DATE

PAGE

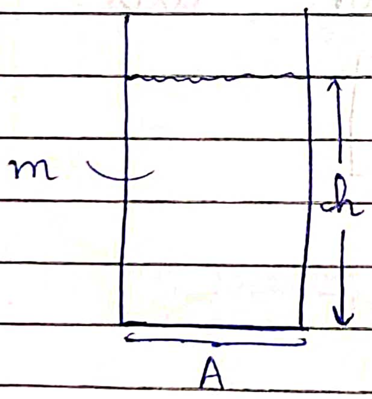
Fluids

Pressure

$$\boxed{P = \left(\frac{F_\perp}{A}\right)} \quad (F_\perp \text{ is force } \perp \text{ to area})$$

Hydrostatic Pressure

Pressure due to liq. column of height 'h'



(Force at Base)

$$= mg = (\rho V) g$$

$$= (\rho A h) g$$

$$\Rightarrow P = F/A$$

$$\Rightarrow \boxed{P = \rho g h}$$

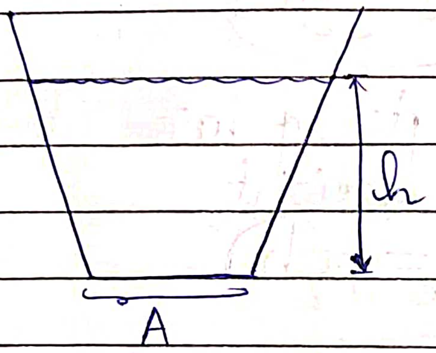
due to liq.

$$(\text{Total Prsre at Base}) = \boxed{P_{hydro.} + P_o}$$

atm prsre.

☆ Prsre is ALWAYS Normal to ~~surface~~ surface.

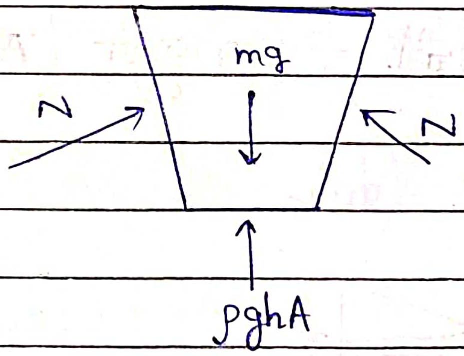
In reality, P is tensor. But we will assume it to be scalar.



(force at base)

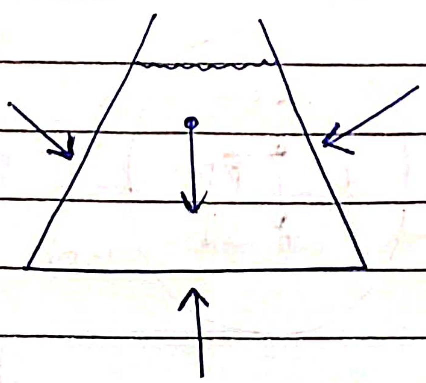
$= PA = \rho ghA \neq mg$

FBD of liq.



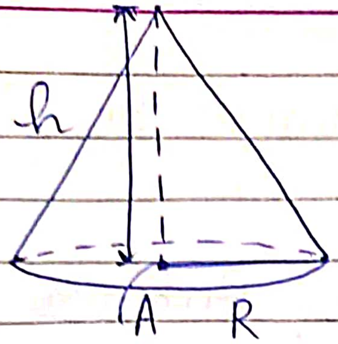
force at base is LESS than 'mg' as normal from walls ~~exerts~~ have upward component.

Similarly,



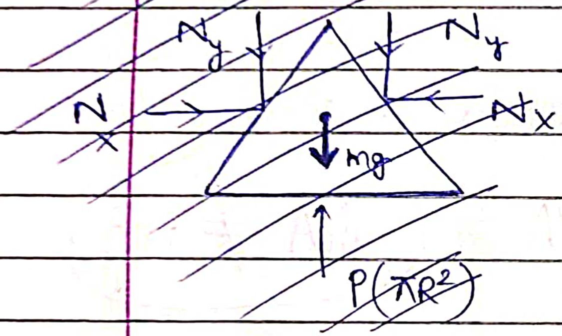
(Force at base) GREATER than 'mg' as Normal has downward component.

Q)



find net force due to side walls.

A) ~~FBD of Liq.~~



Since no wall above pt. A,

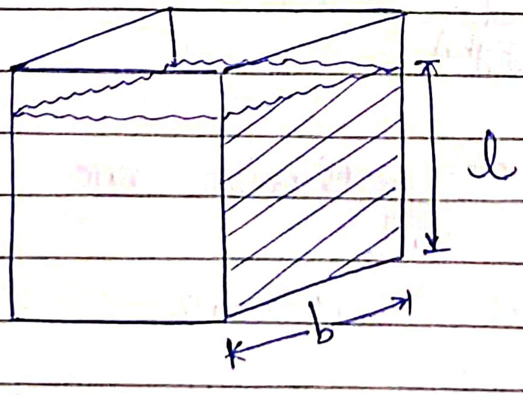
$$P_A = \rho g h$$

Since all pts. of base at same height,

$$P_{base} = \rho g h$$

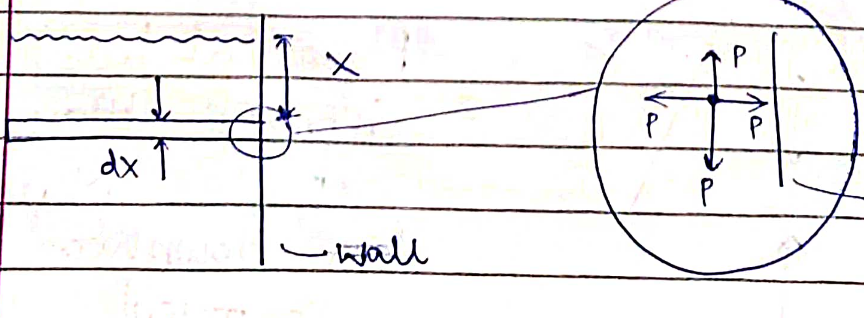
Now, $N_{wall} = (-mg) + (\pi R^2)(\rho g h) \Rightarrow \boxed{N = \frac{2mg}{3}}$

Q)



find ~~press~~ force on wall due to liq.

A)

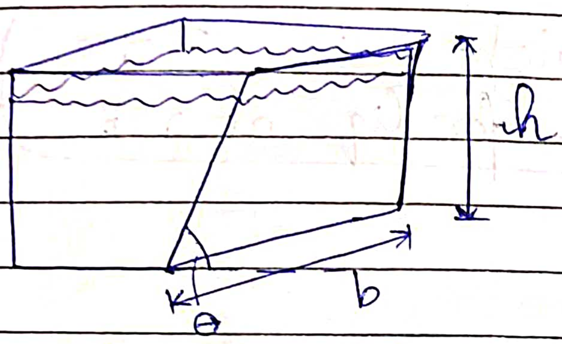


~~$P = \rho g h$~~

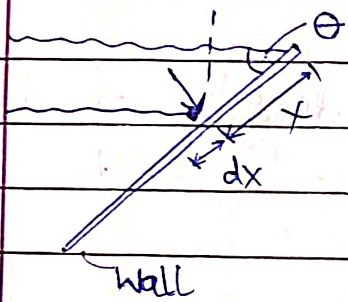
$$dP = \rho g x$$

$$dF = \rho g x (b\,dx)$$

$$\Rightarrow \quad f = \int_0^{l} (\rho g b)\, x\, dx \quad \Rightarrow \quad \boxed{f = \frac{1}{2}\rho g b l^2}$$

Q) 

find force on wall due to liq.

A) 

$$dP = \rho g x \sin(\theta)$$

$$\Rightarrow \quad df = \rho g x\, s_\theta \cdot b\, dx$$

$$\Rightarrow \quad df = (\rho g s_\theta b)\, x\, dx$$

$$\Rightarrow \quad f = (\rho g b s_\theta)\left[\frac{x^2}{2}\right]_0^{(h/s_\theta)}$$

$$\Rightarrow \quad \boxed{f = \cancel{\rho g b s_\theta}\left(\frac{\rho g b h^2}{2 s_\theta}\right)}$$

If we include P_0, (assume air only above), $\quad dP = \rho g x s_\theta + P_0$

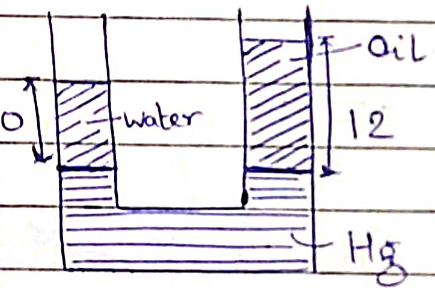
$$\Rightarrow \quad df = (\rho g x s_\theta + P_0)\, b\, dx$$

$$\Rightarrow \quad \boxed{f = \left(\frac{\rho g b h^2}{2 s_\theta}\right) + \left(\frac{P_0 b h}{s_\theta}\right)}$$

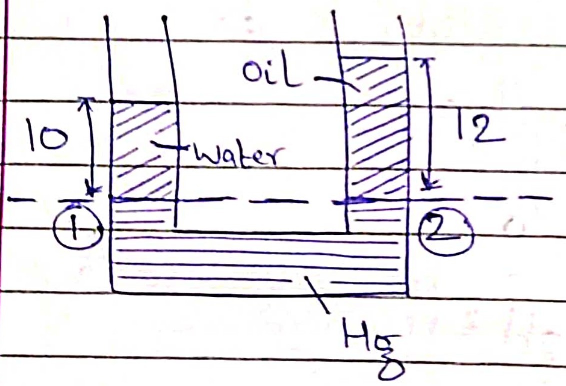
Pascal's Law

Applicable for liq. at rest.

☆ $\boxed{\text{Prsres at same height in same liq. are same. for liq. at rest}}$

Q) 

find rel. density of oil.

A) 

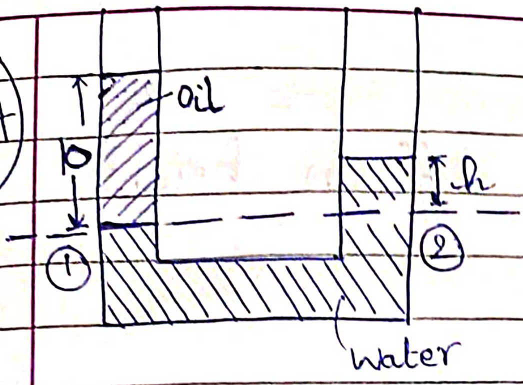
$P_1 = P_2$

$\Rightarrow \rho_w g(10) = \rho_o g(12)$

$\Rightarrow \boxed{(\text{Rel. Density}) = \frac{5}{6}}$

Q) 10 cm of oil $(\rho_{rel} = 0.8)$ is added to U tube filled with water. find diff. in water level.

A)

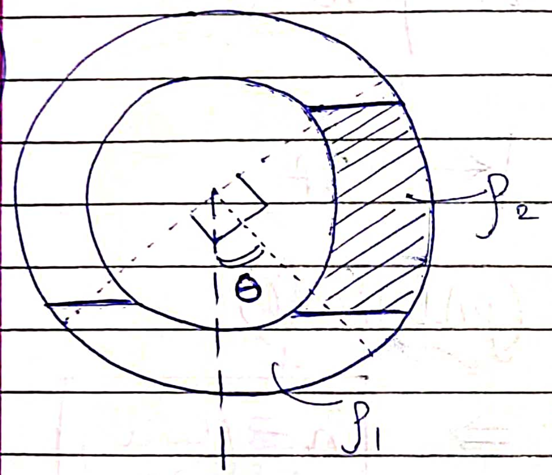


$$P_1 = P_2$$

$$\Rightarrow 10(0.8)\cancel{(10)}\cancel{g} = 1\cdot \cancel{g}\cdot h$$

$$\Rightarrow \boxed{h = 8}$$

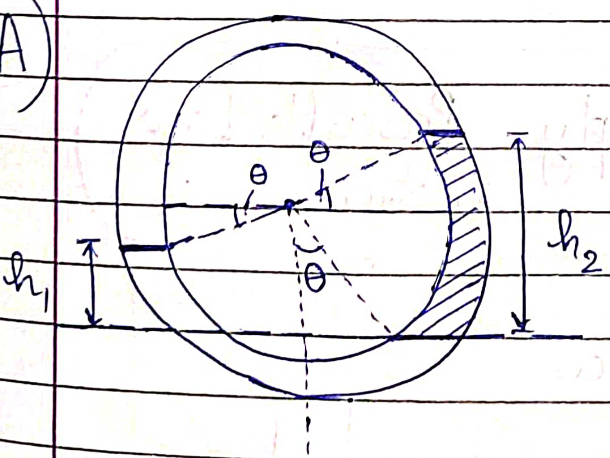
Q)



find $\tan(\theta)$.

$(\rho_1 > \rho_2)$

A)



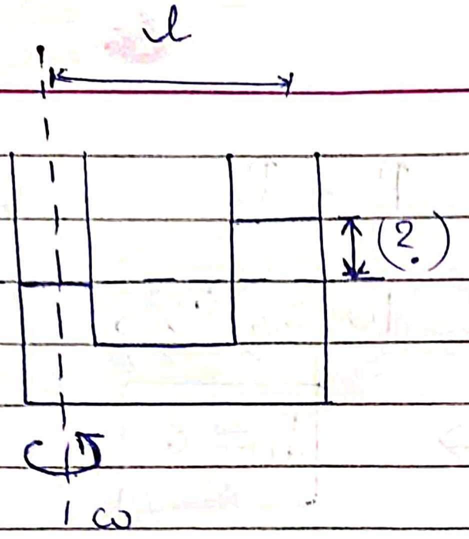
$$h_1 = R(c_\theta - s_\theta)$$

$$h_2 = R(c_\theta + s_\theta)$$

$$\rho_1 g h_1 = \rho_2 g h_2$$

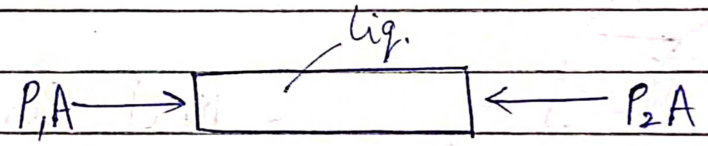
$$\Rightarrow \left(\frac{\rho_1}{\rho_2}\right) = \left(\frac{h_2}{h_1}\right) = \left(\frac{1+t_\theta}{1-t_\theta}\right)$$

$$\Rightarrow \boxed{t_\theta = \left(\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\right)}$$



☆ Q) find diff. in height.

