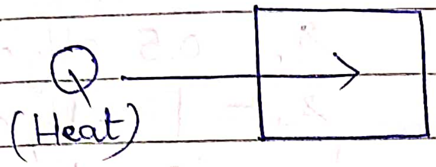
Heat & Thermometry

Calorimetry



If heat supplied to substance & phase NOT change ⇒ Temp. of sub. inc.

$$\boxed{Q = ms(\Delta\theta)}$$

Temp. change

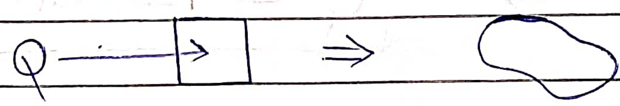
Heat

mass

Specific heat capacity ~~Temp. rais~~

Specific heat Capacity :

Heat req. to raise temp. of 1g of substance by 1°C.



If heat supplied & phase changes ⇒ Temp. NOT change

$$\boxed{Q = mL}$$

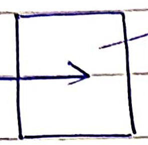
Heat

Latent heat

mass

$$\text{Solid} \xrightarrow{L_f} \text{Liq.} \xrightarrow{L_v} \text{Gas}$$

L_f = Latent heat of fusion ; L_v = Latent heat of vaporisation

Q) Heat  Ice (10 g)

−5° C

$s_{ice} = 0.5$ cal/g°C
$s_w = 1$ cal/g °C
$L_f = 80$ cal/g
$L_v = 540$ cal/g.

Find heat req.
to convert ice into steam at 100°C.

A)

$$\underset{(-5°C)}{\text{Ice}} \xrightarrow{Q_1} \underset{(0°C)}{\text{Ice}} \xrightarrow{Q_2} \underset{(0°C)}{\text{Water}} \xrightarrow{Q_3} \underset{(100°C)}{\text{Water}} \xrightarrow{Q_4} \underset{(100°C)}{\text{Steam}}$$

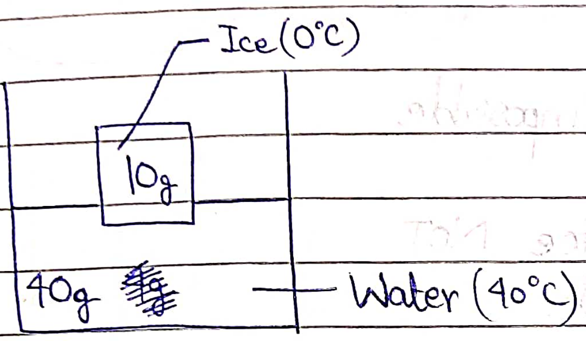
$Q_1 = 10 \cdot 0.5 \cdot 5 = 25$ cal
$Q_2 = 10 \cdot 80 = 800$ cal
$Q_3 = 10 \cdot 1 \cdot 100 = 1000$ cal
$Q_4 = 10 \cdot 540 = 5400$ cal

$Q = \sum Q_i \Rightarrow \boxed{Q = 7225 \text{ cal}}$

Principle of Calorimetry

$$\boxed{\left(\underset{\text{by 1 subs.}}{\text{Heat Loss}}\right) = \left(\underset{\text{by other subs.}}{\text{Heat Gain}}\right)}$$

Q)



$L_f = 80$ cal/g

$s_w = 1$ cal/g °C.

Find final temp. of mix.

A) Let final temp. be T.

$$Q_{\text{gain(ice)}} = m_{ice} L_f + m_w s_w \Delta T$$

$$Q_{\substack{\text{Loss} \\ \text{(water old)}}} = m'_w s_w \Delta T'$$

$$\Rightarrow (10)(80) + (10)(1)(T-0) = (40)(1)(40-T)$$

$$\Rightarrow 80 + T = 160 - \cancel{40}\,4T \Rightarrow \boxed{T = 16\,°C}$$

Q) In above Q, if mass of ice is taken 40 g, find final temp.

A) ☆ Since we do NOT know how much ice melts, assume whole melts

$$\Rightarrow (40)(80) + (40)(1)(T-0) = (40)(1)(40-T)$$

$$\Rightarrow 80 + T = (40 - T) \Rightarrow T = (-20)$$

Obviously, this is impossible

$\Rightarrow$ Whole ice NOT melt.

$$\Rightarrow \boxed{T_{mix} = 0}$$

☆ If 2 subs of temp. T_1 & T_2 mix. & while calc. —

(Whole gas not condense) ~~(Whole ice not melt)~~

$T > \max\{T_1, T_2\} \Rightarrow T_{mix} = \max\{T_1, T_2\}$

$T < \min\{T_1, T_2\} \Rightarrow T_{mix} = \min\{T_1, T_2\}$

(Whole ice not melt)

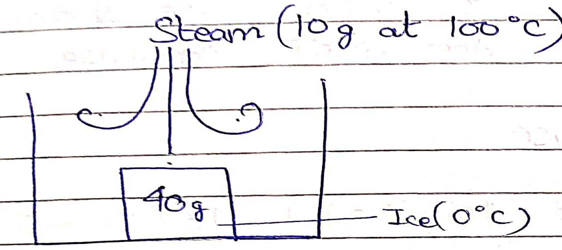
Q) In above Q, find m of ice melted.

A) Let m mass melt. Since $T_{mix} = 0$.

$$\Rightarrow \quad m \cdot L_f = (m_{w,\,old})\, s_w\, \Delta T$$

$$\Rightarrow \quad m\,(80) = (40)(1)(\underbrace{40-0}_{T_{mix}})$$

$$\Rightarrow \quad \boxed{m = 20\ g}$$

Q) 

$L_f = 80$ cal/g
$L_v = 540$ cal/g
$s_w = 1$ cal/g°C
find final temp.

A) Assume both whole ice melts & whole steam condenses.

$$\Rightarrow \quad (4\cancel{0})(80) + (4\cancel{0})(1)(T-0) = (1\cancel{0})(540) + (1\cancel{0})(100-T)$$

$$\Rightarrow \quad 320 + 4T = 540 + 100 - T$$

$$\Rightarrow \quad \boxed{T = 64\ °C}$$

Q) In above Q, if mass of steam is taken 20g, find final temp. Also find mass of steam condensed.

A) Assume whole steam condenses. & whole ice melts.

$$\overset{(2)}{\cancel{(40)}}(80) + \overset{(2)}{\cancel{(40)}}(1)(T-0) = \cancel{(20)}(540) + \cancel{(20)}(100-T)$$

$\Rightarrow \quad 160 + 2T = 540 + 100 - T$

$\Rightarrow \quad T = 160 \quad > \max\{100°C, 0°C\}$

$\Rightarrow \quad \boxed{T_{mix} = 100°C}$ ∴ whole ~~ice~~ steam does NOT condense.

Let m mass condense.

