Elasticity

When force is applied on a body it gets deformed; when force is removed body regains its shape.

Such a body is called **Elastic body**

☆ (Harder obj) ⇒ (Lesser its deformation) ⇒ (More its elasticity) ⇒ (Faster it regains shape.)

Factors affecting Elasticity

1) Material of Obj.

2) Strain (ϵ) : (Longitudnal) $\cancel{\frac{\Delta l}{l}}$ $\boxed{\left(\frac{\Delta l}{l}\right)}$ Δl — Extension, l — Length of obj.

3) Stress (σ) : (Longitudnal) $\boxed{\left(\frac{F}{A}\right)}$ F — Force applied, A — Cross section area

Hooke's Law

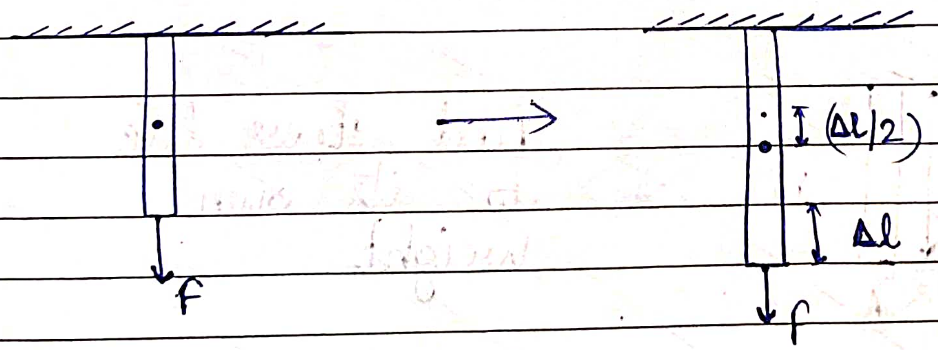
Within ~~elastic~~ proportionality limit, $\boxed{\text{Stress} \propto \text{Strain}}$

$$\Rightarrow \left(\frac{\text{Stress}}{\text{Strain}}\right) = \text{Const.}$$

$$\Rightarrow \boxed{Y = \left(\frac{\text{Stress}}{\text{Strain}}\right)}$$

Young's Modulus

For defining 'Y', force applied should be $\perp$ to area of cross section.



If one end moves by (Δl), then CoM move by $(\Delta l/2)$.

$$\left(\begin{array}{c}\text{Work done}\\ \text{by force}\end{array}\right) = F(\Delta l) \quad ; \quad \left(\begin{array}{c}\text{Work done}\\ \text{on body}\end{array}\right) = F\left(\frac{\Delta l}{2}\right)$$

Observe, $\begin{pmatrix}\text{Work done}\\ \text{by force}\end{pmatrix} \neq \begin{pmatrix}\text{Work done}\\ \text{on body}\end{pmatrix}$

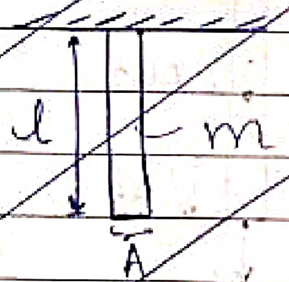
$\Rightarrow$ Some energy is <u>Lost</u>

Now, work done on body is stored at potential energy.

$$U = \frac{1}{2}F(\Delta l) = \frac{1}{2}\left(\frac{F}{A}\right)\left(\frac{\Delta l}{l}\right)(Al) = \frac{1}{2}\left(\frac{F}{A}\right)\left(\frac{\Delta l}{l}\right)(\text{Vol.})$$