A) Make FBD of horiz. part

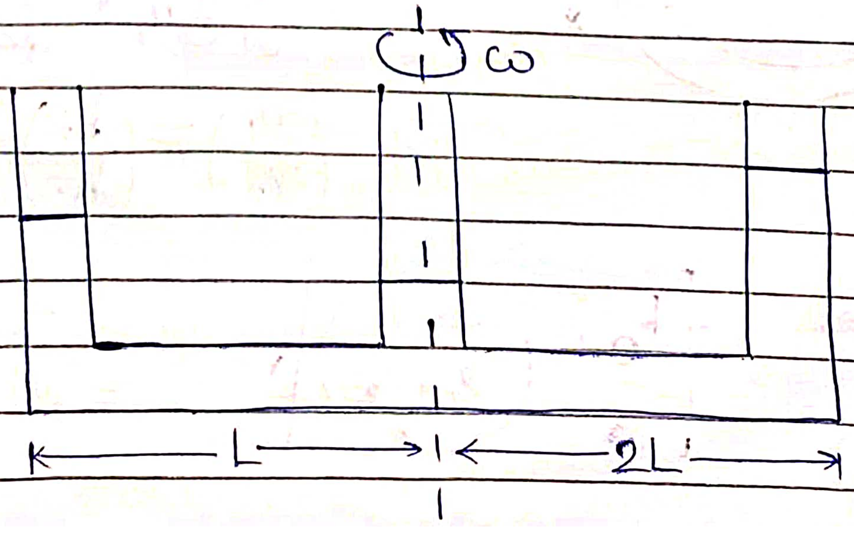


$$(P_2 - P_1)A = m\omega^2 \frac{l}{2} = (\rho A l)\left(\frac{\omega^2 l}{2}\right)$$

$$\Rightarrow \quad \rho g h A = \frac{\rho A \omega^2 l^2}{2} \Rightarrow \boxed{h = \left(\frac{\omega^2 l^2}{2g}\right)}$$

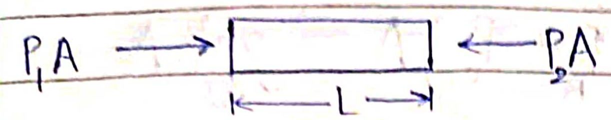
☆ Here, we can't apply Pascal's Law as liq. is in motion.

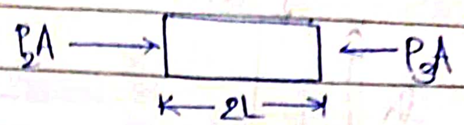
Q)



find diff. in height of extreme columns.

A)





$(P_1 - P_2)A = m\omega^2(L/2)$

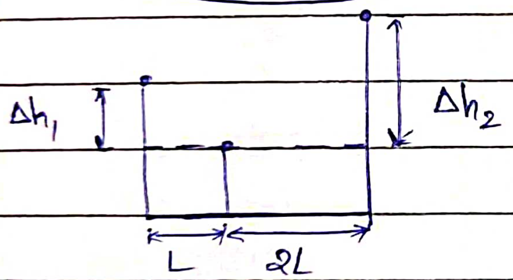
$\Rightarrow (\rho g \Delta h_1) A = (\rho A L)\omega^2\left(\frac{L}{2}\right)$

$\Rightarrow \Delta h_1 = \left(\frac{\omega^2 L^2}{2g}\right)$

$(P_3 - P_2)A = m\omega^2(L)$

$\Rightarrow (\rho g \Delta h_2)A = \rho A(2L)\omega^2 L$

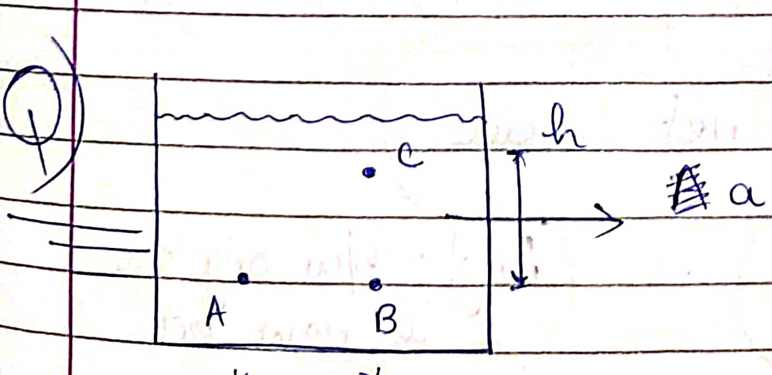
$\Rightarrow \Delta h_2 = \left(\frac{2\omega^2 L^2}{g}\right)$



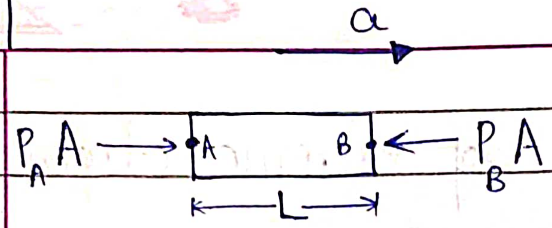
$\Delta h = (\Delta h_2 - \Delta h_1)$

$\Rightarrow$ $\boxed{\Delta h = \left(\frac{3\omega^2 L^2}{2g}\right)}$

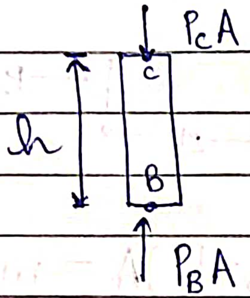
Q)



find $(P_A - P_C)$

A) 

$(P_A - P_B)A = (\rho L A)a$

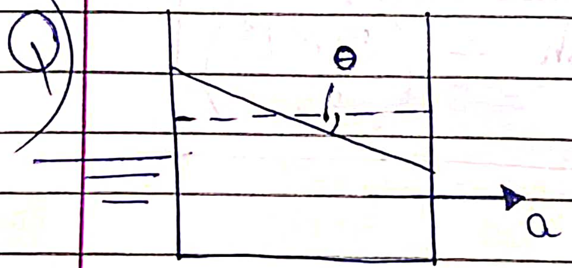


$(P_B - P_C)A = \rho g h A$

$\Rightarrow (P_A - P_B) = \rho L a$

& $(P_B - P_C) = \rho g h$

$\Rightarrow \boxed{(P_A - P_C) = \rho g h + \rho L a}$

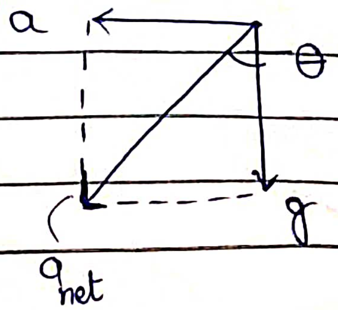
Q) 

find θ.

A) ☆ fluid surface ⊥ to net acc.

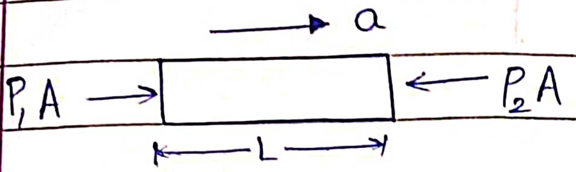
(Angle b/w orig surface & new surface) = (Angle b/w orig acc & new acc.)

Non inertial frame



$\boxed{\theta = \tan^{-1}(a/g)}$

Alternative : Take FBD of ~~box~~ bottom.



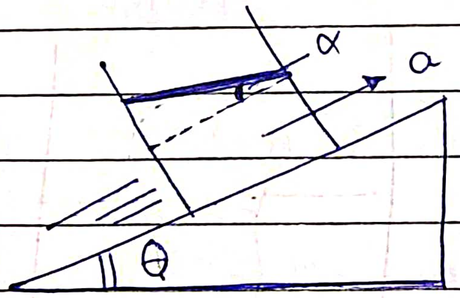
$$(P_1 - P_2)A = (\rho L A)\, a$$

$$\Rightarrow \quad \Delta P = \rho L\, a$$

$$\Delta P = \rho g\, \Delta h \quad \Rightarrow \quad \left(\frac{\Delta h}{L}\right) = \left(\frac{a}{g}\right) = \tan(\theta)$$

$$\Rightarrow \quad \boxed{\theta = \tan^{-1}(a/g)}$$

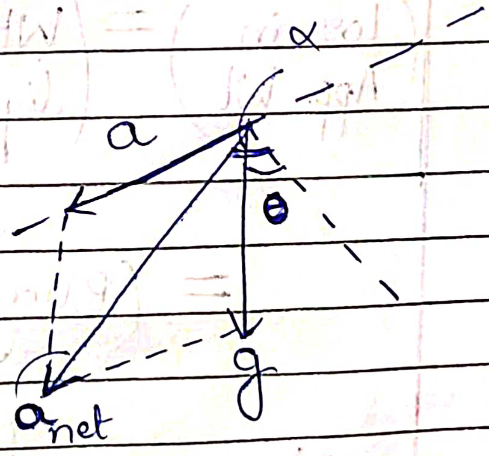
Q)



find $\tan(\alpha)$

A) In non inertial frame,

$$\boxed{t_\alpha = \left(\frac{g s_\theta + a}{g c_\theta}\right)}$$

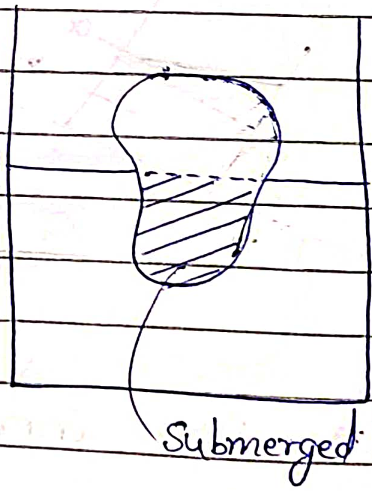


α is angle b/w a_{net} & normal to wedge.

Alternative: Net force on molecules along surface is zero.

Archimedes Principle

When obj. dipped in liq. there is app. loss in wt.



$$\boxed{\begin{pmatrix}\text{Loss in} \\ \text{App. Wt.}\end{pmatrix} = \begin{pmatrix}\text{Wt. of} \\ \text{liq. disp.}\end{pmatrix}}$$

$$= \left(\text{Buoyant force}\right)$$

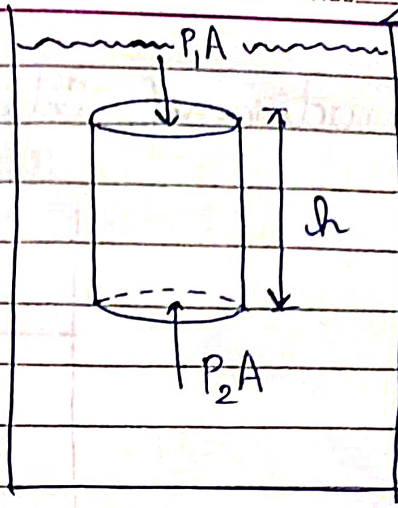
$$\Rightarrow \quad \boxed{F_B = \rho_L V_{\substack{\cancel{\text{sub}} \\ \text{disp.}}} \, g}$$

F_B: Buoyant force

ρ_L: ρ of liq.

$V_{disp.}$: Vol. of liq. disp ~~sub~~

$$f_{net\,(up)} = (P_2 - P_1)A$$
$$= \rho g h A$$
$$\Rightarrow \boxed{f_{net} = \rho g V}$$

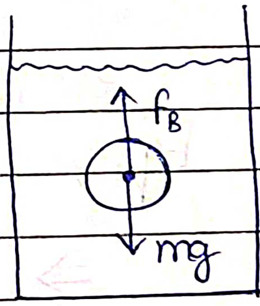


☆ $\boxed{f_B \text{ always up!}}$

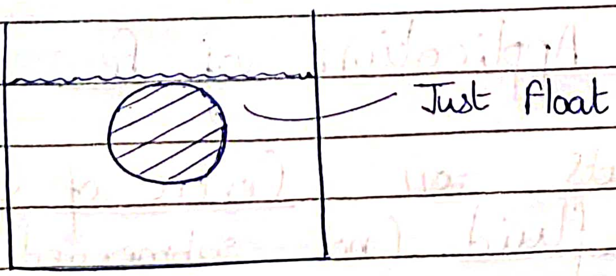
If $mg > (f_B)_{max} \Rightarrow$ Sink

If $mg = (f_B)_{max.} \Rightarrow$ Just Float

If $mg < (f_B)_{max.} \Rightarrow$ Float

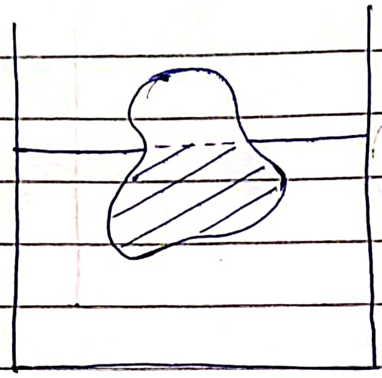


☆ Just Float is Neutral Eq. as $f_{net} = 0$ always.



Floating is Stable Eq.

Fraction of Solid Submerged —



By

$$F_B = \rho_L V_{disp.} g$$

Since Eq., $F_B = mg = \rho_s V g$

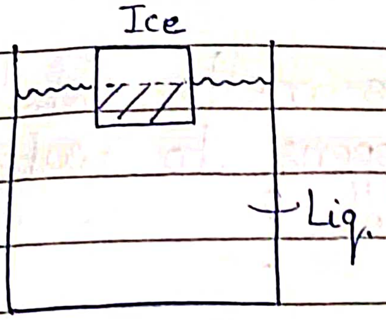
$$\Rightarrow \rho_s V g = \rho_L V_{disp.} g$$

$$\Rightarrow \boxed{\left(\frac{V_{disp.}}{V}\right) = \left(\frac{\rho_s}{\rho_L}\right)}$$

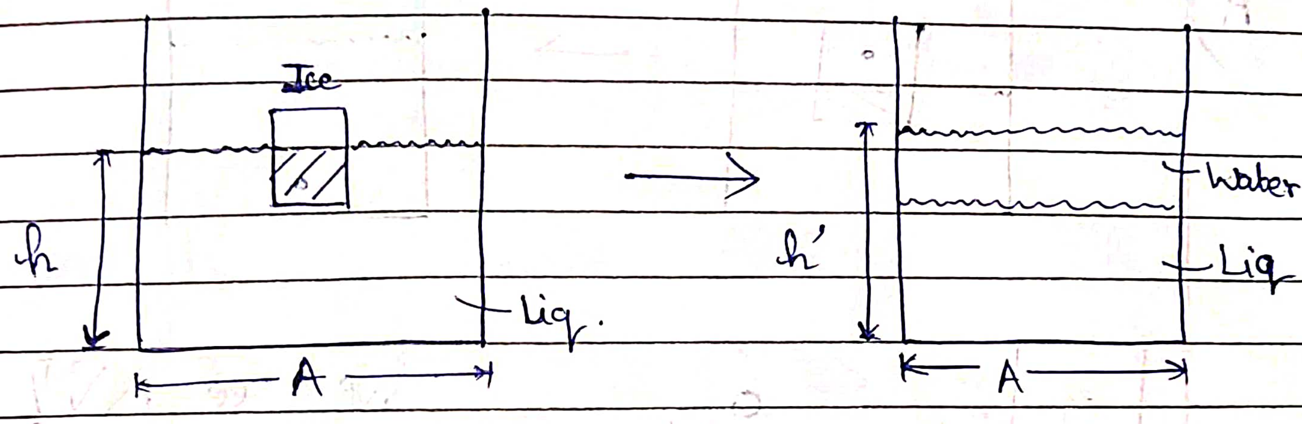
Pt. of Application of F_B —

F_B acts on Centre of Gravity of disp. fluid (not submerged obj.)

Eg :

Q) 

If ice melts, what is the effect on liq. level?
(Assume, $\rho_{water} = \rho_{ice}$)

A) 

$(h' - h)A = (\text{Change in vol})$.