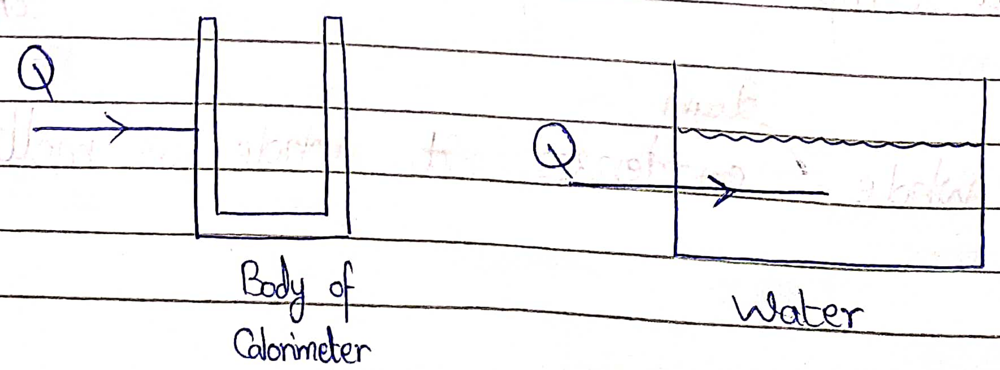
$\Rightarrow \quad (m)(54\cancel{0}) = (4\cancel{0})(80) + (4\cancel{0})(\underbrace{100}_{T_{mix}}-0)$

$\Rightarrow \quad 54m = 320 + 400$

$\Rightarrow \quad \boxed{m = 40/3 \text{ g}}$

Water Equivalent

To accaunt for heat loss due to body of calorimeter.



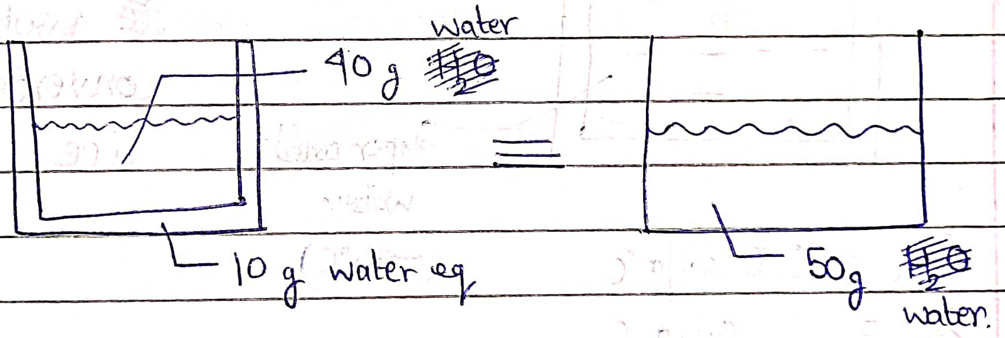
Supply Q heat to both body of calorimeter & water. We take mass of water in such qty. s.t. rise in temp in both cases is same.

$$\Rightarrow \quad Q = m_c s_c \Delta T \quad \& \quad Q = m_w s_w \Delta T$$

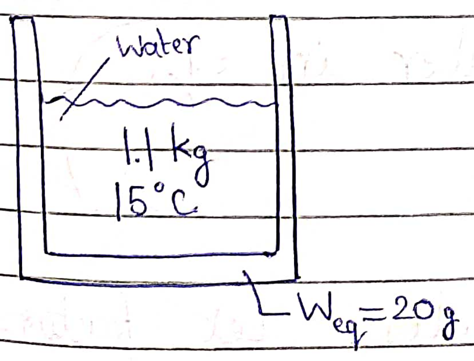
$$\Rightarrow \quad \boxed{m_w = m_c s_c}$$

Water eq.

Usefulness —

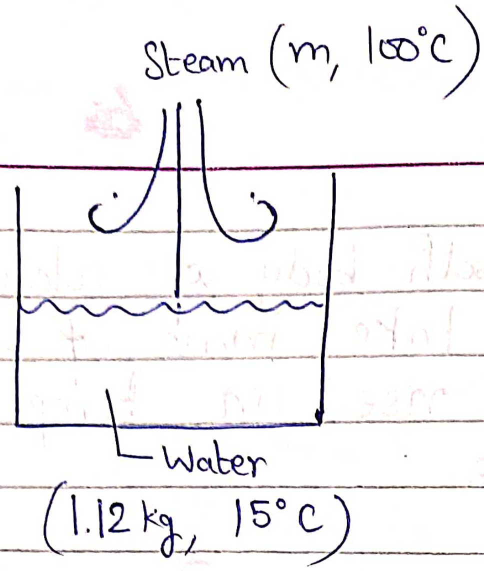


Q)



Water is to be raised to 80°C by passing steam at 100°C.

Find min. amt of steam req.

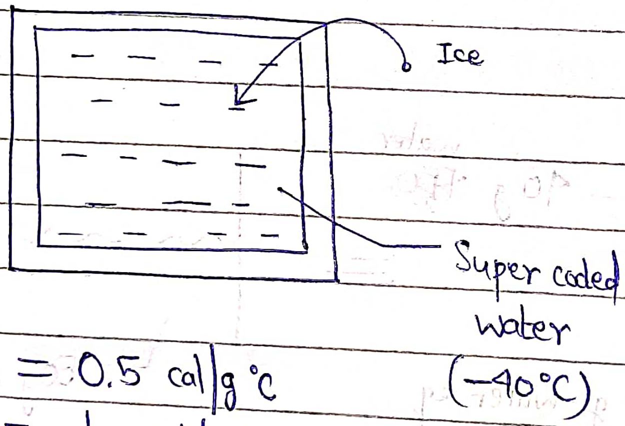


A)

$$(m)(540) + (m)(100-80)$$
$$= (1.12)(80-15) \times 10^3$$
$$\Downarrow$$
$$m = \frac{(1.12)(65) \times 10^3}{(560)}$$

$$\Rightarrow \boxed{m = 130\,g}$$

☆ Q)



find fraction of ~~ice~~ water converted to ice.

$s_{ice} = 0.5$ cal/g °C
$s_w = 1$ cal/g °C
$L_f = 80$ cal/g

A) ☆ Whenever ice & water in eq.,
$T_{mix} = 0°C$

Let m be mass of water. Let fraction 'f' be converted to ice.