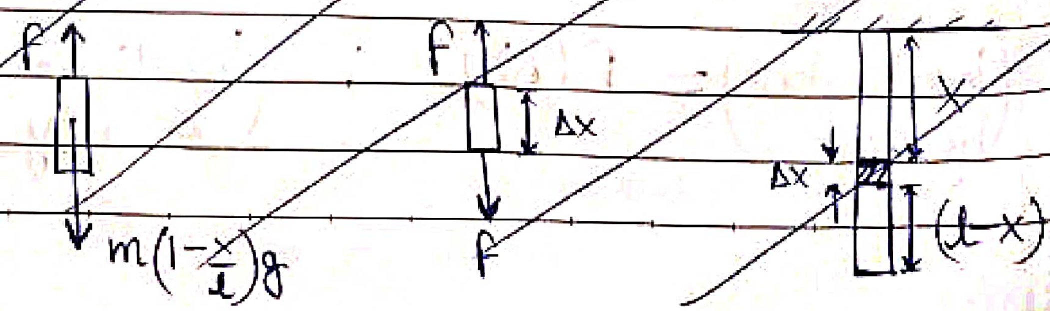
$$\Rightarrow \boxed{u = \left(\frac{U}{V}\right) = \frac{1}{2}(\text{Stress})(\text{Strain})}$$

Energy density

Q) 

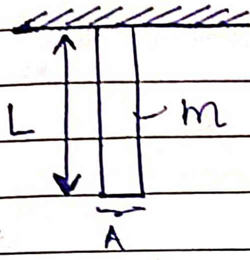
Find stress due to its own weight

A) At any pt 'x' from ceiling.



~~Q) In above Q, if young's modulus is Y, find extension in rod.~~

Q) ★

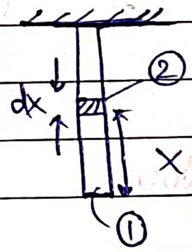


Young's modulus = Y.
find extension in rod due to its own weight.

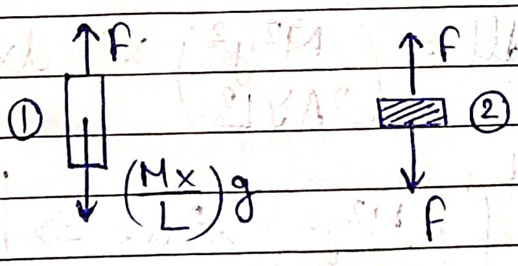
A) Consider a pt. 'x' dist. above bottom.

~~Stress~~ Strain : Consider (δx) extension in (Δx) length of rod.

$$\left(\underset{\text{Strain}}{\cancel{\text{Stress}}}\right) = \left(\frac{\delta x}{\Delta x}\right)$$



~~Strain~~ Stress :



$$\left(\underset{\text{Stress}}{\cancel{\text{Strain}}}\right) = \left(\frac{F}{A}\right) = \left(\frac{Mgx}{AL}\right)$$

Now, $Y = \left(\frac{\text{Stress}}{\text{Strain}}\right) = \left[\frac{(\delta x/\Delta x)}{(Mgx/AL)}\right]^{-1} \Rightarrow \delta x = \left(\frac{Mgx}{ALY}\right) \times (\Delta x)$

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$$\Rightarrow \int \delta x = \Delta L = \left(\frac{Mg}{ALY}\right)\int_0^L x\,dx$$

$$\Rightarrow \boxed{\Delta L = \left(\frac{MgL}{2AY}\right)}$$

Q) Find energy stored in rod in above Q.

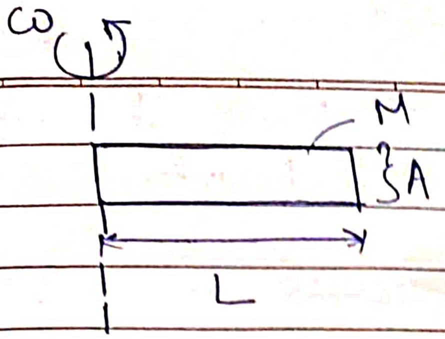
A) $u = \frac{1}{2}(\text{Stress})(\text{Strain}) = \frac{(\text{Stress})^2}{2Y}$

$$= \frac{(Mgx/LA)^2}{2Y} \Rightarrow u = \left(\frac{M^2g^2}{2A^2YL^2}\right)x^2$$