$\Rightarrow \quad (h' - h)A = V - V_{sub}$.

water occupies space $\Rightarrow$ Gain

Sub. part melts $\Rightarrow$ Loss

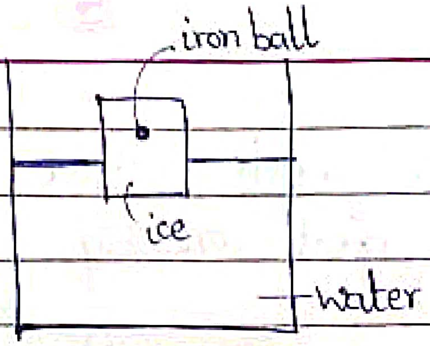
We know, $V\rho_s = V_{sub}\rho_L \Rightarrow V_{sub} = V\left(\frac{\rho_s}{\rho_L}\right)$

$\Rightarrow \quad (h' - h)A = V\left(1 - \frac{\rho_s}{\rho_L}\right)$

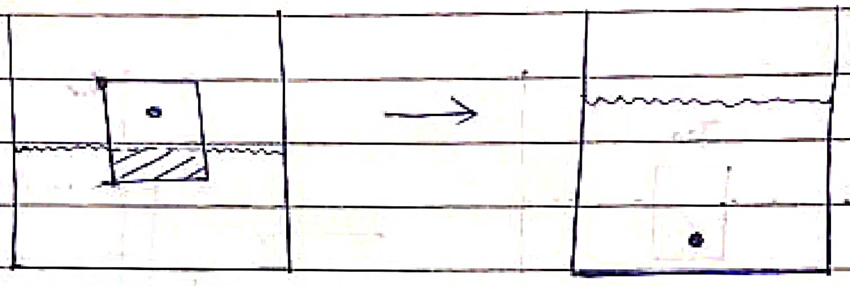
$\Rightarrow \quad \boxed{h' = h + \left(\frac{V}{A}\right)\left(1 - \frac{\rho_s}{\rho_L}\right)}$

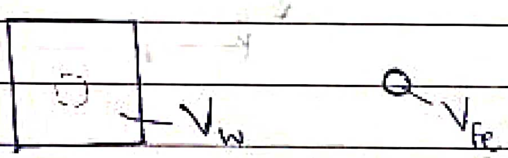
If $\rho_s < \rho_L \Rightarrow$ Level inc.

Since obj. float $\Rightarrow \rho_s < \rho_L \Rightarrow$ $\boxed{\text{Level inc.}}$

Q) 

If ice melts, what happens to water level?

A) 



We see ΔV!

$$\Delta V = V_{Fe} + V_w - V_{sub}$$

$\cancel{\Delta V = V_{Fe} + V_w - V_{sub}}$.

Now, $\rho_w V_{sub}\, g = \rho_{Fe} V_{Fe}\, g + \rho_w V_w\, g$

$$\Rightarrow \quad -\rho_{Fe} V_{Fe} = \rho_w (V_w - V_{sub})$$

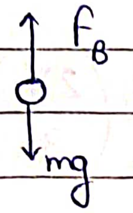
$$\Rightarrow \quad \Delta V = V_{Fe} - \left(\frac{\rho_{Fe}}{\rho_w}\right) V_{Fe}$$

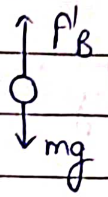
$$\Rightarrow \quad \boxed{\Delta V = V_{Fe}\left(1 - \frac{\rho_{Fe}}{\rho_w}\right) < 0}$$

$\Rightarrow$ Level dec!

☆ If any floating body sinks, then level goes down.

Reason : Before melt After melt





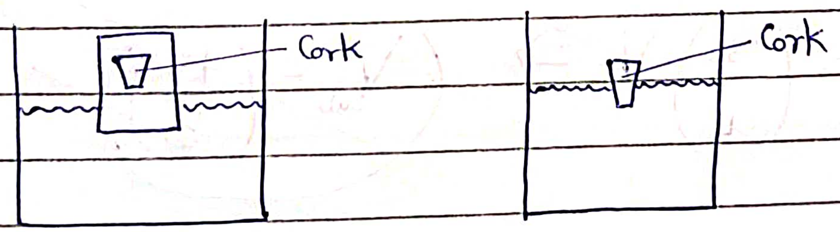
Earlier, $f_B = mg$. Later, $f'_B < mg$.

$(f_B \text{ dec.})$ & $(\rho \text{ \& } g \text{ const.}) \Rightarrow (V_{disp.} \text{ dec})$

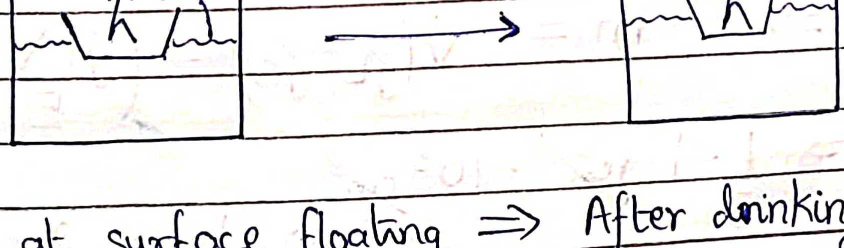
$\Rightarrow$ $\boxed{\text{Level goes down}}$

☆ If any floating body keeps floating, then level remains same.

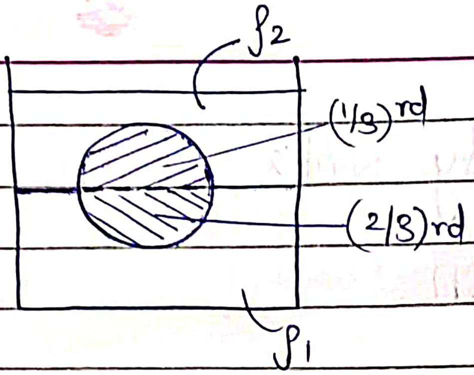
Eg:



Eg:



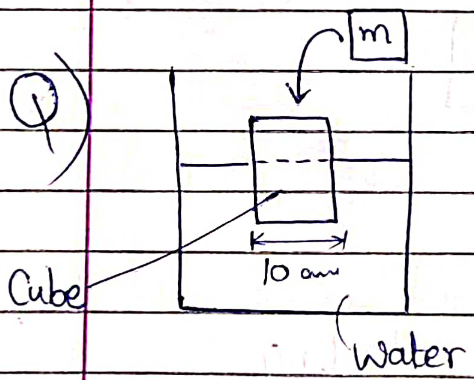
Water at surface floating $\Rightarrow$ After drinking, still float

Q) 

Find density of solid.

A) $\rho_s V g = \rho_2 \left(\frac{V}{3}\right) g + \rho_1 \left(\frac{2V}{3}\right) g$

$$\Rightarrow \quad \boxed{\rho_s = \left(\frac{2\rho_1 + \rho_2}{3}\right)}$$

Q) 

$\rho_{rel} = 0.8$

Init. obj. float. Find 'm' to be put on it s.t. it fully submerge.

A) $\cancel{V_{sub} = \left(\frac{\rho_S}{\rho_L}\right) V \Rightarrow V_{sub} = \left(\frac{4V}{5}\right)}$

After putting 'm', $\rho_L V g = \rho_s V g + mg$

$$\Rightarrow \quad m = V(\rho_L - \rho_s) = \left(\frac{\rho_L V}{5}\right)$$

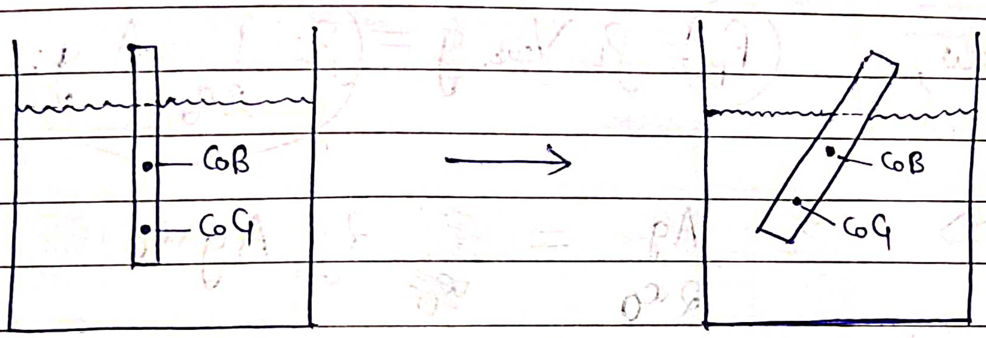
$$\Rightarrow \quad m = \frac{1}{5} \cdot 1\ g\,\cancel{cm^{-3}} \cdot 10^3\ \cancel{cm^3}$$

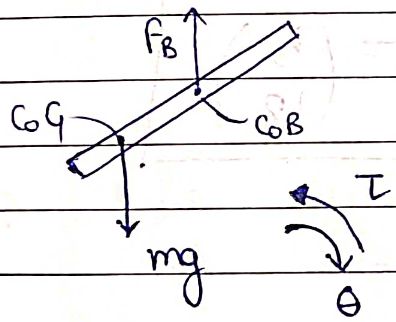
$$\Rightarrow \quad \boxed{m = 200\,g}$$

Equilibrium of Floating Solid

(Centre of Buoyancy) = (CoG of Disp. Liq.)

Let's consider a non uniform rod.



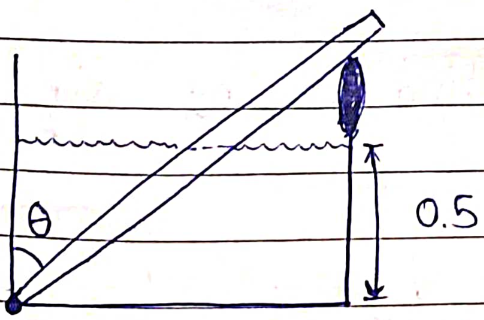


F_B produces τ opp. to disp.

$\Downarrow$

Stable Eq. $\exists$ if CoB above CoG

Q)

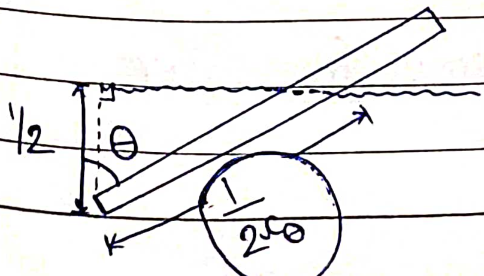


$\rho_{rel} = 0.6$,

Length of Rod = 1

~~$\tan(\theta) = ?$~~ $\cos^2(\theta) = ?$

A)



$V_{sub} = \left(\frac{1}{2\cos\theta}\right)$

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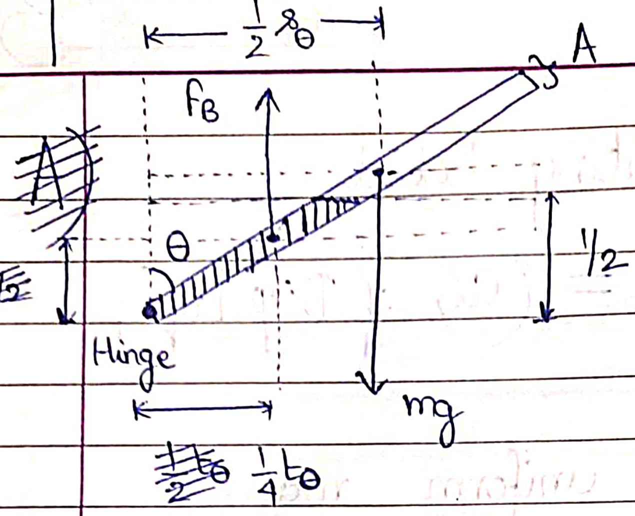


$\tau_{net} = 0$

$\Rightarrow f_B \cdot \frac{1}{4} t_\theta = mg \cdot \frac{1}{2} s_\theta$

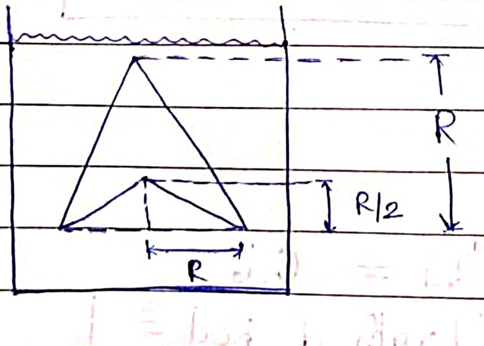
$\Rightarrow \boxed{f_B = \left(\frac{2mg}{\cancel{c_\theta}}\right) c_\theta}$

Now, $f_B = \rho_L V_{sub.} g = \rho_L \cdot \frac{1}{2c_\theta} \cdot A g$

$\Rightarrow \rho_L \cdot \frac{Ag}{2c_\theta} = \cancel{\frac{2}{c_\theta}} 2\rho_s A g c_\theta$

$\Rightarrow c_\theta^2 = \left(\frac{\rho_L}{4\rho_s}\right) \Rightarrow \boxed{c_\theta^2 = \left(\frac{5}{12}\right)}$

Q)



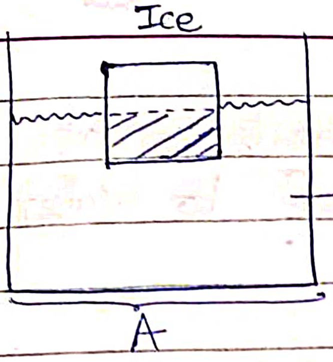
Hollowed cone in eq. in liq.

Find force by liq.

A) ~~Sol~~ By ArP, $\rho_s V = \rho_L V_{sub} = \rho_L V \Rightarrow \rho_s = \rho_L = \rho$ (say)

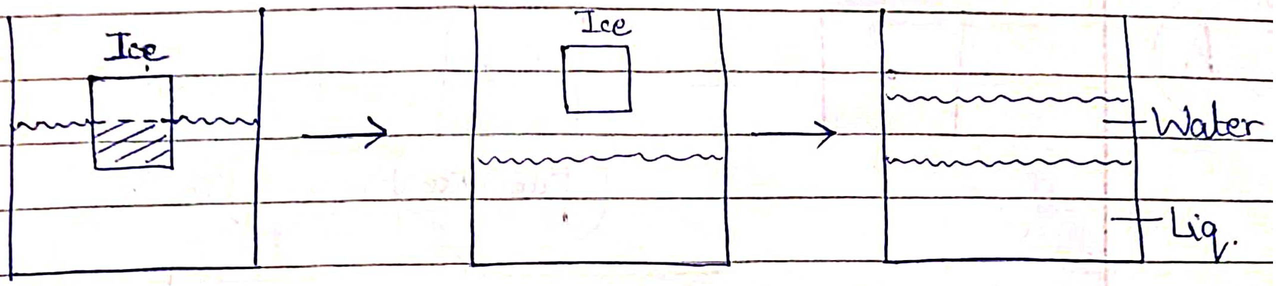
Consider hollowed out cone ~~&~~ water element.

Q)



If ice melts, what is the effect on liq. level?

A)



$$(\Delta h) A = \quad - V_{sub} \qquad\qquad V_w - V_{sub}.$$

Since $m_{ice} = m_w \Rightarrow \rho_{ice} V_{ice} = \rho_w V_w$

By ArP, $\rho_L V_{sub} = \rho_{ice} V_{ice}$

$$\Rightarrow \quad A(\Delta h) = V_w - V_{sub}$$

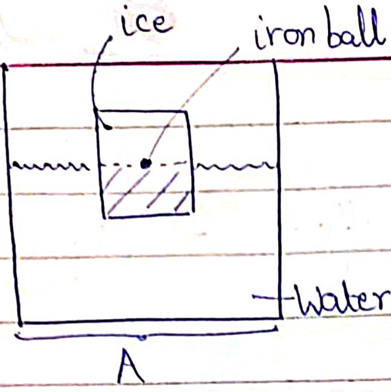
$$= V_w - \frac{\rho_{ice} V_{ice}}{\rho_L} = V_w - \frac{\rho_w V_w}{\rho_L}$$

$$\Rightarrow \quad \boxed{\Delta h = \left(\frac{V_w}{A}\right)\left(1 - \frac{\rho_w}{\rho_L}\right)}$$

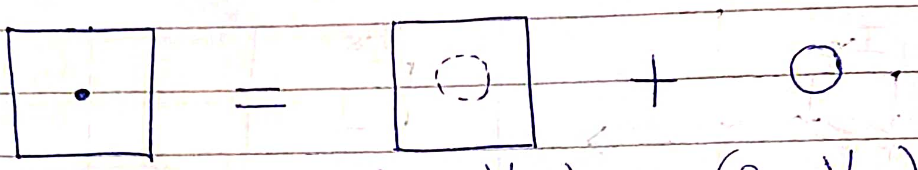
If $\rho_L > \rho_w \Rightarrow$ Level inc.