$Q_{loss\,(by\ ice)} = (mf)(80)$

$Q_{gain\,(by\ ice\ \&\ water)} = (mf)(0+40)^{(1/2)} + m(1-f)(0+40)(1)$

$\Rightarrow \quad m(80f) = m(20f + 40 - 40f)$

$\Rightarrow \quad 4f = f + 2 - 2f \Rightarrow \boxed{f = 2/5}$

~~Graphs~~

~~Consider we are heating ice at $-5°C$~~

~~for $-5°C \leq T < 0°C$, $Q = m s_{ice}(T+5)$~~

~~$\Rightarrow T = (-5) + \frac{Q}{m s_{ice}}$ (given by us)~~

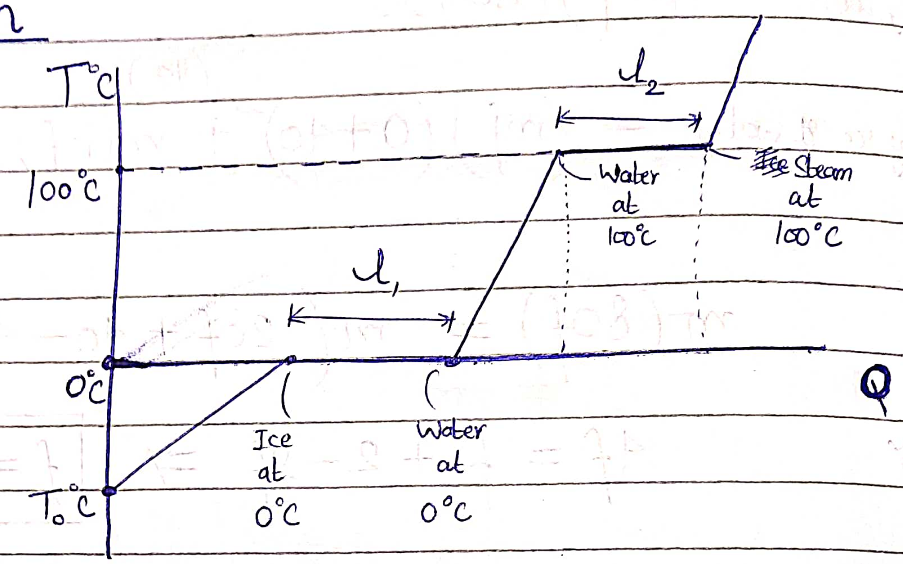
~~for $T = 0°C$, (Phase change)~~

~~$Q \in [m s_{ice} \cdot 5,\ m s_{ice} \cdot 5 + mL_f]$~~
~~$T = 0°C$~~

~~for $T > 0°C$, $Q = m s_{ice} \cdot 5 + mL_f + m s_w T$~~

DATE
PAGE

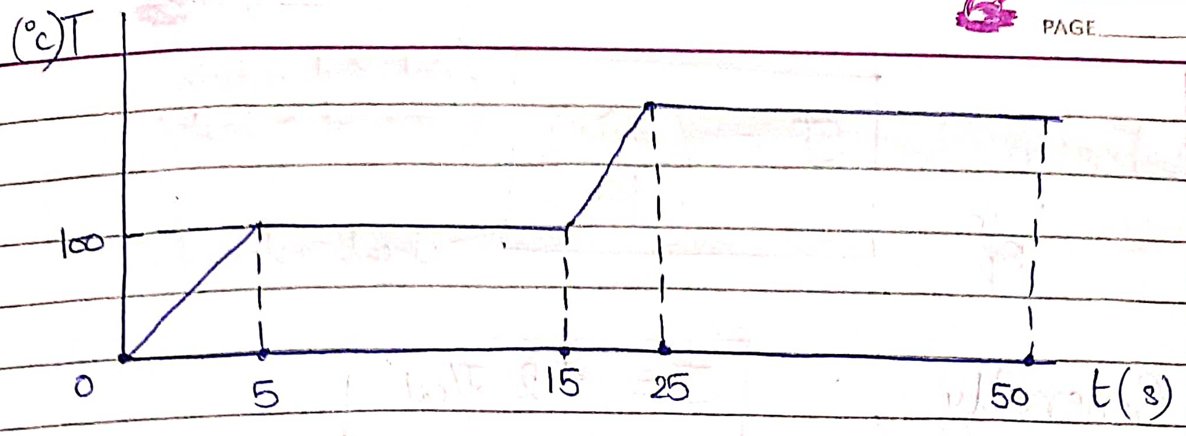
Graph



for ice starting at temp T_0,

$$\boxed{\begin{matrix}\text{Ice}\\ @\ T_0\,°C\end{matrix}} \longrightarrow \boxed{\begin{matrix}\text{Ice}\\ @\ 0°C\end{matrix}} \longrightarrow \boxed{\begin{matrix}\text{Water}\\ @\ 0°C\end{matrix}}$$

Q)



Rate of heating is const. $m = 1\,kg$. $s_{solid} = 0.6\ \frac{kCal}{g\,°C}$

find L_f & L_v.

A) Let rate of heating be 'r'.

$\Rightarrow \quad Q_1 = (r)(5) = (1)(0.6)(100)$

$\Rightarrow \quad r = 12\ kCal/s$

Now, $Q_f = (1)L_f = (12)(15-5) \Rightarrow \boxed{L_f = 120\ kCal/g}$

$Q_v = (1)L_v = (12)(50-25) \Rightarrow \boxed{L_v = 300\ kCal/g}$

Mechanical Equivalent

If some work done ON system is fully converted to heat (NOT the other way around as converting heat fully to work is NOT possible), then amt. of heat produced is called its ~~tme~~ mech. eq.

done ON system

$$\boxed{J = \left(\frac{W}{H}\right)}$$

Mech. Eq.

(Work that is fully converted to heat)

(heat released)

Generally, $\boxed{J = 4.2 \text{ J/cal}}$

Q) 100 m high water fall. On coming to ground, 50% of work is converted into heat. Find rise in temp. of water.

A) $W = mgh = (10^3)m \cdot m^2/s^2$

$W_{\left(\substack{\text{converted fully}\\ \text{to heat}}\right)} = 50\% \text{ of } (10^3)m \ \ m^2/s^2$

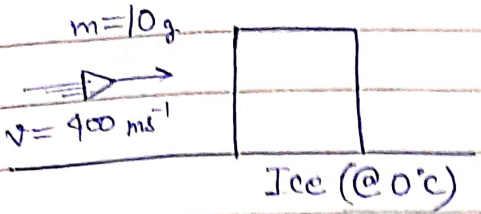
$= (5\times 10^2)m \ \ m^2/s^2$

$H = \left(\frac{W_{(\text{fully converted to heat})}}{J}\right) = \frac{(5\times10^2)m \ \ m^2/s^2}{4.2 \ \ J/cal}$

Now, $Q = ms(\Delta T)$

$\Rightarrow (\Delta T)(\cancel{m})\left(1 \ \frac{\cancel{cal}}{g\,°c}\right) = \frac{(5\times 10^2)\cancel{m}}{4.2} \ \frac{\cancel{cal}}{kg}$

$\Rightarrow \boxed{\Delta T = \left(\frac{5}{42}\right) °C}$

Q) 

Bullet gets embedded.
Combination NOT moving.

$(L_f = 80 \text{ cal/g})$

Find amt. of ice melted

A) $\underset{\text{(fully converted to heat)}}{W} = \Delta K = \frac{1}{2}mv^2 = \frac{1}{2}(10\times10^{-3})(4\times10^2)^2 = 8\times10^2 \text{ J}$

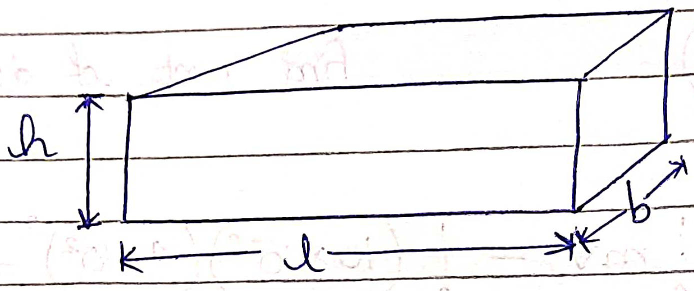
$$H = \left(\frac{\overset{\text{(fully converted to heat)}}{W}}{J}\right) = \left(\frac{8\times10^2}{4.2}\right) \text{cal}$$

Now, $$(m_{melt})\left(80 \frac{\cancel{cal}}{g}\right) = \left(\frac{8\times10^2}{4.2}\right)\cancel{cal}$$

$$\Rightarrow \boxed{m_{melt} = 50/21 \text{ g}}$$

☆ All done in this topic is useless.
Just consrv. energy & include heat also, while taking care of units!
J is just a unit conversion factor.