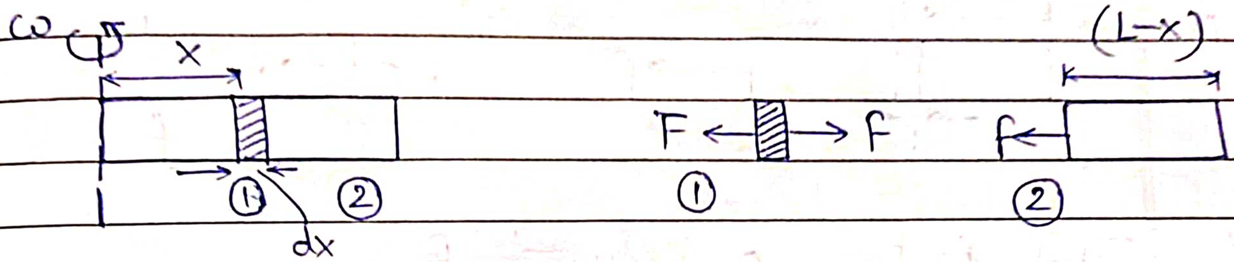
Now, $dU = u\,dV = uA\,dx$

$$\Rightarrow dU = \left(\frac{M^2g^2}{2AYL^2}\right)x^2\,dx$$

$$\Rightarrow U = \int_0^L \left(\frac{M^2g^2}{2AYL^2}\right)x^2\,dx \Rightarrow \boxed{U = \left(\frac{M^2g^2L}{6A}\right)}$$

Q) 

Find extension in rod.

A) 

'f' acts as centripetal force for ②. $\Rightarrow f = \left[\frac{m(L-x)}{L}\right]\omega^2\left[\frac{x+L}{2}\right]$

$$\Rightarrow f = \left(\frac{m\omega^2}{2L}\right)\left(L^2-x^2\right)$$

Consider δx elongement in Δx part.

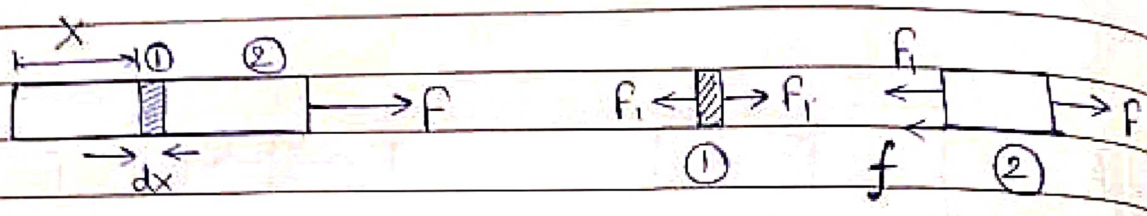
$$\Rightarrow Y = \frac{(F/A)}{(\delta x/\Delta x)} \Rightarrow \delta x = \left(\frac{m\omega^2}{2ALY}\right)\left(L^2-x^2\right)dx$$

$$\Rightarrow \Delta L = \left(\frac{m\omega^2}{2ALY}\right)\int_0^L \left(L^2-x^2\right)dx$$

$$\Rightarrow \boxed{\Delta L = \left(\frac{m\omega^2L^2}{3AY}\right)}$$

Q) 

If block at rest, find extension.

A) 

Total friction on body = F

$\Rightarrow$ Friction of ② $= \left(\frac{F(L-x)}{L}\right)$ {as $f \propto m$}

$\Rightarrow \quad f = F\left(1 - \frac{x}{L}\right)$

$\Rightarrow \quad F_1 = \left(\frac{Fx}{L}\right)$

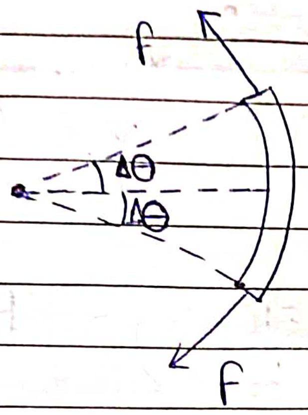
Consider δx elongement in Δx portion.