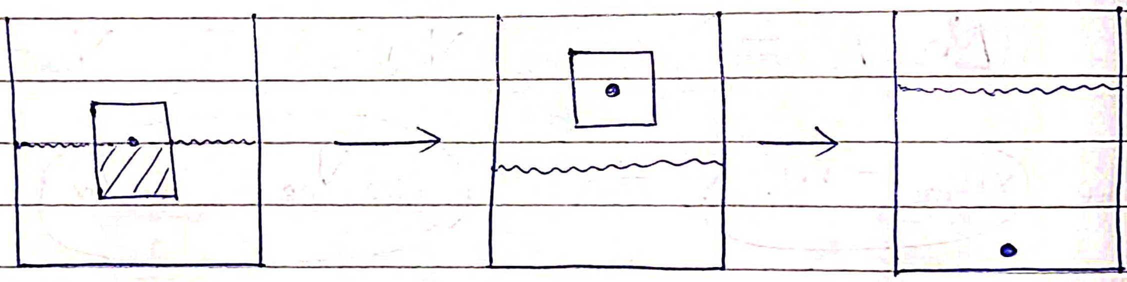
If $\rho_{ice} < \rho_L < \rho_w \Rightarrow$ Level dec.

30

Q) 

If ice melts, what happens to water level?

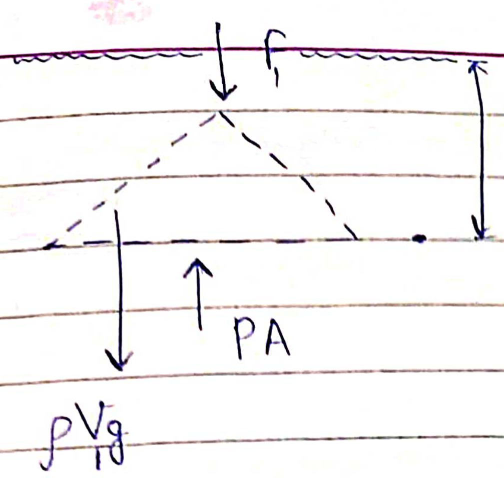
A) 

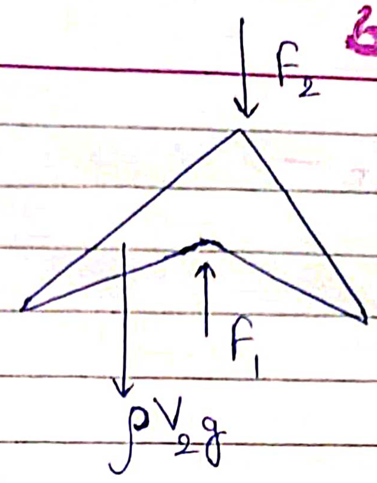


$$A(\Delta h) = -V_{sub} \qquad V_{w,new} + V_{Fe} - V_{sub}$$

Since, $m_i = m_{w,new} \Rightarrow \rho_i V_i = \rho_{w,new} V_{w,new}$

By ArP, $\rho_w V_{sub.} = \cancel{\rho_{w,new}} \rho_i V_i + \rho_{Fe} V_{Fe}$





Now, $V_1 = \frac{1}{3}\pi R^2\left(\frac{R}{2}\right) \Rightarrow V_1 = \pi R^3/6$

& $V_2 = \frac{1}{3}\pi R^2\left(R - \frac{R}{2}\right) \Rightarrow V_2 = \pi R^3/6$

Since eq.,
$$F_1 + \rho V_1 g = PA$$

$$\Rightarrow F_1 + \rho\left(\frac{\pi R^3}{6}\right) g = (\pi R^2)(\rho R g)$$

$$\Rightarrow F_1 = \left(\frac{5\pi R^3 g}{6}\right)\rho$$ — [Force on upper surface]

Also,
$$F_2 + \rho V_2 g = F_1$$

$$\Rightarrow F_2 + \left(\frac{\pi R^3 g}{6}\right)\rho = \left(\frac{5\pi R^3 g}{6}\right)\rho$$

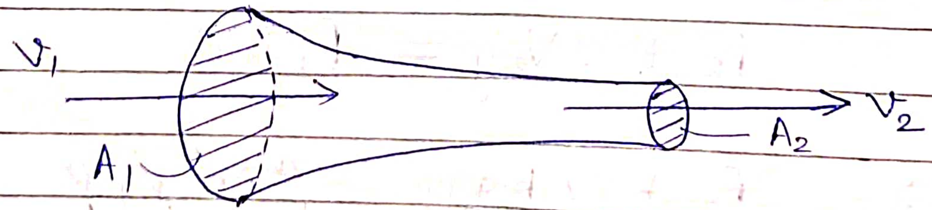
$$\Rightarrow F_2 = \left(\frac{2}{3}\pi R^3 g\right)\rho$$ — [Force on lower surface]

Alternate -

Eqn of Continuity

$$\boxed{\left(\begin{array}{c}\text{Rate of flow}\\ \text{of mass in}\end{array}\right) = \left(\begin{array}{c}\text{Rate of flow}\\ \text{of mass out}\end{array}\right)}$$

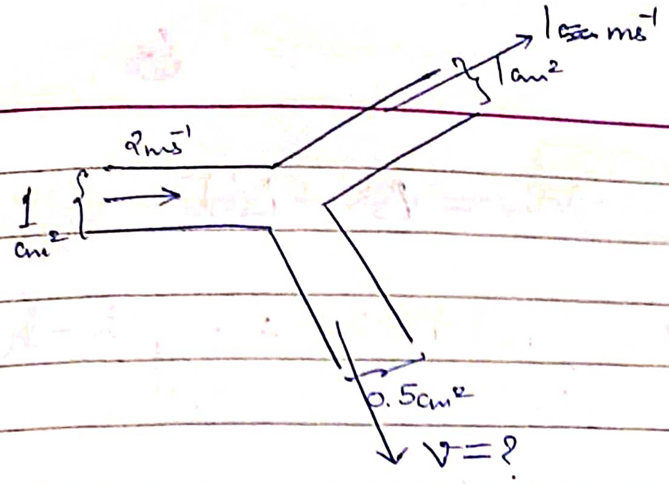
This is due to Consrv. of Mass.



Mass flow Rate $= \left(\frac{m}{t}\right) = \left(\frac{\rho V}{t}\right) = \rho A \frac{l}{t} = \rho A v$

$\Rightarrow \quad \rho A_1 v_1 = \rho A_2 v_2$

$\Rightarrow \quad \boxed{A_1 v_1 = A_2 v_2}$

Q) 

find v.

A) $\left(r_m \ \underline{IN} \right) = \left(r_m \ \underline{OUT} \right)$; r_m = Rate of flow of mass

$$\Rightarrow \rho(1)(2) = \rho(1)(1) + \rho(1/2)(v) \Rightarrow \boxed{v = 2 ms^{-2}}$$

Bernoulli's Theorem

Based on Consrv. of Energy.

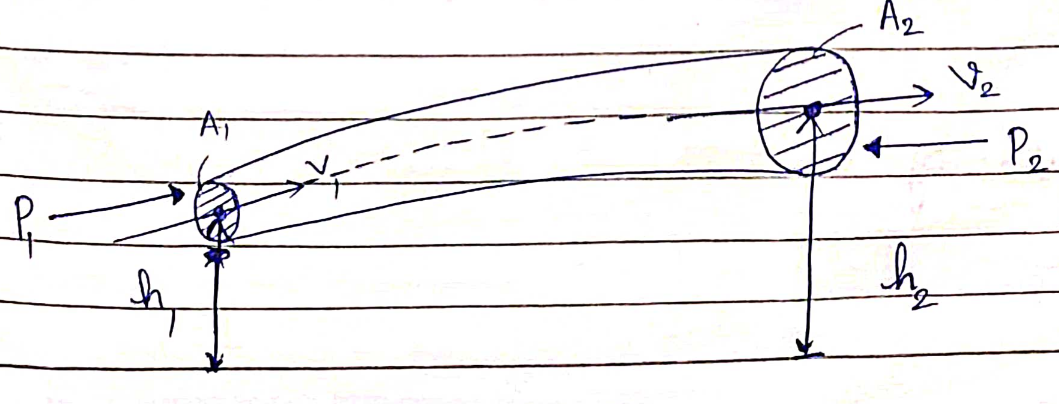
$$\boxed{P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2}$$

P.E. / Vol.

K.E. / Vol.

Pressure Energy / Vol.

Proof:



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By Consrv. of Energy, $W_p = \Delta K + \Delta U$

$$\Rightarrow (P_1 - P_2)\Delta V = \frac{1}{2} m (v_2^2 - v_1^2) + mg(h_2 - h_1)$$

(as $P_1 > P_2$ as liq. from high P to low P)

$$\Rightarrow (P_1 - P_2) = \frac{1}{2}\left(\frac{m}{\Delta V}\right)(v_2^2 - v_1^2) + \left(\frac{m}{\Delta V}\right) g (h_2 - h_1)$$

$$\Rightarrow (P_1 - P_2) = \frac{1}{2}\rho (v_2^2 - v_1^2) + \rho g (h_2 - h_1)$$

$$\Rightarrow \boxed{\underbrace{P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1}_{\text{Just before entering}} = \underbrace{P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2}_{\text{Just after exiting.}}}$$

Corollary : $\boxed{\left(\frac{P}{\rho g}\right) + \left(\frac{v^2}{2g}\right) + h = \text{Const.}}$

Pressure Head Velocity Head

Pressure Head − Height reached by fluid if only P pressure applied.

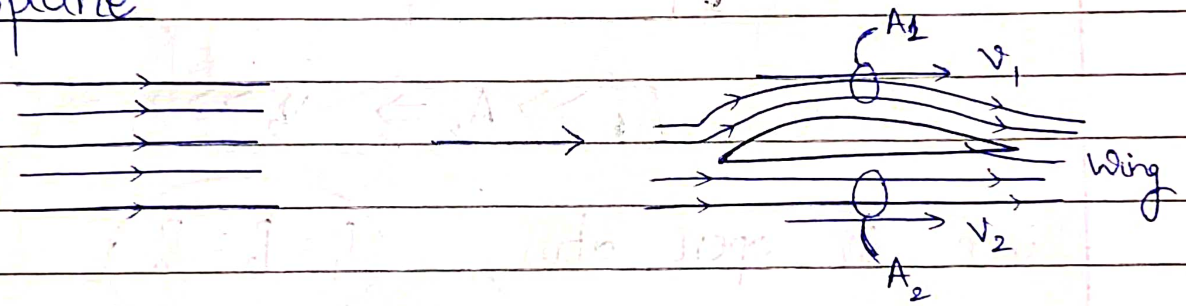
Velocity Head −

If $h = \text{Const.}$ $\Rightarrow$ $\boxed{P + \frac{1}{2}\rho v^2 = \text{Const.}}$

$\Rightarrow$ faster the fluid flows, lower the P!

Application

1) Aeroplane –



Aeroplane wing comes b/w flowing air

$\Downarrow$

Air above wing compresses $\Rightarrow$ $A_1 < A_2$

$\Rightarrow$ $v_1 > v_2$

$\Rightarrow$ $\boxed{(P \text{ above wing}) < (P \text{ below wing})}$

$\Rightarrow$ Plane gets Lift!

If $A_1 = A_2$ $\Rightarrow$ $\text{Lift} = (P_2 - P_1)\,A$ (area of wing's surface)

$\Rightarrow$ $\boxed{\text{Lift} = \frac{1}{2}\rho\,(v_1^2 - v_2^2)\,A}$

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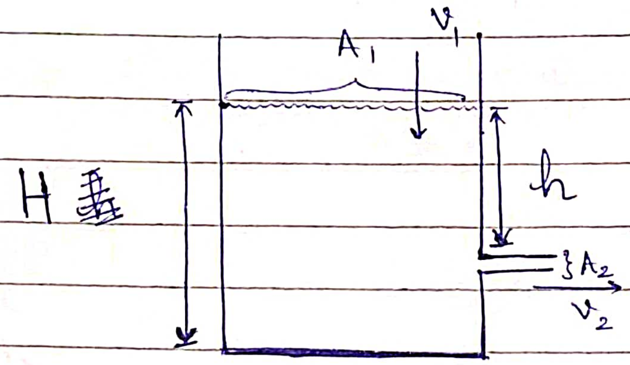
2) Torcelli's Theorem –

Gives velocity of efflux thru small orifice in water container.



We have $A_1 >> A_2 \Rightarrow v_2 >> v_1 \Rightarrow v_1 \sim 0$

Since in open atm, $P_1 = P_2 = P_0$

Applying Bernoulli's Theorem,

$$\cancel{P_1} + \cancel{\frac{1}{2}\rho v_1^2} + \rho g h_1 = \cancel{P_2} + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$\Rightarrow \quad \rho g H = \frac{1}{2}\rho v_2^2 + \rho g(H-h)$$

$$\Rightarrow \quad \boxed{v_2 = \sqrt{2gh}}$$

vel. of efflux

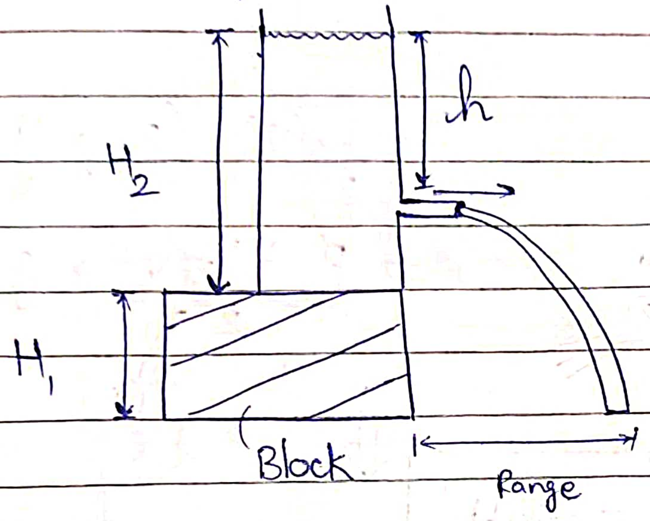
time of flight

$$\text{Range} = v_2 \cdot t = \sqrt{2gh} \cdot \sqrt{\frac{2(H-h)}{g}}$$

$$\Rightarrow \quad \boxed{\text{Range} = 2\sqrt{h(H-h)}}$$

At $h = H/2$, $(\text{Range})_{max.} = H$.