Thermal Expansion



If uniform heating & temp. of body inc. by (ΔT), then

$$\boxed{\begin{aligned} l &= l_0(1+\alpha\cdot\Delta T) \\ b &= b_0(1+\alpha\cdot\Delta T) \\ h &= h_0(1+\alpha\cdot\Delta T) \end{aligned}}$$

where $\alpha = \left(\frac{1}{L}\right)\left(\frac{dL}{dT}\right)$ (inc. in length of 1m rod by 1°C)

(Coeff. of linear exp^n)

where L is ANY length (linear measure)

Now, $A = lb = l_0 b_0 (1+\alpha\cdot\Delta T)^2$

$\Rightarrow A = l_0 b_0 (1+2\alpha\cdot\Delta T + \alpha^2\cdot\cancel{(\Delta T)^2})$ (0)

$\Rightarrow \boxed{A = A_0(1+(2\alpha)\cdot\Delta T)}$

Similarly, $\boxed{V = V_0(1 + (3\alpha)\cdot \Delta T)}$

In general,

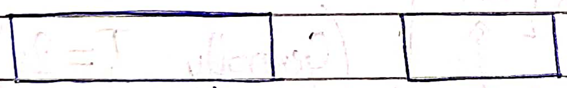
Linear measure — $L = L_0(1 + \alpha \cdot \Delta T)$ — Coeff. of Linear exp^n

Areal measure — $A = A_0(1 + \beta \cdot \Delta T)$ — Coeff. of Areal exp^n

Vol. measure — $V = V_0(1 + \gamma \cdot \Delta T)$ — Coeff. of Vol. exp^n

where $\boxed{\alpha = \dfrac{\beta}{2} = \dfrac{\gamma}{3}}$

Q)



(L_1, α_1) (L_2, α_2)

On heating to produce same temp. diff., change in length is same

Find rel^n b/w given qtys.

A) $\Delta L_1 = L_1 \alpha_1 \Delta T$

$\Delta L_2 = L_2 \alpha_2 \Delta T$

Given, $\Delta L_1 = \Delta L_2$

$\Rightarrow \boxed{L_1 \alpha_1 = L_2 \alpha_2}$

DATE
PAGE

Change in Time in Pendulum Clock

$$T_L = 2\pi\sqrt{\frac{L}{g}} \Rightarrow T_L' = 2\pi\sqrt{\frac{L'}{g}}$$

Time period

$$\Rightarrow \left(\frac{T_L'}{T_L}\right) = \sqrt{\frac{L'}{L}}$$

$$\Rightarrow \left(\frac{T_L'}{T_L}\right) = \sqrt{\frac{\cancel{L}(1+\alpha\cdot\Delta T)}{\cancel{L}}}$$ temp. diff.

$$= \sqrt{1+\alpha\cdot\Delta T}$$

$$\Rightarrow \quad T_L' = T_L\left(1+\frac{\alpha}{2}\cdot\Delta T\right)$$

$$\Rightarrow \quad \boxed{\Delta T_L = \frac{1}{2}\alpha T_L \Delta T}$$

Orig. Time period

Change in Time period

Change in temp.

(Generally, $T_L = 2s$)

☆ $\left(\Delta T_L / T_L\right) = (\#s \text{ lost in a day})/(\#s \text{ in a day}) = \frac{1}{2}\cdot\alpha\cdot\Delta T$

Q) At 5°C, a pendulum clock gains 10 s in a day. At 15°C, it loses 20 s in a day. Find temp. at which it gives correct time.

A) Let it give correct time at T °C.

$$\boxed{\begin{array}{l} \underline{\text{Gaining}}\ \text{Time} \iff \text{Time Period } \underline{\text{dec}} \\ \underline{\text{Losing}}\ \text{Time} \iff \text{Time Period } \underline{\text{inc}} \end{array}}$$

$$\cancel{(-10) = \frac{1}{2}\cdot\alpha\cdot T\cdot(T-5)\,(5-T)}$$

$$\cancel{(+20) = \frac{1}{2}\cdot\alpha\cdot T\cdot(T-15)}$$

$$\left.\begin{array}{l} 10 = \frac{1}{2}\cdot\alpha\cdot T_L\cdot(T-5) \\ 20 = \frac{1}{2}\cdot\alpha\cdot T_L\cdot(15-T) \end{array}\right\} \quad \frac{1}{2} = \left(\frac{T-5}{15-T}\right)$$

$$\Rightarrow \boxed{T = 25/3\ ^\circ C}$$

Alternative:

ΔT_L :	-10	0	$+20$
ΔT :	5	T	15

(-10) $(+20)$

0

$$20 : 10 = (15-T) : (T-5) \Rightarrow \boxed{T = 25/3\ ^\circ C}$$

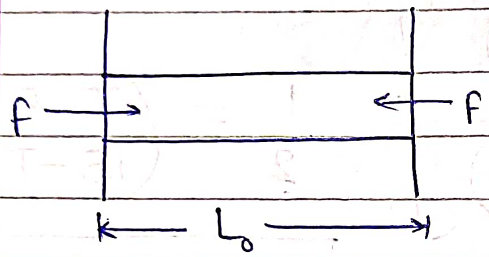
DATE

PAGE

Thermal Stress

If a ~~wall~~ rod, fix b/w walls is heated, its expansion is opposed by the wall

$\Rightarrow$ Thermal stress generated.



$$L = L_0(1 + \alpha \cdot \Delta T)$$

$$\Downarrow$$

$$\Delta L = \alpha L_0 \cdot \Delta T$$

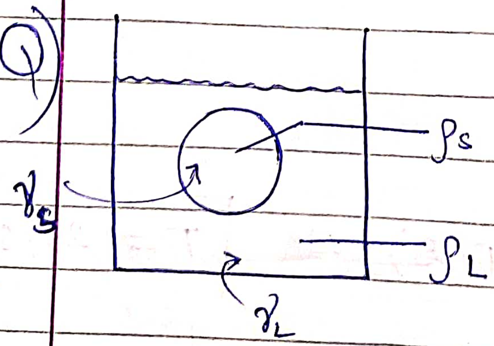
$$\Rightarrow \left(\frac{\Delta L}{L_0}\right) = \alpha \Delta T$$

Strain.

Now, Stress = Y (Strain)

$\Rightarrow$ $\boxed{\text{Stress} = Y\alpha \cdot \Delta T}$

Q)



If everything heated, find change in buoyant force.
(Assume full obj. submerged.)

A) ☆ Liq. ALWAYS expand volumetrically.

$V_s' = V_s(1+\gamma_s \cdot \Delta T)$ & $\rho_s' V_s' = \rho_s V_s$

$V_L' = V_L(1+\gamma_L \cdot \Delta T)$ & $\rho_L' V_L' = \rho_L V_L$

(Vol. expn) (Mass. Const.)

Now, $F_B = \rho_L V_s g$ & $F_B' = \rho_L' V_s' g$

$\Rightarrow \quad \Delta F_B = (\rho_L' V_s' - \rho_L V_s) g$

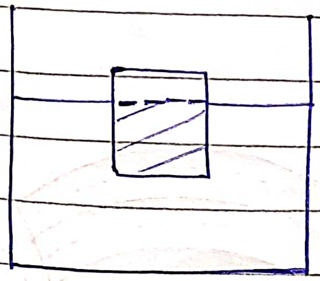
$= (\rho_L V_s g)\left(\dfrac{\rho_L'}{\rho_L} \cdot \dfrac{V_s'}{V_s}\right) \cancel{g} - (\rho_L V_s) g$

$= (\rho_L V_s g)\left(\dfrac{(V_s'/V_s)}{(V_L'/V_L)}\right) - (\rho_L V_s g)$

$= (\rho_L V_s g)(1+\gamma_s \cdot \Delta T)(1+\gamma_L \cdot \Delta T)^{-1} - (\rho_L V_s g)$

$\Rightarrow \quad \boxed{\Delta F_B = (\rho_L V_s g)(\gamma_s - \gamma_L)(\Delta T)}$

Q)



Find change in fraction of solid submerged, if everything heated.