$\Rightarrow \quad Y = \frac{(F_1/A)}{(\delta x/\Delta x)} \Rightarrow \quad \delta x = \left(\frac{F}{ALY}\right) \times dx$

$\Rightarrow \quad \Delta L = \left(\frac{F}{ALY}\right)\int_0^L x\,dx \Rightarrow \boxed{\Delta L = \left(\frac{FL}{2AY}\right)}$

Q) A uniform ring of radius R is rotating with ω abt central axis, placed on smooth horizontal surface. If σ is breaking stress of ring, then find ω_{min} s.t. ring breaks.

A) Consider a small element.



$m = \rho A R (2\Delta\theta)$

as $\Delta\theta$ very small

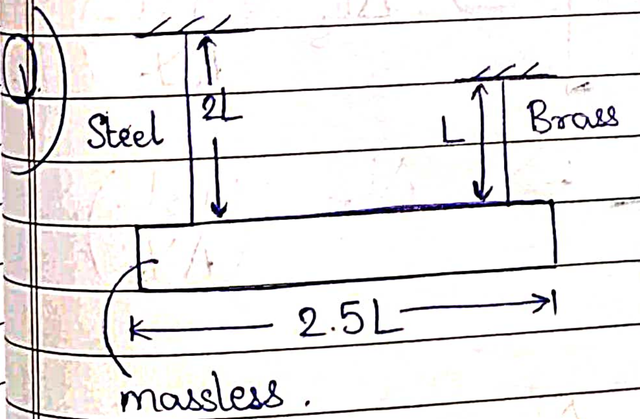
Fnet along centre provides centripetal force.

$\Rightarrow \quad 2F \sin\Delta\theta = m\omega^2 R \approx 2F(\Delta\theta)$

$\Rightarrow \quad \rho A R(2\Delta\theta)\,\omega^2 R = 2F(\Delta\theta)$

$\Rightarrow \quad \left(\frac{F}{A}\right) = \rho\omega^2R^2 \leq \sigma \Rightarrow \boxed{\omega \leq \sqrt{\frac{\sigma}{\rho R^2}}}$

Q)



Young's modulus of Steel $= Y_s$
Young's modulus of Brass $= Y_B$

Find dist. from centre where force F should be applied to produces same ext^n in both rods.

Consider wires to be very thin, massless & cross section area $= A$.

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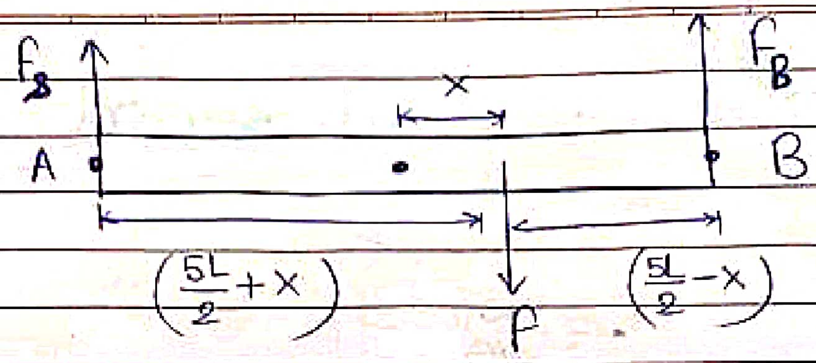
A)



$$\tau_A = 0 \Rightarrow F\left(\frac{5L}{2}+x\right) = f_B\left(\frac{5L}{2}\right)$$

$$\Rightarrow f_B = F\left(1+\frac{2x}{5L}\right)$$

$$\tau_B = 0 \Rightarrow F\left(\frac{5L}{2}-x\right) = f_S\left(\frac{5L}{2}\right)$$

$$\Rightarrow f_S = F\left(1-\frac{2x}{5L}\right)$$

Now, $Y_S = \left[\frac{(\Delta L_S/2L)}{(f_S/A)}\right]^{-1} \Rightarrow \Delta L_S = \left(\frac{2f_S L}{AY_S}\right)$

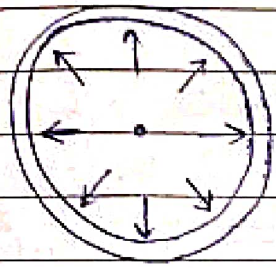
Now, $Y_B = \frac{(f_B/A)}{(\Delta L_B/L)} \Rightarrow \Delta L_B = \left(\frac{f_B L}{AY_B}\right)$