If this config.,

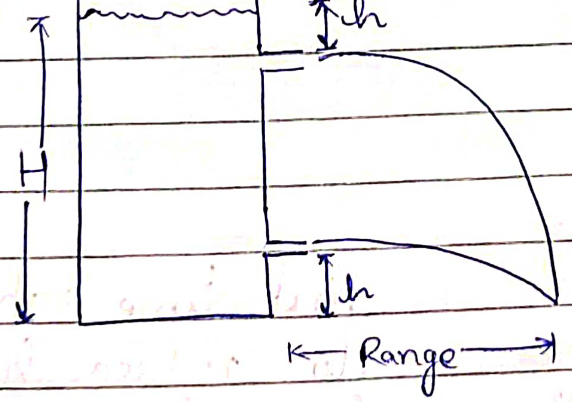


Range is Max. at

$$\boxed{h = \left(\frac{H_1 + H_2}{2}\right)}$$

$$\boxed{(\text{Range})_{max.} = (H_1 + H_2)}$$

If we take 2 pts. one 'h' above bottom & other 'h' below top, then.



Range is Same.

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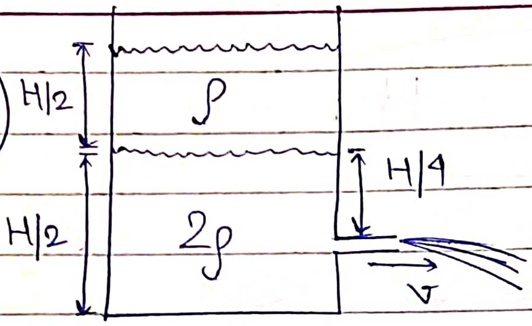
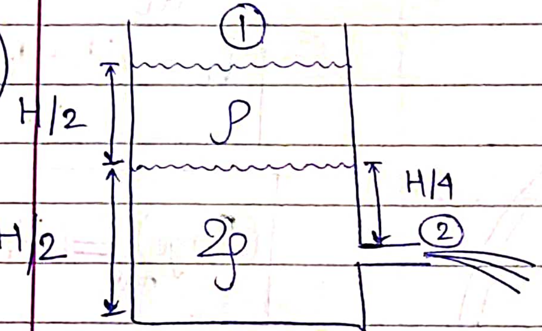
☆ Q)



find v.

A)



$$\cancel{P_1} + \frac{1}{2}\rho \cancel{v_1^2} + \left(2\rho \cdot g \cdot \frac{H}{2} + \rho \cdot g \cdot \frac{H}{2}\right)$$

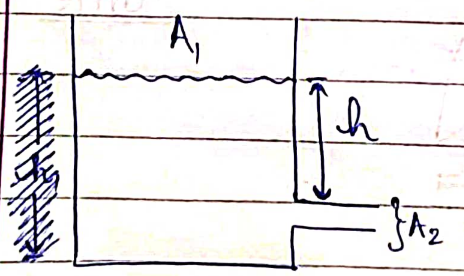
$$= \cancel{P_2} + \frac{1}{2}(2\rho) v_2^2 + 2\rho \cdot g \cdot \frac{H}{4}$$

$$\Downarrow$$

$$\boxed{v_2 = \sqrt{gH}}$$

☆ Observe in P.E./vol. at 1, we have added $\boxed{\rho_{fluid} \cdot (\text{Height of column})_{fluid}}$ for diff. fluids.

Q)



find time taken to reduce height from h_1 to h_2

A) By Bernoulli's Eqⁿ, $\frac{1}{2}\rho(\dot{h})^2 + \rho g h + \cancel{P_0} = \cancel{P_0} + \frac{1}{2}\rho v^2$ —(1)

By Eqⁿ of Continuity, $A_1 \dot{h} = A_2 v$

$$\Rightarrow \quad v = (A_1/A_2)(\dot{h}) \quad \text{— (2)}$$

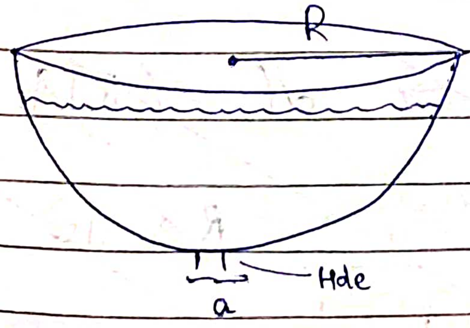
Into (1), $\rho g h + \frac{1}{2}\rho(\dot{h})^2 = \frac{1}{2}\rho\left(\frac{A_1}{A_2}\right)^2(\dot{h})^2$

$$\Rightarrow \quad \dot{h} = -\sqrt{\frac{2gA_2^2}{A_1^2 - A_2^2}}\,\sqrt{h}$$

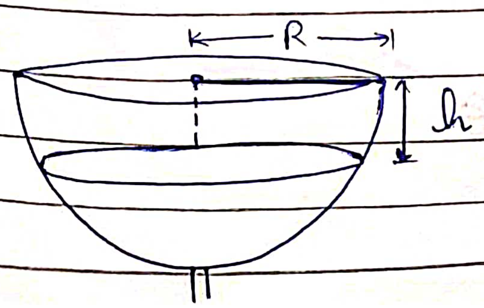
(as h dec.)

$$\Rightarrow \quad \left[2h^{1/2}\right]_{h_2}^{h_1} = \left[\sqrt{\frac{2gA_2^2}{A_1^2 - A_2^2}}\right]_0^t$$

$$\Rightarrow \quad \boxed{t = \left[\frac{2\left(\sqrt{h_1} - \sqrt{h_2}\right)}{\sqrt{2g\left(\frac{A_2^2}{A_1^2 - A_2^2}\right)}}\right]}$$

Q) 

Hemispherical bowl find time to empty bowl.

A) 

Eqⁿ of Continuity –

$$\pi(R^2 - h^2)\,\dot{h} = av$$

Bernoulli's Theorem – $\frac{1}{2}\rho(\dot{h})^2 + \rho g(R-h) = \frac{1}{2}\rho v^2$

$$\Rightarrow \quad (\dot{h})^2 + 2g(R-h) = \left(\frac{\pi^2(R^2-h^2)^2}{a^2}\right)(\dot{h})^2$$

$$\Rightarrow \quad \boxed{\dot{h} = \left(\frac{+\ a\sqrt{2(R-h)}}{\sqrt{(\pi^2)(R^2-h^2)^2 - a^2}}\right)}$$

If small dec. in height $\Rightarrow$ (a << (Area of water surface))

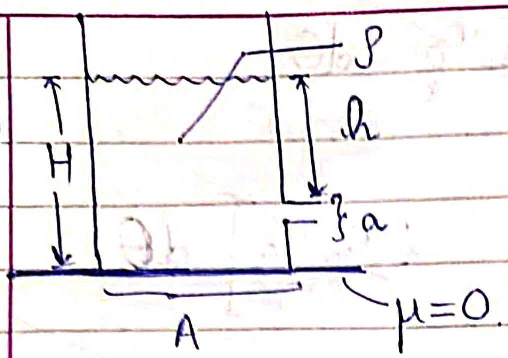
$$\Rightarrow \quad \dot{h} \approx \left(\frac{a\sqrt{2(R-h)}}{\pi(R^2-h^2)}\right)$$

$$\Rightarrow \quad \int_{0}^{h} \frac{(R^2-h^2)}{\sqrt{R-h}}\,dh = \int_{0}^{t} \frac{a\sqrt{2}}{\pi}\,dt$$

Now, $h = R\cos(\theta) \Rightarrow dh = (-R)\,s_\theta\,d\theta$

$$\Rightarrow \quad \left(\frac{a\sqrt{2}}{\pi}\right)t = \int_{\pi/2}^{\phi} \frac{R^2 s_\theta^2}{\sqrt{R}\sqrt{1-c_\theta}} \cdot -R\,s_\theta\,d\theta$$

$$= \left(\frac{R^{5/2}}{\sqrt{2}}\right)\int_{\phi}^{\pi/2} \frac{s_\theta^3}{s_{\theta/2}}\,d\theta$$

Q) 

find init. acc.

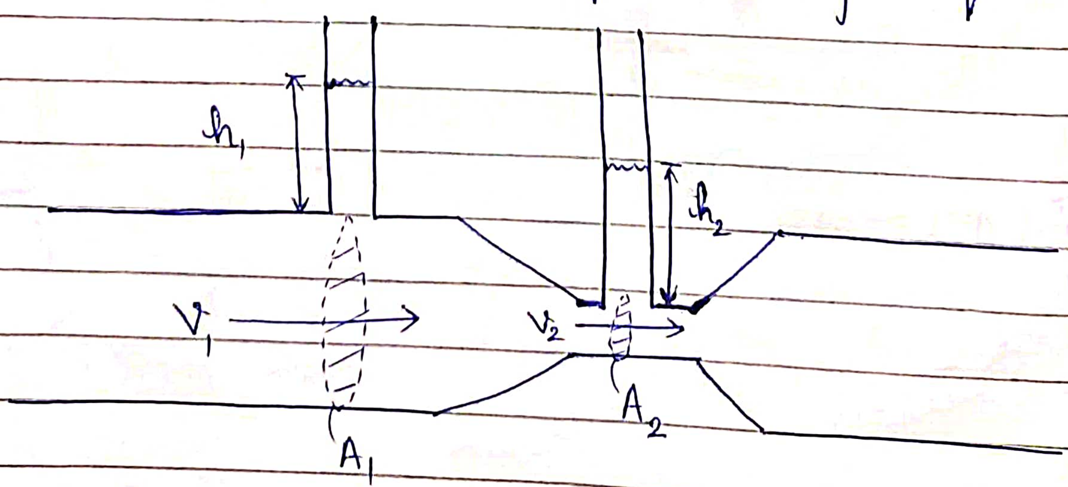
A) $f = v\left(\frac{dm}{dt}\right) = a\rho v^2 \Rightarrow \cancel{\rho A h}(\rho A H)(\ddot{x})$

$= a\rho \cdot 2gh$

$\Rightarrow \boxed{\ddot{x} = 2\left(\frac{a}{A}\right)\left(\frac{h}{H}\right)g}$

3) Venturimeter —

To measure ~~flow~~ rate of ~~rate~~ flow of liq. in a pip



$A_1 v_1 = A_2 v_2$ & $\frac{1}{2}\rho v_1^2 + P_1 = P_2 + \frac{1}{2}\rho v_2^2$

We need 'v_1' as it is vel. inside pipe.

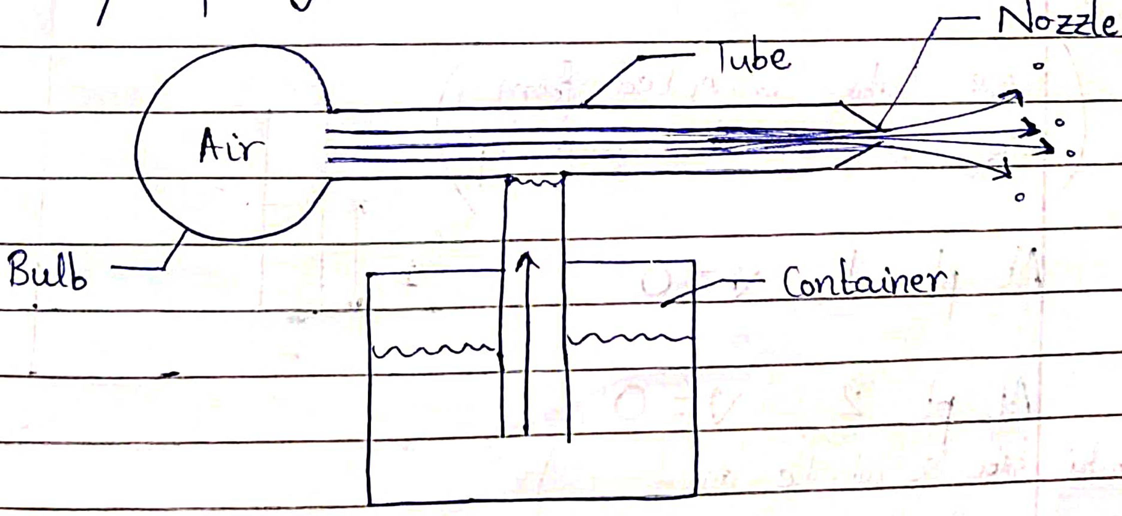
$$\Rightarrow \frac{1}{2}\rho v_1^2 + P_1 = P_2 + \frac{1}{2}\rho \cdot \left(\frac{A_1}{A_2}\right)^2 \cdot v_1^2$$

$$\Rightarrow \frac{1}{2}\rho \left[\left(\frac{A_1}{A_2}\right)^2 - 1\right] v_1^2 = (P_1 - P_2)$$

$$= \rho g (h_1 - h_2)$$

$$\Rightarrow \boxed{v_1 = \sqrt{\frac{2g(h_1 - h_2)}{\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]}}}$$

4) Atomiser / Sprayer —



When bulb pressed $\Rightarrow$ P in Tube reduces.

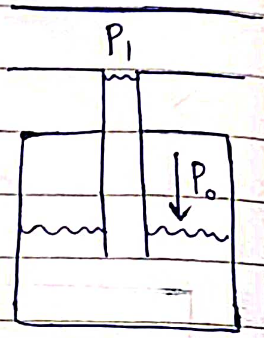
Now, P in container $= P_0$

$\Rightarrow$ Liq. in container moves up $\Rightarrow$ Air carries it in fine droplets

Q) In atomiser tube, $v_{air} = v_a$ & $\rho_{air} = \rho_a$. If liq. filled in container, find vel. when it travels in tube.

A) for air,

$$P_1 + \frac{1}{2}\rho_a v_a^2 = P_0$$



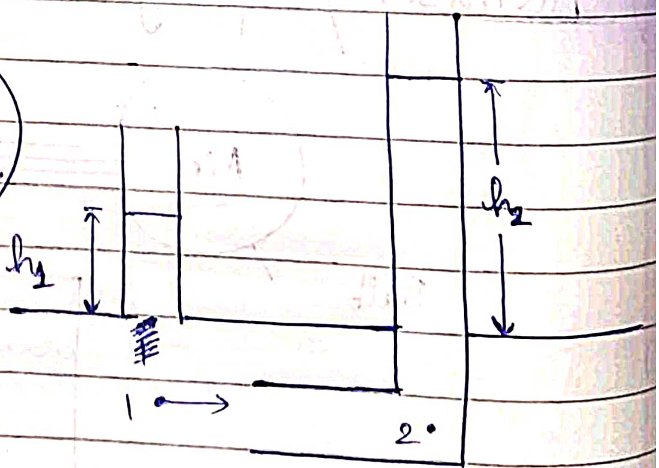
for liq.,

$$P_1 + \frac{1}{2}\rho_L v_L^2 = P_0$$

$$\boxed{v_L = v_a\sqrt{\frac{\rho_a}{\rho_L}}}$$

4) Pitot Tube —

(One tube connected from side, one inserted into liq.)



