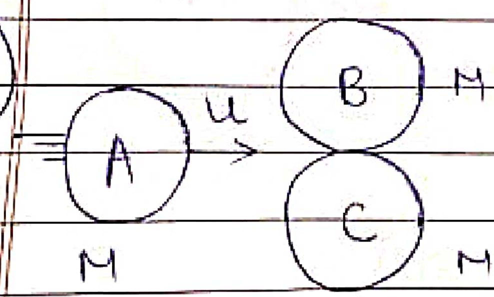
Along LoC: $mu\left(\sqrt{3}/2\right) = mv_2 + Mv_3$

also $e = 1 \Rightarrow v_3 - v_2 = \left(\frac{u\sqrt{3}}{2}\right)$

$\Rightarrow \boxed{v_2 = \left(\frac{m-M}{m+M}\right)\left(\frac{u\sqrt{3}}{2}\right)}, \boxed{v_3 = \left(\frac{2m}{m+M}\right)\left(\frac{u\sqrt{3}}{2}\right)}$

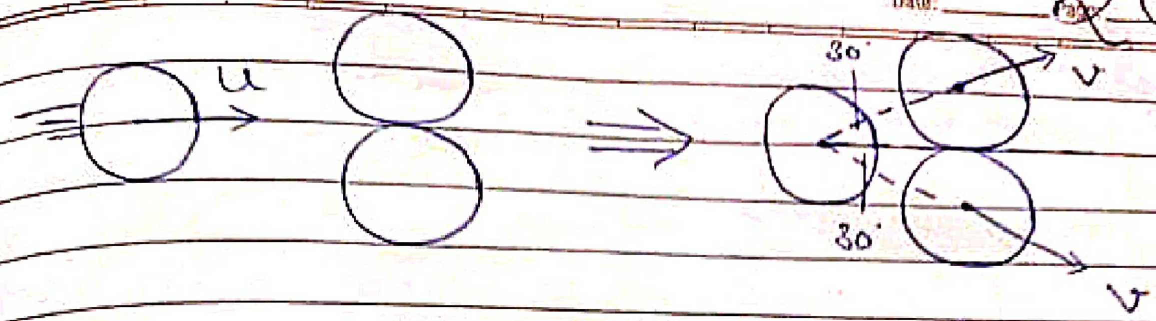
☆ Use this method for both Elastic & Inelastic.

Q)



All mass identical. A comes to rest after striking

Find e.

A) 

Consrv. $\underline{P}$ along : $Mu = Mv\left(\frac{\sqrt{3}}{2}\right) + Mv\left(\frac{\sqrt{3}}{2}\right)$

$\Rightarrow$ $u = v\sqrt{3}$

Along LoC : $e = \dfrac{v}{u(\sqrt{3}/2)}$ $\Rightarrow$ $\boxed{e = 2/3}$