We have, $\Delta L_S = \Delta L_B$

$$\Rightarrow \left(\frac{2L}{A}\right)\left(\frac{1}{Y_S}\right)(F)\left(1-\frac{2x}{5L}\right) = \left(\frac{L}{A}\right)\left(\frac{1}{Y_B}\right)(F)\left(1+\frac{2x}{5L}\right)$$

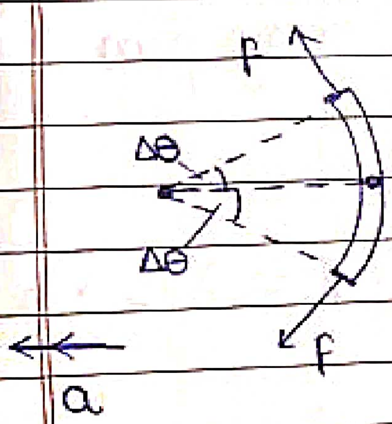
$$\Rightarrow \left(\frac{2Y_B}{Y_S}\right) = \left(\frac{5L+2x}{5L-2x}\right) \Rightarrow \boxed{x = \left(\frac{5L}{2}\right)\left(\frac{2Y_B+Y_S}{2Y_B-Y_S}\right)}$$

Q)



forces applied radially outward & inc. radius by ΔR.

A) Let radius change by 'r'. Consider a small element of this new ring.



$$\left(\frac{\Delta m}{M}\right) = \left(\frac{2\Delta\theta}{2\pi}\right)$$

$$\Rightarrow \left(\frac{\Delta m}{\Delta\theta}\right) = \frac{M}{\pi}$$

$$Y = \left(\frac{\sigma}{\epsilon}\right) = \frac{(F/A)}{(r/R)} \Rightarrow F = \left(\frac{AY}{R}\right) r$$

Now, $a = \left(\frac{2F S_{\Delta\theta}}{\Delta m}\right) \sim 2F\left(\frac{\Delta\theta}{\Delta m}\right)$

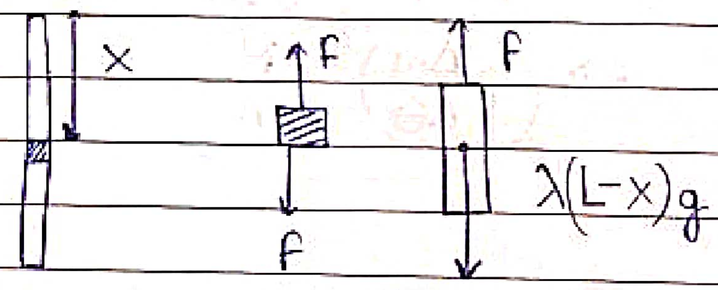
$$\Rightarrow \boxed{\ddot{r} = a = \left(\frac{2\pi}{M}\right)\left(\frac{AY}{R}\right) r}$$

We get $\boxed{\ddot{\vec{r}} \propto (-\vec{r})} \Rightarrow$ S.H.M.!

Apply eqⁿs of S.H.M. to get ans.

Q) Chain :- Breaking Stress $= 2 \times 10^{11}\ N/m^2$
Mass per unit length $= 2\ kg/cm$
Area of Cross Section $= 10^{-2}\ cm^2$
Find max. length that can be hanged w/o breaking.

A) Consider a pt. 'x' dist. away from ceiling.