At pt. 1, $v \neq 0$

At pt 2, $v = 0$

(kyun ki tube se takrake aa raha hai & 90° turn kar raha hai)

By Bernoulli Eqn,

$$\rho g h_1 + \frac{1}{2}\rho v^2 = \rho g h_2$$

$$\Rightarrow \boxed{\frac{1}{2}\rho v^2 = \rho g(\Delta h)}$$

Viscosity

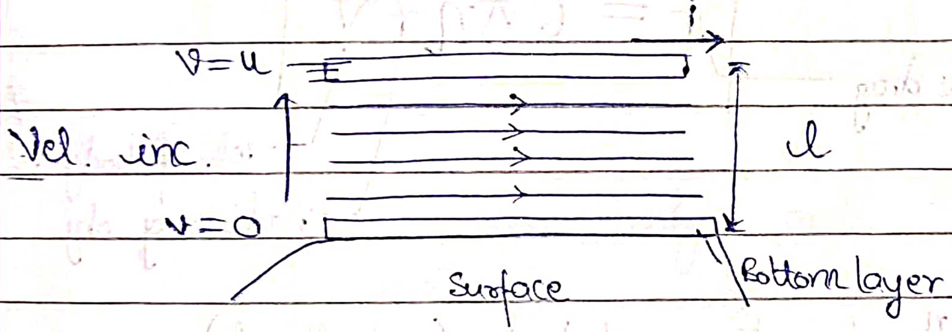
Newton's Law of Viscosity

$$\boxed{f = (-\eta A)\left(\frac{dv}{dx}\right)}$$

Area of contact

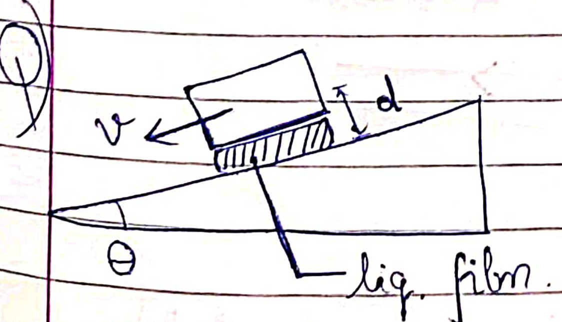
Vel. gradient.

Coeff. of viscosity.



$$\left(\frac{dv}{dx}\right) = \left(\frac{u}{l}\right)$$

Viscous force opposes motion b/w layers of liq.

Q) 

Mass of obj. 'm'.
Area of contact 'A'.
Coef. of viscosity 'η'.
Find ~~....~~ vel if obj. move with const. vel.

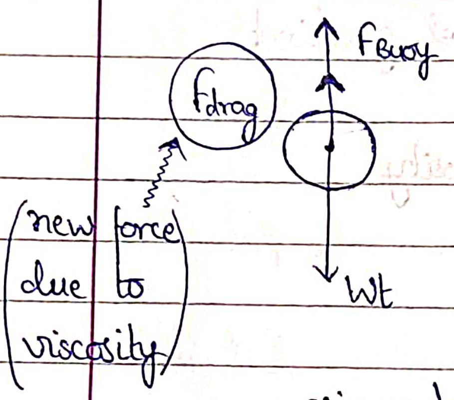
A) Const. vel. ⇒ $f_{net} = 0$ ⇒ $mg\,s_\theta = \eta A (v/d)$

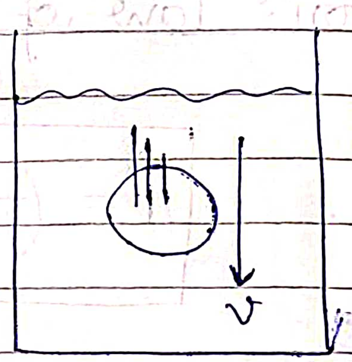
$$\Rightarrow \boxed{v = \left(\frac{mg\,s_\theta\,d}{A\eta}\right)}$$

Stoke's formula

Only for spherical bodies.

If spherical body falls in viscous liq,





$$\boxed{F = 6\pi\eta r v}$$

viscous drag — F

r — radius of obj.

v — vel. of body rel.

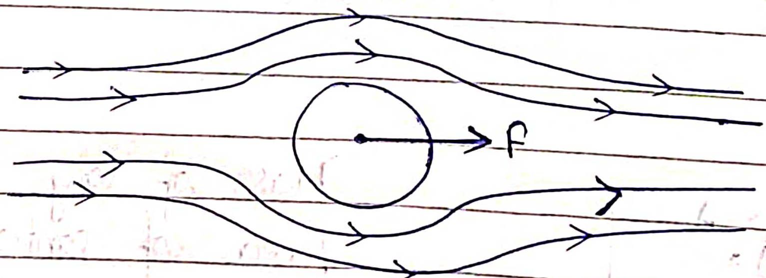
It acts —

Opp. of dirx^n of motion of body (wrt fluid)

OR

In dirx^n of flow of liq.

Eg —



Now, if obj. fall in a long tube

$$F_{net} = mg - F_B - 6\pi\eta r v$$

Since $\left(v\uparrow \Rightarrow f_{drag}\uparrow\right)$ & $\left(mg \text{ \& } f_B = \text{Const.}\right)$,

at some pt. $f_{net} = 0$.

Vel. at that pt. is called <u>terminal vel.</u>

$$\Rightarrow v_T = \left(\frac{mg - f_B}{6\pi\eta r v}\right)$$

$$\left\{m = \frac{4}{3}\pi r^3 \cdot \rho_s\right\}$$

$$\Rightarrow \boxed{v_T = \frac{2}{9} r^2 \frac{(\rho_s - \rho_L) g}{\eta}}$$

(This is when $\rho_s > \rho_L$)

If $\rho_s < \rho_L$, then obj. gain terminal vel. when obj. is moving up.

Now, $f_{net} = m\dot{v} = \left(mg - f_B\right) - 6\pi\eta r v$

$$\Rightarrow m\dot{v} = 6\pi\eta r\left(v_T - v\right)$$

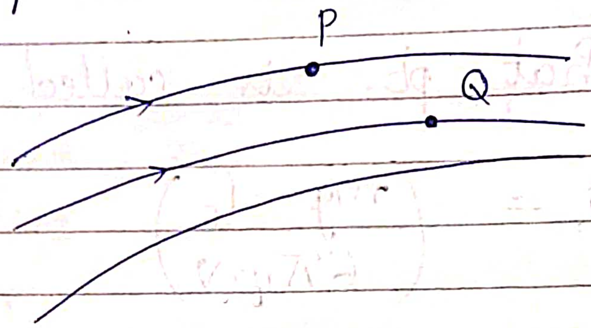
$$\Rightarrow \int_{v_0}^{v(t)} \left(\frac{\dot{v}}{v_T - v}\right) = \int_0^t \left(\frac{6\pi\eta r}{m}\right)$$

~~$v(t) =$~~

$$\Rightarrow \boxed{v(t) = v_T\left[1 - e^{-\left(\frac{6\pi\eta r}{m}\right)t}\right] + v_0\left[e^{-\left(\frac{6\pi\eta r}{m}\right)t}\right]}$$

Types of flow

1) Laminar / Streamline –



v_P may NOT be = to v_Q

Let us pick a fix. pt. P in space.

If vel. of every particle passing thru P is same, then flow at P is said to be laminar.

2) Turbulent –

Flow which is NOT laminar.

Reynold's No.

$$\boxed{N_{Re} = \left(\frac{D\rho v}{\eta}\right)}$$

D – diameter of pipe

ρ – density of liq.

v – vel. of liq.

η – coef. of viscosity of liq.

This is defined for liq. flowing in pipe.

$$N_{Re} < 2000 \iff \text{Laminar Flow.}$$

$$2000 < N_{Re} < 3000 \iff \text{Transition Phase}$$

$$N_{Re} > 3000 \iff \text{Turbulent flow.}$$

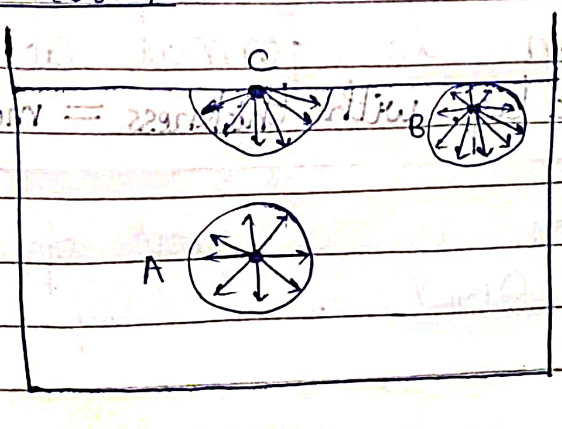
Critical Vel.

Vel. of fluid below which flow is laminar.

for critical vel., $N_{Re} = 2000 = \left(\frac{D\rho v_c}{\eta}\right)$

$$\Rightarrow \boxed{v_c = \left(\frac{2000\,\eta}{D\rho}\right)}$$

Surface Tension



Sphere of influence of pts. A, B, C

If ~~we~~ see attractive forces ~~on~~ b/w particles

$$F_A = 0$$

$$F_B \text{ \& } F_C = \text{Downward.}$$

[If surface area is increased]

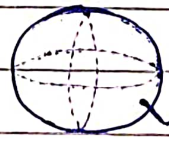
A liq. particle moves from inside liq. (where $F_{net}=0$) to surface (where $F_{net} \neq 0$)

⇒ (It has moved **AGAINST** an opposing force.)

⇒ (In Surface particles, a **POTENTIAL ENERGY** is stored.)

If surface is let free, it will try to minimise its energy.

⇒ (It will try to attain spherical shape (min. surface energy)



Liq. drop.)

Hence, a film is formed at liq. surface.

Uppermost layer of ~~surface~~ liq. with thickness = molecular range is surface

★ Surface Tension is a prop^t of liq. & depends ONLY on liq. & temp.

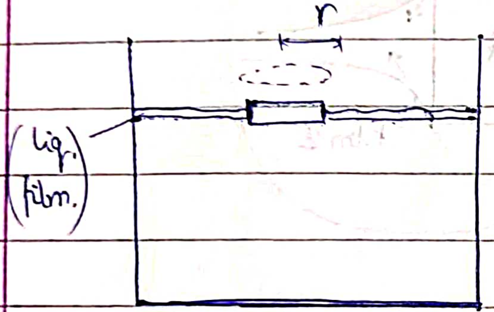
Consider a solid obj. kept on liq. surface.

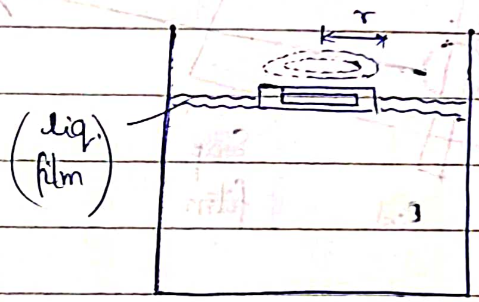
$$\boxed{T = \left(\frac{F}{l}\right)}$$

Surface Tension

(Force ~~experienced~~ experienced by obj.)

(Length (of solid) in contact with surface film)

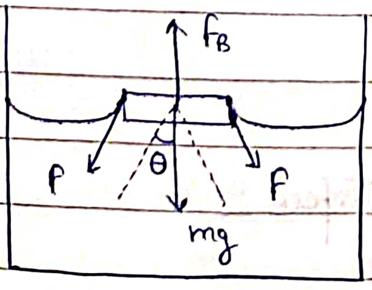


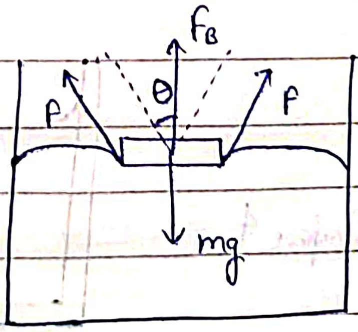


For disc, $2\pi r$ (only external surface in contact)

For thin ring, $4\pi r$ (both external & internal surface in contact)

Consider two liqs. s.t.,



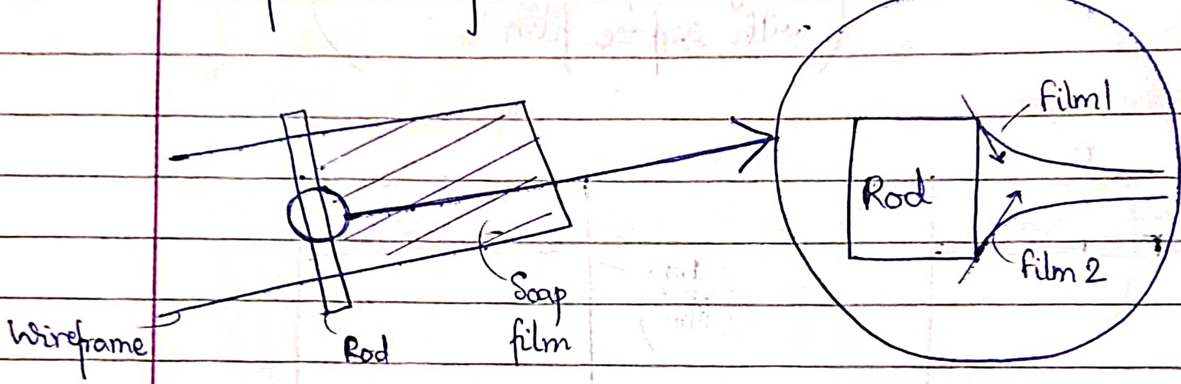


$$\cancel{F_{net}} \; \underset{F_B}{\cancel{mg}} = \underset{mg}{\cancel{F_B}} + (2\pi r)T\cos\theta$$

$$mg = F_B + (2\pi r)T\cos\theta$$

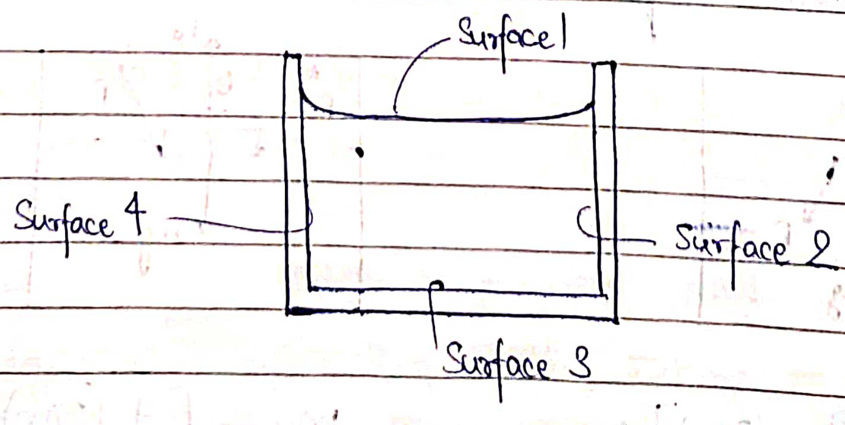
Consider a wireframe dipped in soap film. A movable rod is attached to one end of film.

Here, ∃ 2 films as there are 2 free surfaces.

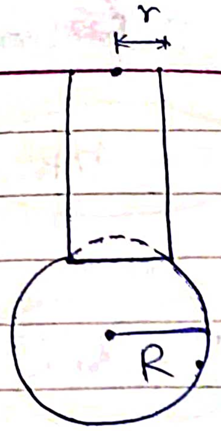


If a liq. is kept is a container, it has 4 surfaces (3 in contact with wall, 1 in contact with air).

But only 1 is FREE surface (the one which is in contact with air)

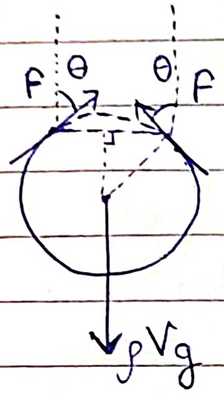


Q)



Find R_{min} s.t. drop. falls.

A)



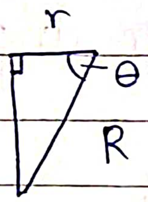
$l = 2\pi r$

$$V = \frac{4}{3}\pi R^3$$

$$\sin\theta = r/R.$$

Now, $\rho V g = (2\pi r) T \sin\theta$

$$\Downarrow$$



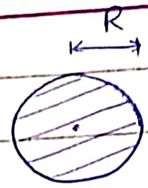
$$\rho \cdot \frac{4}{3}\pi R^3 \cdot g = (2\pi r) T \cdot \left(\frac{r}{R}\right)$$

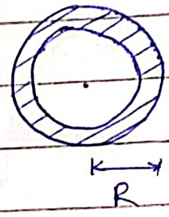
$$\Rightarrow \boxed{R = \left(\frac{3T r^2}{2\rho g}\right)^{1/4}}$$

Surface Energy

$$\boxed{(\text{Surface Energy}) = \left(\begin{array}{c}\text{Surface}\\ \text{Tension}\end{array}\right)\left(\begin{array}{c}\text{Surface Area}\\ \text{of Liq. film}\end{array}\right)}$$

It is $= \sum$ PE of particles on surface.