$(\rho_s < \rho_L)$

A) $V'_s = V_s(1+\gamma_s \cdot \Delta T)$ & $\rho'_s V'_s = \rho_s V_s$

$V'_L = V_L(1+\gamma_L \cdot \Delta T)$ & $\rho'_L V'_L = \rho_L V_L$

Now, $\rho_L \cdot V_{sub} = \rho_s \cdot V_s$

$$\Rightarrow \left(\begin{matrix}\text{Fraction of Solid}\\ \text{Submerged}\end{matrix}\right) = \left(\frac{\rho_s}{\rho_L}\right)$$

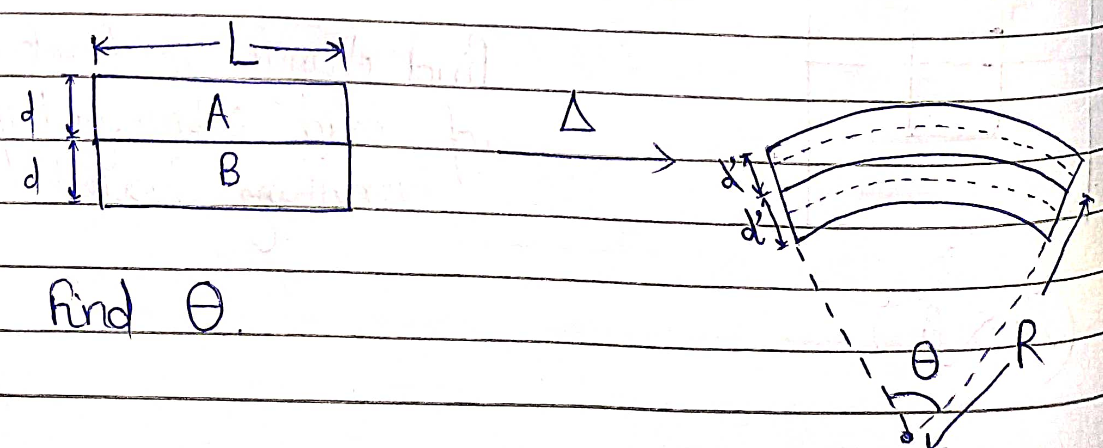
$$\Rightarrow \left(\begin{matrix}\text{New fraction of}\\ \text{Solid submerged}\end{matrix}\right) = \left(\frac{\rho'_s}{\rho'_L}\right) = \left(\frac{\rho_s}{\rho_L}\right)\left(\frac{(V'_L/V_L)}{(V'_s/V_s)}\right)$$

$$= \left(\frac{\rho_s}{\rho_L}\right)(1+\gamma_L \cdot \Delta T)(1+\gamma_s \cdot \Delta T)^{-1}$$

$$= \cancel{\rho_s}\left(\frac{\rho_s}{\rho_L}\right)(1+(\gamma_L-\gamma_s)\Delta T)$$

$$\Rightarrow \boxed{\left(\begin{matrix}\text{Change in}\\ \text{fraction}\end{matrix}\right) = \left(\frac{\rho_s}{\rho_L}\right)(\gamma_L-\gamma_s)(\Delta T)}$$

Q)



Find θ.

A) $L'_A = L_A(1+\alpha_A\cdot\Delta T)$ & $L'_B = L_B(1+\alpha_B\cdot\Delta T)$.

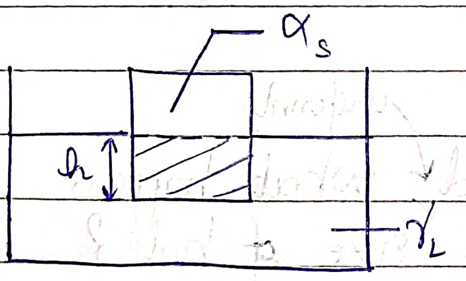
Now, $\left(R+\frac{d'}{2}\right)\theta = L'_A$ & $\left(R-\frac{d'}{2}\right)\theta = L'_B$

$\Rightarrow$ $\cancel{d = \frac{L'_A}{R+d}}$ $\left(\frac{d'}{2}\right) = \left(\frac{L'_A}{\theta} - R\right)$ & $\left(\frac{d'}{2}\right) = \left(R - \frac{L'_B}{\theta}\right)$

$$\Rightarrow \frac{L'_A}{\theta} + \frac{L'_B}{\theta} = 2R = \frac{L_A(1+\alpha_A\cdot\Delta T)}{\theta} + \frac{L_B(1+\alpha_B\cdot\Delta T)}{\theta}$$

$$\Rightarrow 2R = \left(\frac{L}{\theta}\right)\left(2 + (\alpha_A+\alpha_B)\cdot\Delta T\right)$$

$$\Rightarrow \boxed{\theta = \left(\frac{L}{2R}\right)\left(2 + (\alpha_A+\alpha_B)\cdot\Delta T\right)}$$

Q) 

If on heating, h (height of solid inside liq.) remains same, find relⁿ b/w α_s & γ_s

A) Let cross section area of solid be A_s.

$\Rightarrow$ $A_s' = A_s(1+2\alpha_s\cdot\Delta T)$.

Now, $V_L' = V_L(1+\gamma_L\cdot\Delta T)$ & $\rho_L' V_L' = \rho_L V_L$

DATE
PAGE

Now, $mg = f_B = f'_B$

$$\Rightarrow (A_s)(\cancel{h})\rho_L \cancel{g} = (A'_s)(\cancel{h})\rho'_L \cancel{g}$$

$$\Rightarrow \left(\frac{A'_s}{A_s}\right)\left(\frac{\rho'_L}{\rho_L}\right) = 1$$

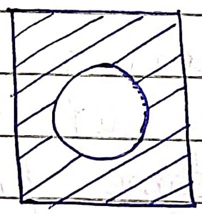
$$\cancel{\Rightarrow (1+2\alpha_s \cdot \Delta T)\,V = (V'/V)}$$

$$\cancel{\Rightarrow 1+}$$

$$\Rightarrow \left(\frac{A'_s}{A_s}\right) = \left(\frac{V'_L}{V_L}\right) \Rightarrow (1+2\alpha_s \cdot \Delta T) = (1+\gamma_L \cdot \Delta T)$$

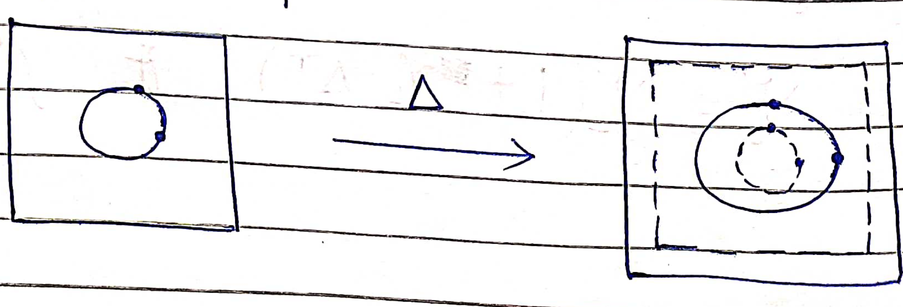
$$\Rightarrow \boxed{\gamma_L = 2\alpha_s}$$

☆ Q)



If heated uniformly, what happens to size of hole?