$$f = \lambda(L-x)g$$

Now, for max. stress $x = 0 \Rightarrow f_{max.} = \lambda g L$

$$\Rightarrow \sigma_{max.} = \left(\frac{\lambda g}{A}\right) L$$

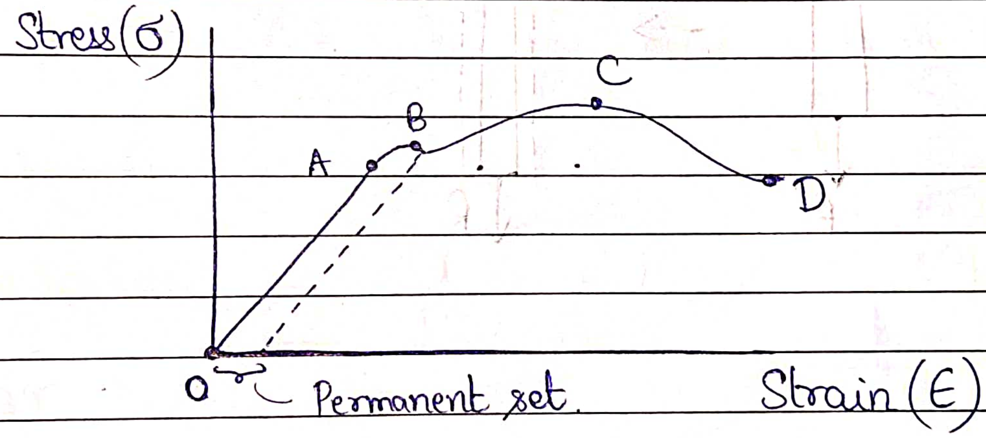
$$\Rightarrow \frac{(2 \cdot 10^{2})(10)\,L}{(10^{-6})} \leqslant (2 \times 10^{11})$$

$$\Rightarrow \boxed{L \leqslant 100\,m}$$

☆ Breaking Stress does NOT depend on length or area of cross section.

It depends on material.

Stress - Strain Curve

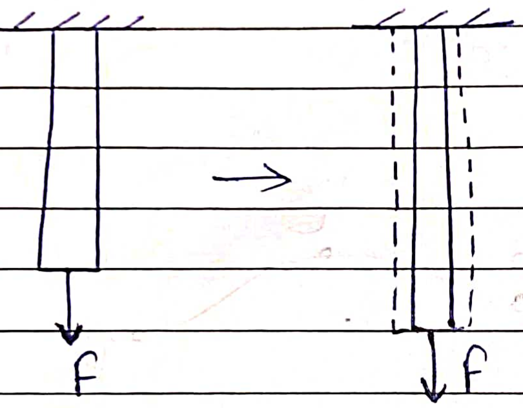


A - Proportionality limit
B - Elastic limit
C - Breaking Stress
D -

Poisson's Ratio

When obj. extendend, its cross section area decreases.

Let obj. extend by ΔL & its cross section radius dec. by ΔR.



☆ Q) Wire :- $\sigma = 0.5$. (% inc. in length) = 2%.

Find change in volume.

A) $V = \pi R^2 L$ & $(V+\Delta V) = \pi(R+\Delta R)^2(L+\Delta L)$

$$\Rightarrow \Delta V = \pi(R+\Delta R)^2(L+\Delta L) - \pi R^2 L$$

$$\Rightarrow \Delta V = (\pi R^2 L)\left(1+\left(\frac{\Delta R}{R}\right)\right)^2\left(1+\frac{\Delta L}{L}\right) - \pi R^2 L$$

$$\Rightarrow \quad \Delta V \approx (\pi R^2 L)\left(1 + 2\left(\frac{\Delta R}{R}\right)\right)\left(1 + \frac{\Delta L}{L}\right) - \pi R^2 L$$

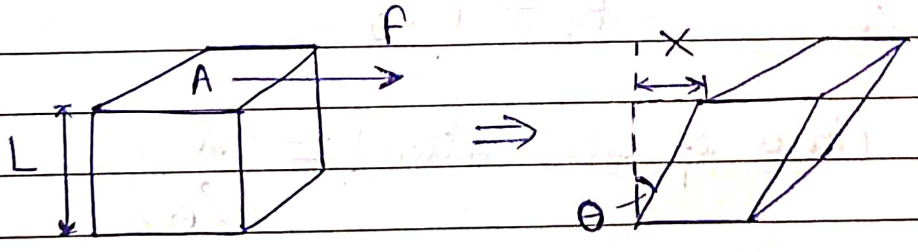
$$\Rightarrow \quad \Delta V = (\pi R^2 L)\left(2\left(\frac{\Delta R}{R}\right) + \frac{\Delta L}{L}\right) \Rightarrow \Delta V = 0$$

$$\sigma = 1/2 = \frac{-(\Delta R/R)}{(\Delta L/L)} \Rightarrow 2\left(\frac{\Delta R}{R}\right) + \left(\frac{\Delta L}{L}\right) = 0$$