Eg : for liq. drop,  $T \cdot 4\pi R^2$

for soap bubble,  $T \cdot\cdot 8\pi R^2$

$$\boxed{(\text{Work done}) = (SE_f - SE_i)}$$

Q) Find work done to break bigger drop into 'n' smaller drops.

A) Let init. radius be 'R' & final radius be 'r'.

$$V_i = V_f \Rightarrow \frac{4}{3}\pi R^3 = n \cdot \frac{4}{3}\pi r^3$$

$$\Rightarrow R^3 = nr^3.$$

$$(\text{Work done}) = (SE_f - SE_i)$$

$$= (4\pi r^2)(n)T - (4\pi R^2)T$$

$$= \boxed{(4\pi R^2 T)(n^{1/3} - 1)}$$

☆ If a drop breaks w/o taking energy from environment, it does so by utilising its internal energy.

Since, $W > 0 \Rightarrow \Delta U < 0 \Rightarrow$ (Temp. of drop reduces)

Similarly, if two small drops combine w/o taking energy from outside, their temp. inc.

Q) 2 soap bubbles with radius R_1 & R_2 combine to form new soap bubble of radius R, under isothermal condⁿs.

find R.

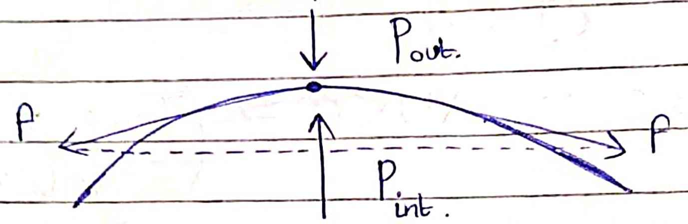
A) Isothermal condⁿs $\Rightarrow$ (Surface Energy) Const.

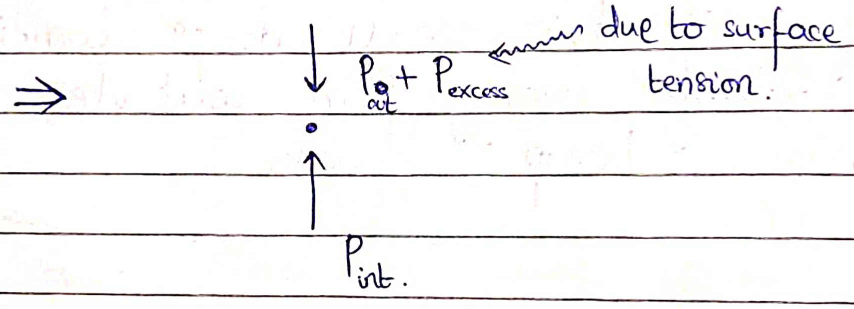
$$\Rightarrow (8\pi R_1^2)(T) + (8\pi R_2^2)(T) = (8\pi R^2)(T)$$

$$\Rightarrow \boxed{R = \sqrt{R_1^2 + R_2^2}}$$

Excess Pressure

Consider a drop or liq,



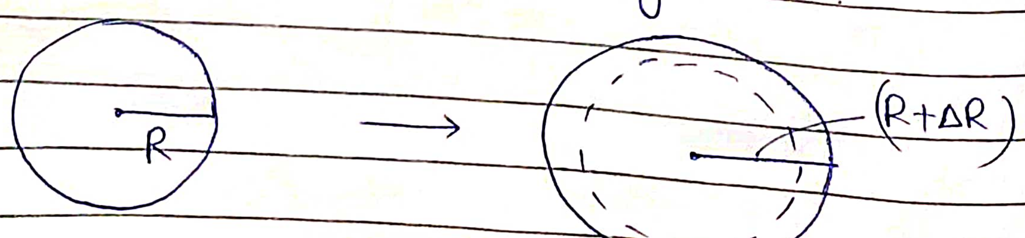


$$\Rightarrow \quad \boxed{P_{excess} = (P_{int.} - P_{out})}$$

Pressure in inside (i.e. at concave surface) is GREATER than outside (i.e. at convex surface)

1) Liq. Drop –

Consider a liq. drop with excess pressure 'p'
Let its radius inc. by ΔR.



Work done by P_{excess} is stored as surface energy.

$$\Rightarrow \left(P \cdot 4\pi R^2 \cdot \Delta R\right) = \left(SE_f - SE_i\right)$$

{as P always ⊥ to surface}

$$\Rightarrow P \cdot 4\pi R^2 \cdot \Delta R = \cancel{4\pi R^2}\, T\left[4\pi(R+\Delta R)^2 - 4\pi R^2\right]$$

$$\Rightarrow 4\pi R^2 \cdot P \cdot \Delta R = T \cdot 8\pi R \cdot \Delta R + \underbrace{4\pi T \cdot (\Delta R)^2}_{\text{ignore}}$$

$$\Rightarrow \boxed{P = \left(\frac{2T}{R}\right)}$$

2) Soap Bubble —

Doing as above, we find $\boxed{P = \left(\frac{4T}{R}\right)}$

In general, Drop : $P = T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

Bubble : $P = (2T)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

where R_1 & R_2 are radii of curvature of drop or liq. is 2 ⊥ dirxⁿs.

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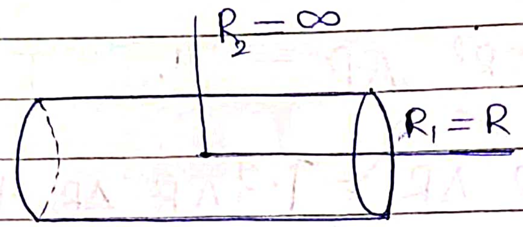
PAGE

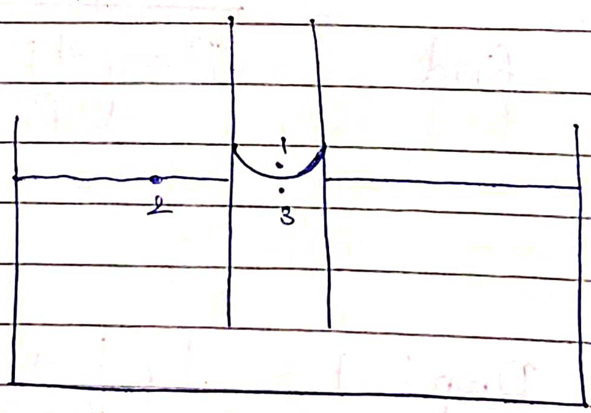
3) Cylindrical ~~Bubble~~ Drop or Bubble –

$R_1 = R$, $R_2 = \infty$ $\Rightarrow$ $\boxed{P_{Bubble} = \left(\frac{2T}{R}\right)}$ $\boxed{P_{drop} = \left(\frac{T}{R}\right)}$



Capillary

Let us put a tube in liq. as follows.



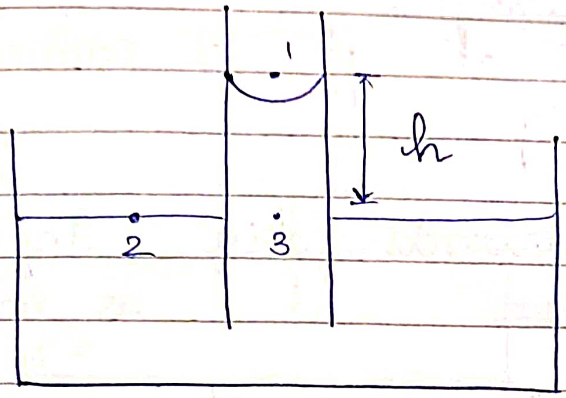
By excess Pressure, $P_1 > P_3$

Since height at 2 & 1 same, $P_2 = P_1$.

$\Rightarrow$ $P_2 > P_3$

But 2 & 3 at same height. This is against Pascal's Law.

Hence, liq. rises to = pressures at 2 & 3.



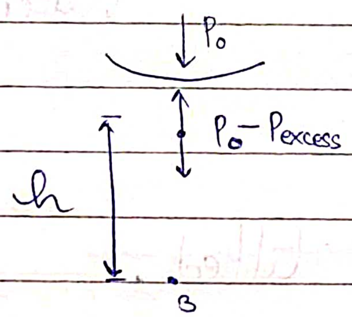
Now, $P_2 = P_3$ (for Eq.)

$\Rightarrow \cancel{P_0 = P_{excess} + \rho g h + P_0}$

$\Rightarrow \cancel{P_0 = P_0 - 21}$

★ Another way to derive is to balance forces on liq.! (See Pg 67)

At 3,

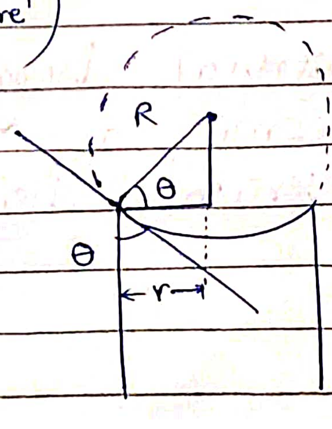


$P_3 = (P_0 - P_{excess}) + \rho g h$

Hence, $P_0 = P_0 + \rho g h - (2T/R)$

$\Rightarrow \boxed{h = \left(\frac{2T}{\rho g R}\right)}$ (Radius of Curvature)

Now, $\cos(\theta) = r/R$

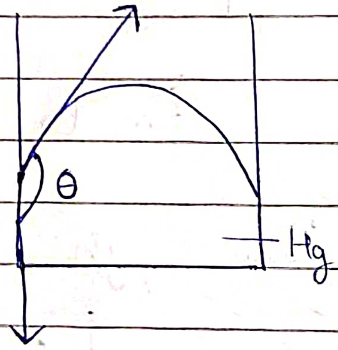


$\Rightarrow \boxed{h = \left(\frac{2T\cos(\theta)}{\rho g r}\right)}$ (Angle of Contact) (Radius of Capillary)

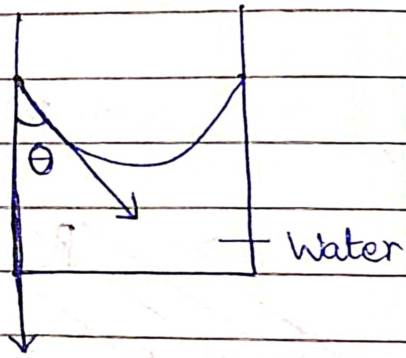
Angle of Contact –

Make tangent at pt. of contact towards liq.

~~Make~~ Take line towards liq. Angle in b/w is angle of contact

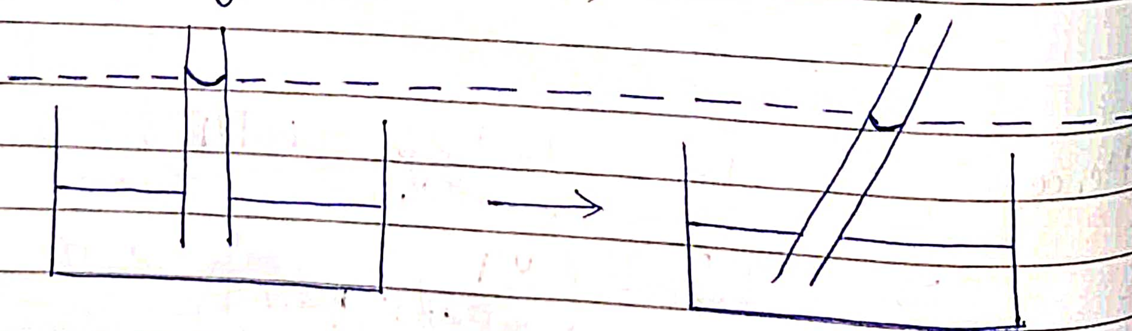


Hg – Glass



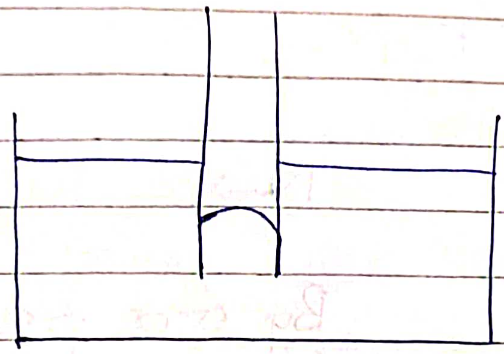
Water – Glass

If capillary is tilted,



vertical height of liq. column remains same as P depends only on vertical height.

For liq. like Hg, liq. level in capillary goes DOWN



But the formula for height still applicable.

Since $\theta > 90^\circ \Rightarrow \cos\theta < 0 \Rightarrow h < 0 \Rightarrow$ Height dec.

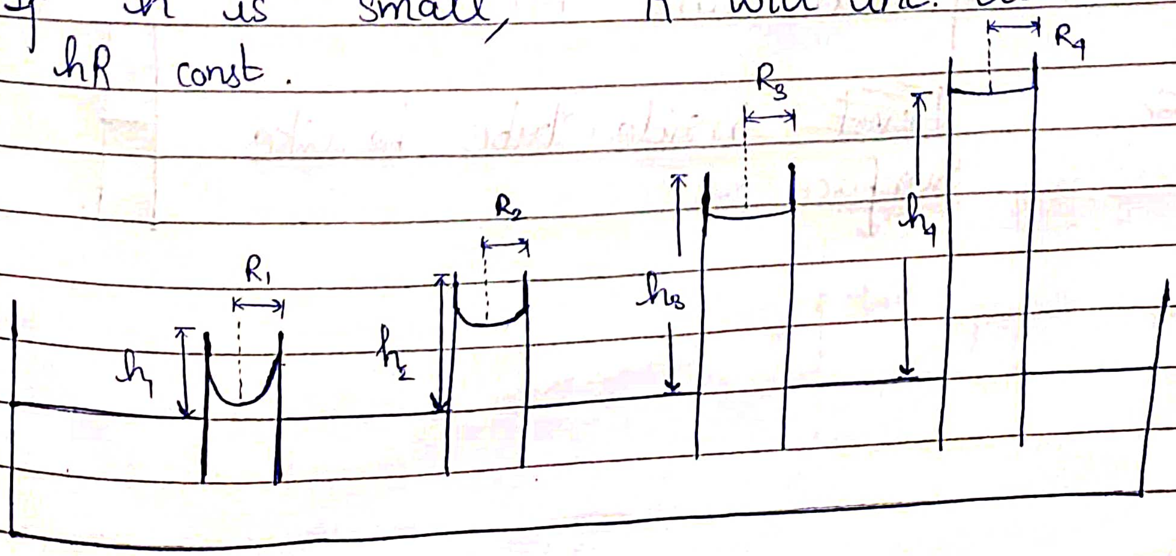
Tube of Insufficient Height

Liq. NOT overflow !

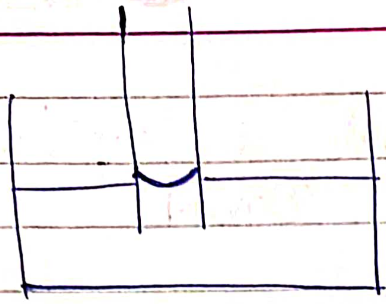
Observe,

$$\boxed{hR = \left(\frac{2T}{\rho g}\right) = \text{Const}}$$

If h is small, R will inc. to make hR const.