A) On heating, avg. dist. b/w any 2 molecules inc. Consider 2 pts on circumference of hole.



Dist. b/w them should inc. ⇒ Radius inc.

Thermometry (Old)

Standard fix Pts.

- Lower (LFP) — Freezing Pt. of Water
- Upper (UFP) — Boiling Pt. of Water

Temp. Scale —

	L.F.P	U.F.P
1) Celcius (°C)	0°C	100°C
2) Farenheit (°F)	32°F	212°C

$$\boxed{\left[\frac{(\text{Reading})-(\text{LFP})}{(\text{UFP})-(\text{LFP})}\right] = \left(\frac{C}{100}\right) = \left(\frac{F-32}{180}\right)}$$

(In general for any scale)

(Rel^n b/w Temp. diff) : $\boxed{\frac{1^\circ C}{5} = \frac{1^\circ F}{9}}$

Q) faulty Therm. LFP = 5°C UFP = 90°C
If temp. of body on this scale is 50°C. find actual temp.

A) Let us denote faulty by $°C^*$

$$\left(\frac{50°C^* - \cancel{0}^{5}°C^*}{90°C^* - \cancel{0}_{5}°C^*}\right) = \left(\frac{X - 0°C}{100°C - 0°C}\right)$$

$\Rightarrow$ $X = 100 \cdot \frac{45}{85}$ $\Rightarrow$ $\boxed{X \approx 53°C}$

★ For any ~~therm.~~ thermometric propt (i.e. $\propto$ temp.)

$$\boxed{(\text{Temp.}) = \left(\frac{X_{T°C} - X_{0°C}}{X_{100°C} - X_{0°C}}\right) \cdot 100}$$

Where $X_{T°C}$ is value of therm. propt at T°C.

Eg - Resistance, Prsre, Length, ...

Thermometry (Modern)

Only 1 fix pt, $\boxed{\left(\begin{array}{c}\text{Triple Pt.}\\ \text{of Water}\end{array}\right) = 273.16\ K}$

$$\boxed{(\text{Temp.}) = \left(\frac{P_{TK}}{P_{273.16\,K}}\right) \times 273.16\ K}$$

Where P_{TK} is prsre of gas taken in const. vol. therm. at $T\ K$.

Heat Transfer

Slow

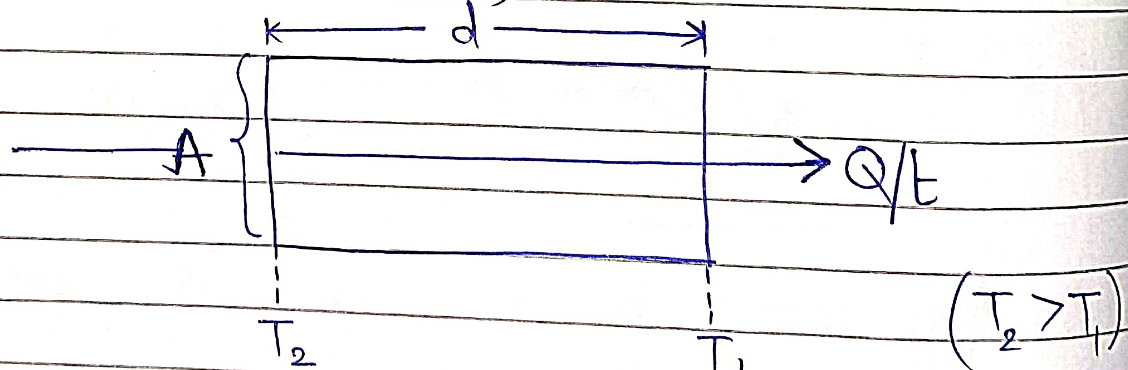
Fast

<u>Conduction</u> — In solids. Sometimes in liq.

<u>Convection</u> — In fluids.

<u>Radiation</u> — No medium req. Can happen in solid, liq, gas, vaccum

<u>Conduction (in 1 D)</u>



Rate of Flow of Heat = Q/t

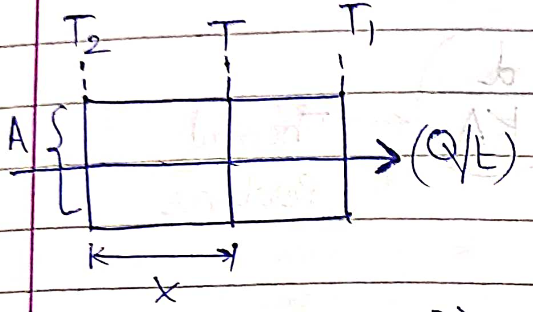
Thickness of Slab, in dirx^n of heat flow = d

Area of Cross Section, normal to heat flow = A

for Steady State, (if K = Const.) (if A = Const.)

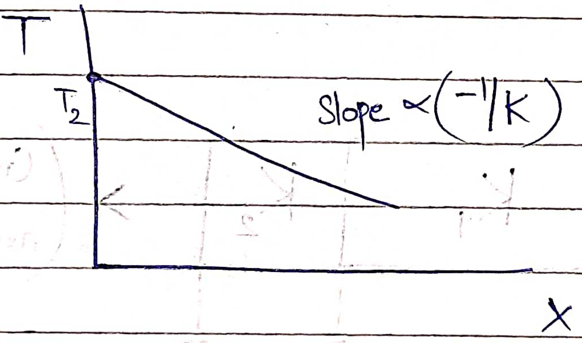
$$\boxed{\left(\frac{Q}{t}\right) = \left(\frac{KA(T_2 - T_1)}{d}\right)}$$

K — Coeff. of Thermal Conductivity (depends of material)

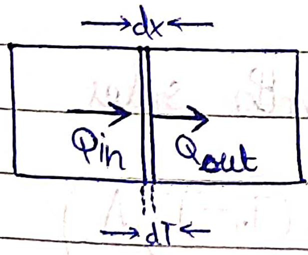


$$\left(\frac{Q}{t}\right) = \frac{KA(T_2 - T)}{x}$$

$$\Rightarrow \quad T = T_2 - \left(\frac{1}{KA} \cdot \frac{Q}{t}\right)x$$



In heat supply started,



$Q_{in} \neq Q_{out}$ as cross section is ~~heating~~ getting heated.