Alternate : $V = \pi R^2 L \Rightarrow \left(\frac{\Delta V}{V}\right) = 2\left(\frac{\Delta R}{R}\right) + \left(\frac{\Delta L}{L}\right)$

$$\Rightarrow \Delta V = 0$$

Shear Stress



$(\text{Shear Stress}) = \left(\frac{F}{A}\right)$ $\qquad (\text{Shear Strain}) = \theta$

Since θ small $\Rightarrow$ $\boxed{\theta = x/L}$

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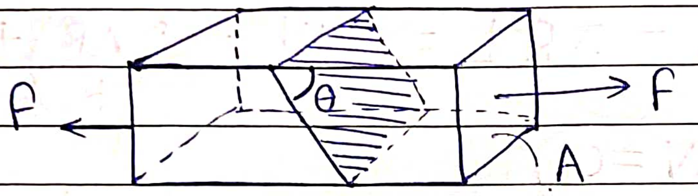
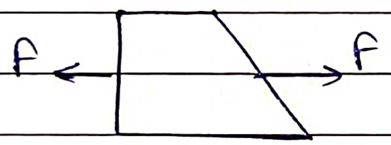
Modulus of Elasticity / Rigity:

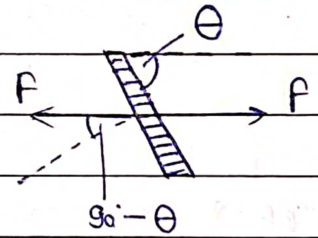
Shear Modulus

$$\boxed{\eta = \left(\frac{\text{Shear Stress}}{\text{Shear Strain}}\right) = \frac{(F/A)}{\theta}}$$

Q) 

Find shear & longitudnal stress.

A) 



$F_{\perp} = F s_\theta$, $F_{\parallel} = F c_\theta$

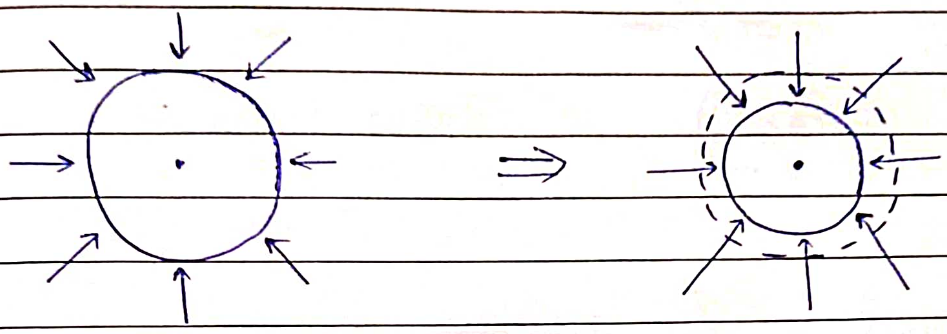
Now, $(\text{Area of Cross Section}) = \left(\frac{A}{s_\theta}\right)$

$$(\text{Shear Stress}) = \cancel{F_{\parallel}}\ \frac{(F c_\theta)}{(A/s_\theta)} = \boxed{\left(\frac{F}{A}\right) s_\theta c_\theta}$$

$$(\text{Longitudnal Stress}) = \frac{(F s_\theta)}{(A/s_\theta)} = \boxed{\left(\frac{F}{A}\right) s^2_\theta}$$

Bulk Stress

Consider a body immersed in fluid applying pressure P on it.



$(\text{Bulk Stress}) = P$ $\qquad$ $(\text{Bulk Strain}) = -\left(\frac{\Delta V}{V}\right)$

$$\boxed{B = \frac{(\text{Bulk Stress})}{(\text{Bulk Strain})} = \frac{P}{(-\Delta V/V)}}$$

Bulk Modulus