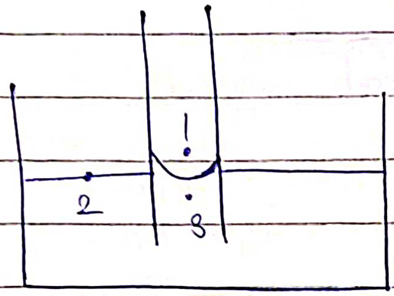


☆ Q)



What happens in gravity free space?

A)



Now, $P_1 = P_2$

By excess pressure,

$P_1 > P_3$

Since $g = 0$, ~~how~~ no matter how much liq. rises P_3 will remain as it is (since $\rho g h$ term $= 0$).

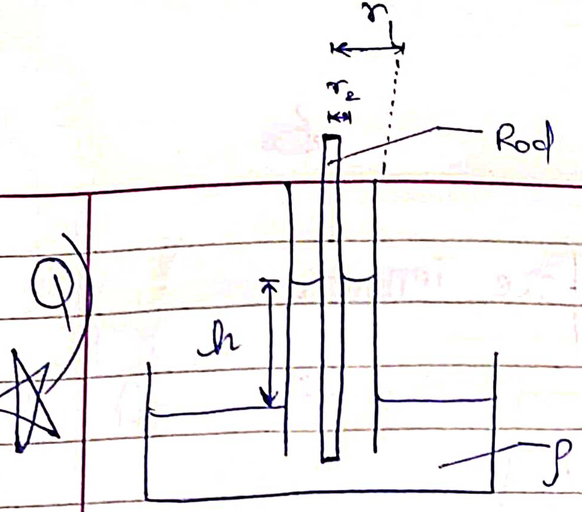
$\Rightarrow$ $P_3 < P_2$ (always)

$\Rightarrow$ Liq. keeps rising until top of tube.

At top of tube, it becomes case of tube of insufficient height.

Radius of curvature becomes ∞.

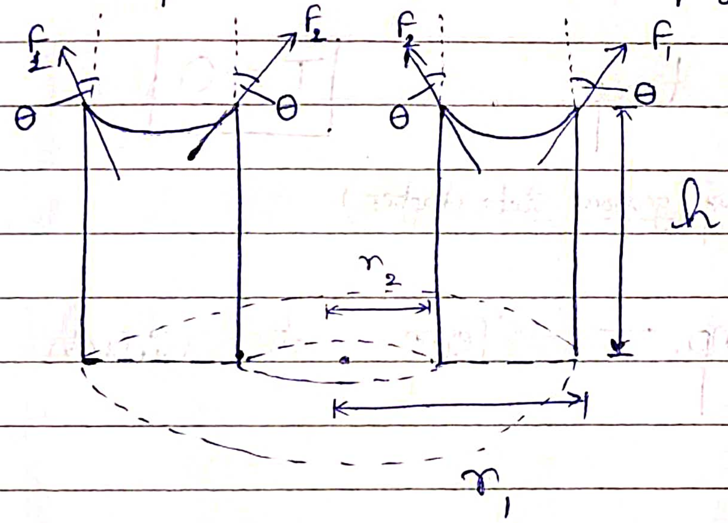
$\Rightarrow$ ~~Level~~ Surface inside tube be like



Q) Rod placed inside capillary of long length.

Find height in capillary.
(Ignore buoyant force due to rod).

A) Balance forces on liq!



By eq.,
$$mg = T(2\pi r_2)\cos\theta + T(2\pi r_1)\cos\theta$$

Now,
$$m = \rho(\pi r_1^2 - \pi r_2^2)h$$

$$\Rightarrow \rho g = \frac{(2\cancel{\pi}T\cos\theta)(r_2+r_1)}{(\cancel{\pi})(r_1^2-r_2^2)h}$$

$$\Rightarrow h = \left(\frac{2T\cos\theta}{\rho g(r_1-r_2)}\right)$$

We can assume $\theta = 0 \Rightarrow$ $\boxed{h = \left(\frac{2T}{\rho g(r_1-r_2)}\right)}$

Effect of Temp. on Surface Tension

$$\boxed{T = T_0(1 - \alpha t)}$$

Temp. diff.

Surface Tension

Temp. coeff.

At critical temp., $\boxed{T=0}$

(NOT as of gaseous state chapter)

Critical temp. — Temp. at which $T=0$.

Effect of Impurity on Surface Tension

1) Soluble Impurity — T inc.

Eg — Sugar water, ...

2) Partially Soluble — T dec.

Eg — Soap solⁿ, ...

Cohesive forces

force b/w 2 similar (same substance) molecules.

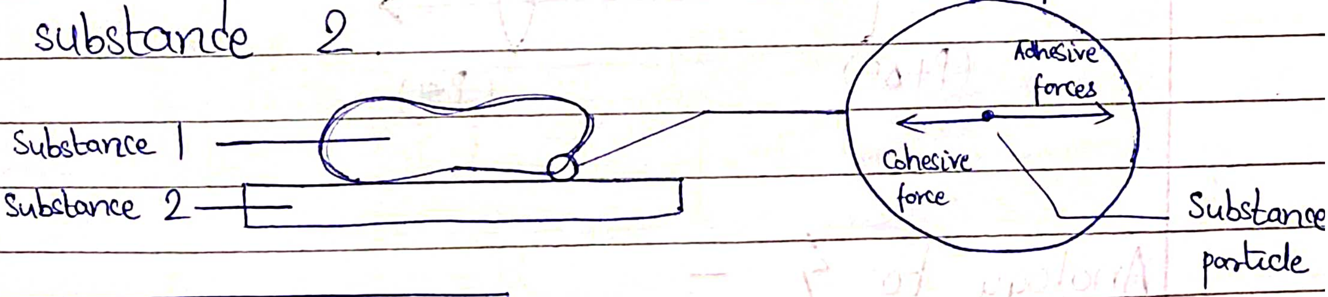
It is attractive in nature. (assumption)

Adhesive forces

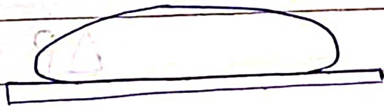
force b/w 2 dissimilar (diff. substances) molecules.

It is attractive in nature. (assumption)

Let us put substance 1 on a plate of substance 2.

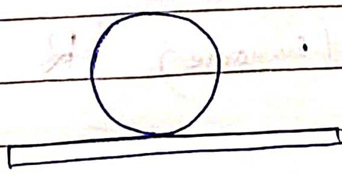


If $\boxed{\text{Adhesive} > \text{Cohesive}}$,



Substance 1 spreads

If $\boxed{\text{Adhesive} < \text{Cohesive}}$,



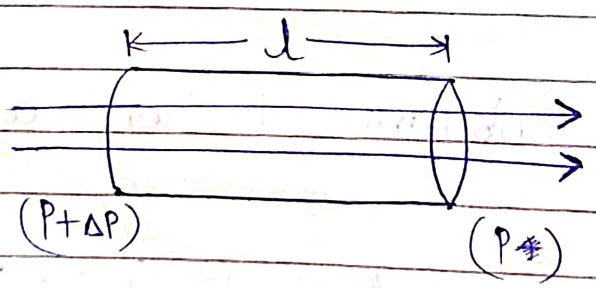
Substance 1 in form of droplets.

★ Generally, if $\boxed{\rho_{surface} > \rho_{liq}}$

↳ $\boxed{F_{adhesive} > F_{cohesive}}$

Huygen Poiseulli's Eq^n

$$\boxed{\underbrace{Q}_{\text{Volumetric flow rate}} = \left(\frac{\pi \overbrace{r^4}^{\text{radius of cross section of tube}} \overbrace{(\Delta P)}^{\text{Pressure diff.}}}{8\eta \underbrace{l}_{\text{length of tube}}}\right)}$$



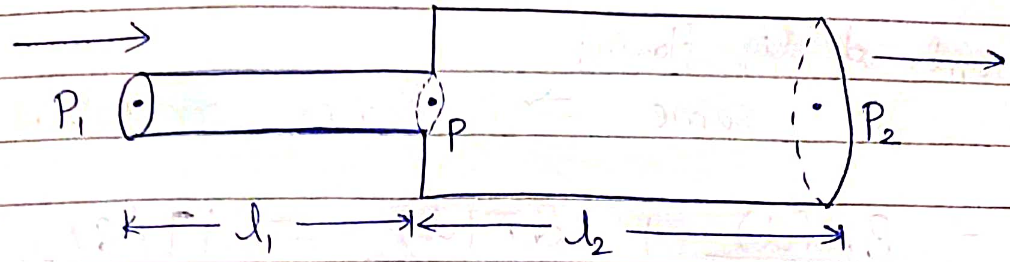
Analogy to ⚡ –

$\Delta P \equiv V$ — (Potential)

$Q \equiv i$ — (Current)

(Resistance) — $R \equiv \left(\dfrac{8\eta l}{\pi r^4}\right)$

Series Connection –



Q for both pipes is same $(A_1 v_1 = A_2 v_2)$

$$\Rightarrow \left(\frac{\pi r_1^4 (P_1 - P)}{8\eta l_1}\right) = \left(\frac{\pi r_2^4 (P - P_2)}{8\eta l_2}\right)$$

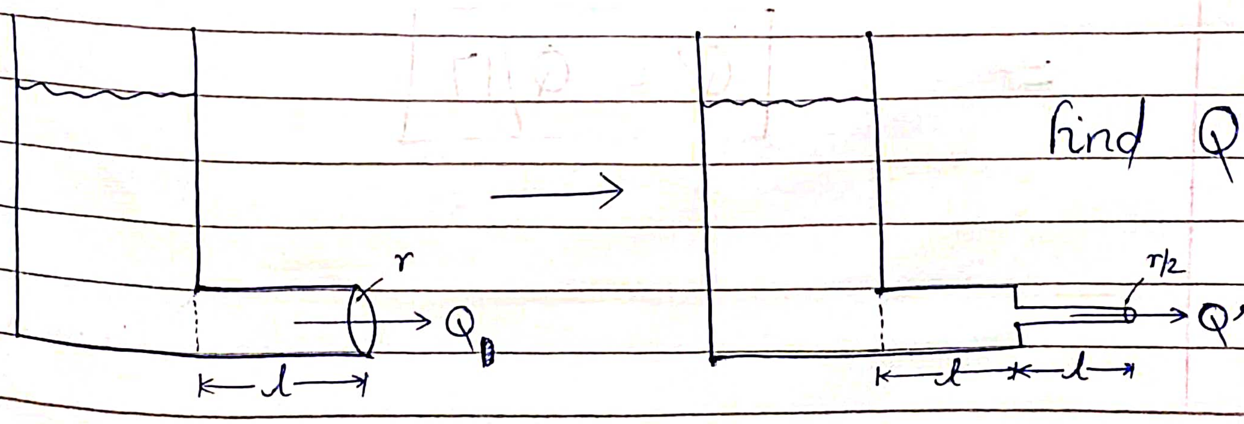
Now, $R_1 \equiv \left(\frac{8\eta l_1}{\pi r_1^4}\right)$ & $R_2 \equiv \left(\frac{8\eta l_2}{\pi r_2^4}\right)$

$$\Rightarrow R_{eq} \equiv \left(\frac{8\eta}{\pi}\right)\left(\frac{l_1}{r_1^4} + \frac{l_2}{r_2^4}\right)$$

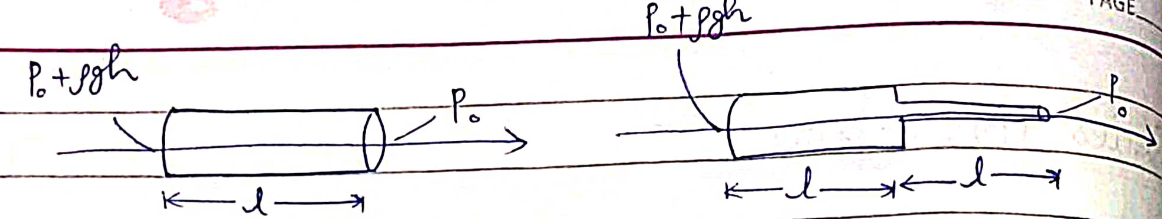
Now, $V \equiv (\Delta P) \Rightarrow$ $Q = \frac{\Delta P}{\left(\frac{8\eta}{\pi}\right)\left(\frac{l_1}{r_1^4} + \frac{l_2}{r_2^4}\right)}$

(apply $V = iR$)

Q) find Q'



A)



Same amt. of liq. flowing

~~ΔP~~ is same $\Rightarrow$ Series connection.

$$R' = \frac{8\pi(l)}{\pi(r)^4} + \frac{8\pi(l)}{\pi(r/2)^4} = 17\left(\frac{8\pi l}{\pi r^4}\right)$$

$$R = \frac{8\pi(l)}{\pi(r^4)} = \left(\frac{8\pi l}{\pi r^4}\right)$$

~~Now,~~ $\cancel{Q = Q'}$ $\cancel{\Delta P = \Delta P}$

$$\cancel{\Rightarrow \frac{\Delta P}{R} = \frac{\Delta P}{R'}}$$

$\cancel{\Rightarrow}$

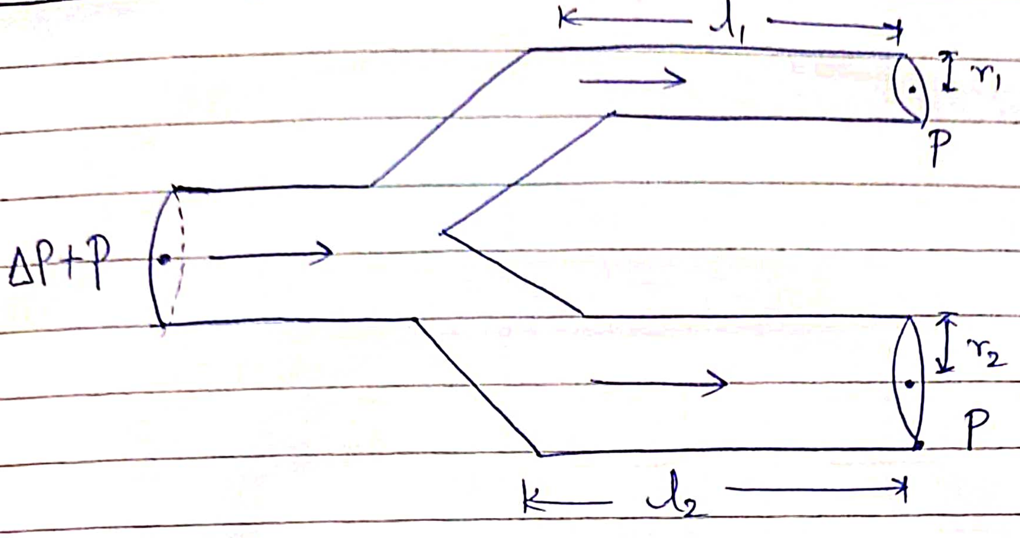
Now, $\Delta P = \Delta P' = \rho gh$

$$\Rightarrow Q R = Q' R'$$

$$\Rightarrow Q\left(\frac{8\pi l}{\pi r^4}\right) = Q'\left(17 \cdot \frac{8\pi l}{r^4}\right)$$

$$\Rightarrow \boxed{Q' = Q/17}$$

Parallel Connection –



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\Rightarrow \frac{1}{R_{eq.}} = \left(\frac{\pi r_1^4}{8\eta l_1}\right) + \left(\frac{\pi r_2^4}{8\eta l_2}\right)$$

~~As~~ (This is bcoz) $Q = Q_1 + Q_2$ & ΔP same.