After some time, $Q_{in} = Q_{out} \Rightarrow$ Steady State

for Steady State,

$$\boxed{\left(\frac{dT}{dx}\right)(-KA) = \left(\frac{Q}{t}\right)}$$

(Diff. eq^n for heat flow)

Analogy to ⚡ —

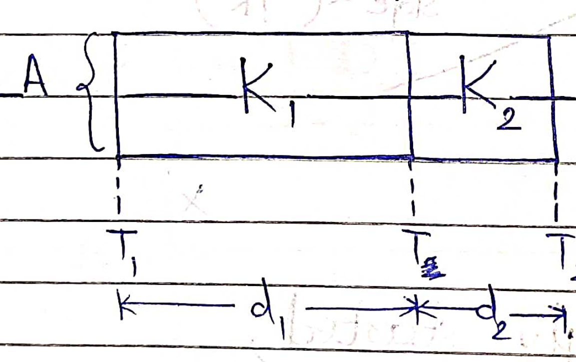
$$i \Longleftrightarrow Q/t$$

$$V \Longleftrightarrow \Delta T$$

$$R \Longleftrightarrow \frac{d}{KA} \leftarrow \text{Thermal Resistance}$$

Combination of Slabs —

1) Series —



(Cross Section Area need NOT be same)

Q/t across junction, same on both sides,

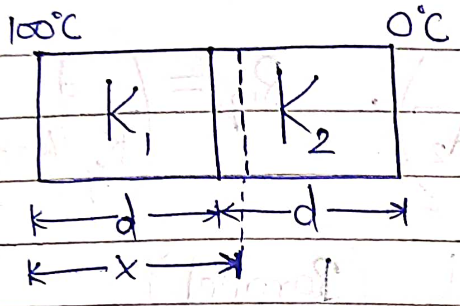
$$\left(\frac{Q}{t}\right) = \left(\frac{K_1(T-T_1)A}{d_1}\right) = \left(\frac{K_2(T_2-T)A}{d_2}\right)$$

Q) If $K_1/K_2 = 2$, $d_1/d_2 = 1/2$, $(T_1 - T_2) = 24$ K

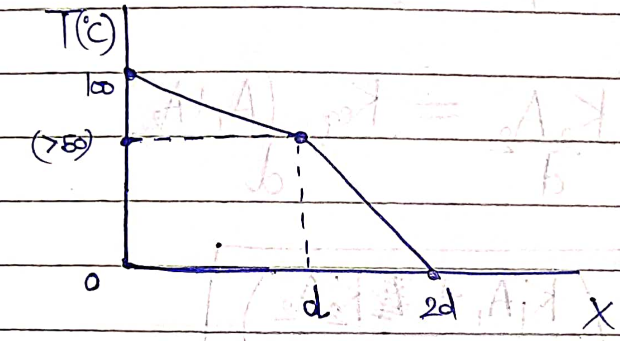
find $(T_1 - T_{\text{junction}})$

A) $$\frac{K_1(T-T_1)}{d_1} = \frac{K_2(T_2-T)}{d_2} \Rightarrow \frac{4}{1} = \left(\frac{24-(T_1-T)}{(T_1-T)}\right)$$

$$\Rightarrow \frac{(96/5)}{(24/5)} = \left(\frac{24-(T_1-T)}{(T_1-T)}\right) \Rightarrow \boxed{(T_1-T) = 4.8\,K}$$

Q) 

$K_1 > K_2$. Draw T vs x graph.

A) 

$K_1 > K_2$

$\Downarrow$

$1/K_1 < 1/K_2$

$\Downarrow$

$|\text{Slope}: 0-d| < |\text{Slope}: d-2d|$

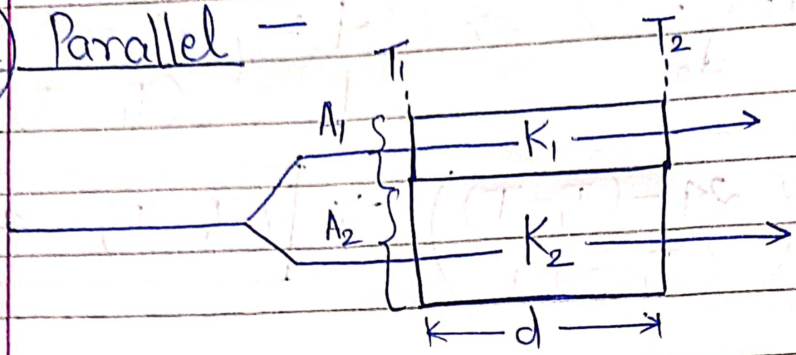
1.1) Equivalent K :

$$R_1 = \left(\frac{d_1}{K_1 A}\right), \quad R_2 = \left(\frac{d_2}{K_2 A}\right), \quad R_{eq} = \left(\frac{(d_1+d_2)}{K_{eq} \cancel{A_{eq}}}\right)$$

Now, $R_1 + R_2 = R_{eq}$ (Series)

$$\Rightarrow \boxed{(d_1+d_2)/K_{eq}\cancel{A_{eq}} = d_1/K_1 + d_2/K_2}$$

2) Parallel –



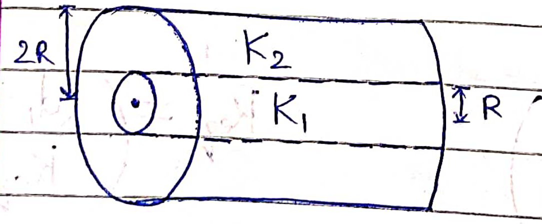
$R_1 = \left(\frac{d}{K_1 A_1}\right)$, $R_2 = \left(\frac{d}{K_2 A_2}\right)$, $R_{eq} = \left(\frac{d}{K_{eq}(A_1+A_2)}\right)$

Now, $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$ (Parallel)

$\Rightarrow \frac{K_1 A_1}{d} + \frac{K_2 A_2}{d} = \frac{K_{eq}(A_1+A_2)}{d}$

$\Rightarrow \boxed{K_{eq} = \left(\frac{K_1 A_1 + K_2 A_2}{A_1 + A_2}\right)}$

Q)

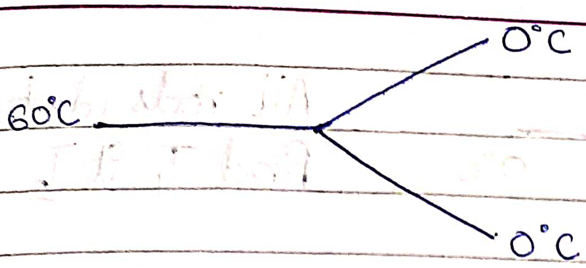


find K_{eq}.

A) $K_{eq} = \frac{K_1(\pi R^2) + K_2(4\pi R^2 - \pi R^2)}{4\pi R^2}$

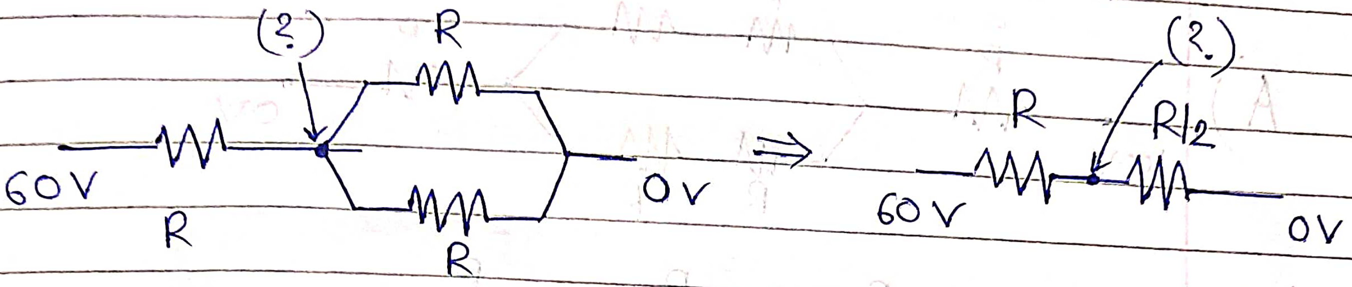
$\Rightarrow \boxed{K_{eq} = \left(\frac{K_1 + 3K_2}{4}\right)}$

Q)



Find temp. at junction. All rods identical.

A)

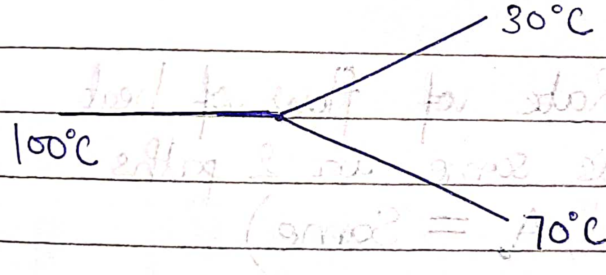


$$i = \frac{60}{3R/2} \Rightarrow i = 40/R$$

By K.V.L, $60 - R(40/R) = (?)$

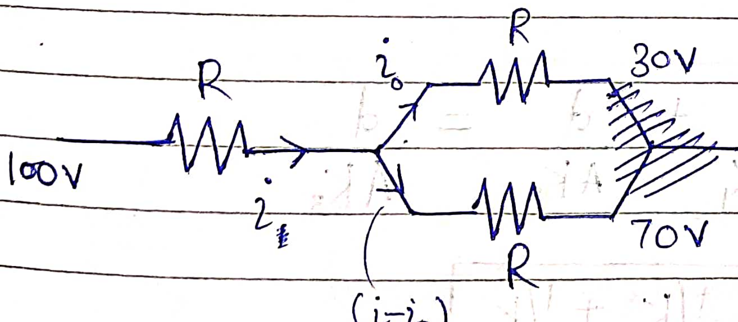
$\Rightarrow$ $(?) = \cancel{40}$ 20 V $\Rightarrow$ $\boxed{T_{junction} = 20°C}$

Q)



Find temp. at junction All rods identical.

A)



$100 - iR - i_0R = 30$ —①

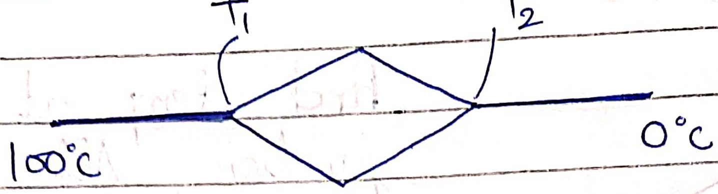
$100 - iR - (i-i_0)R = 70$

$\Rightarrow 100 - 2iR + i_0R = 70$ —②

① + ② $\Rightarrow$ $200 - 3iR = 100 \Rightarrow iR = 100/3$

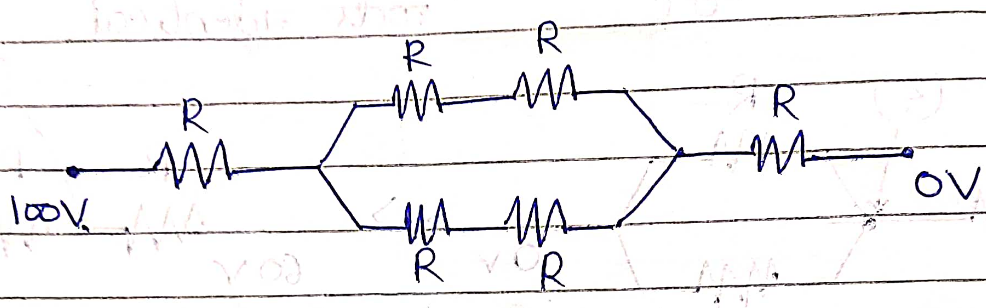
By KVL, $100 - iR = V \Rightarrow V = 200/3 \Rightarrow$ $\boxed{200/3\ °C}$

Q)

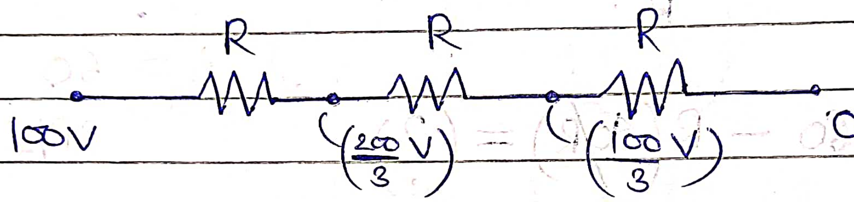


All rods identical
find T_1 & T_2

A)

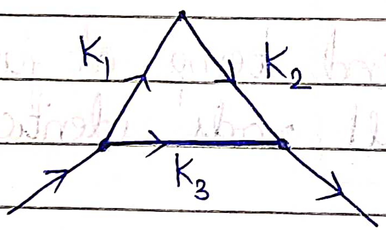


$\Rightarrow$



$\Rightarrow$ $\boxed{T_1 = \left(\frac{200}{3}\right)^\circ C}$ $\boxed{T_2 = \left(\frac{100}{3}\right)^\circ C}$

Q)

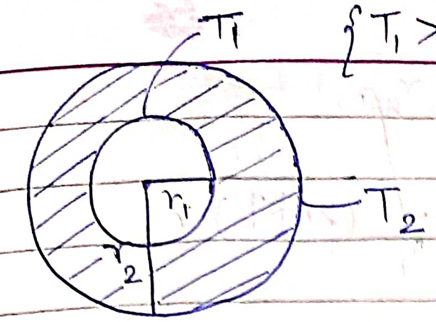


Rate of flow of heat is same in 2 paths
(d, A = Same)

A) $R_1 = R_2 \Rightarrow \frac{d}{AK_1} + \frac{d}{AK_2} = \frac{d}{AK_3}$

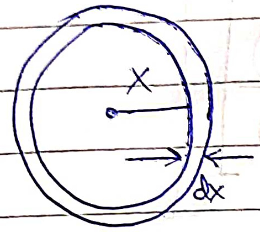
$\Rightarrow$ $\boxed{1/K_3 = 1/K_1 + 1/K_2}$

$\{T_1 > T_2\}$

Q) 

A hollow sphere. find Q/t.

A) Consider a shell of radius 'x'.



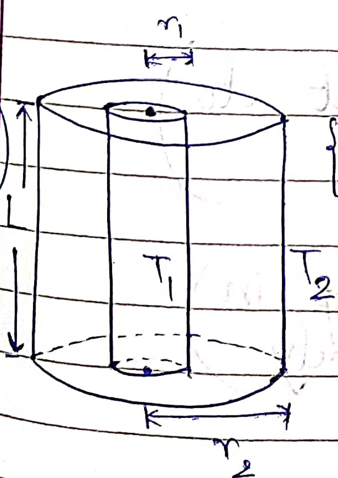
$$dR = \frac{dx}{K(4\pi x^2)}$$

$$\Rightarrow R = \int_{r_1}^{r_2} \frac{1}{4\pi K} \cdot x^{-2}\,dx$$

$$\Rightarrow R = \frac{1}{4\pi K} \cdot \left(\frac{1}{x}\right)_{r_2}^{r_1} \Rightarrow R = \left(\frac{1}{4\pi K}\right)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Hence, ~~Note~~

$$\boxed{\left(\frac{Q}{t}\right) = \frac{(T_1 - T_2)(4\pi k)(r_1 r_2)}{(r_2 - r_1)}}$$

Q) 

$\{T_1 > T_2\}$

A hollow cylinder. find Q/t

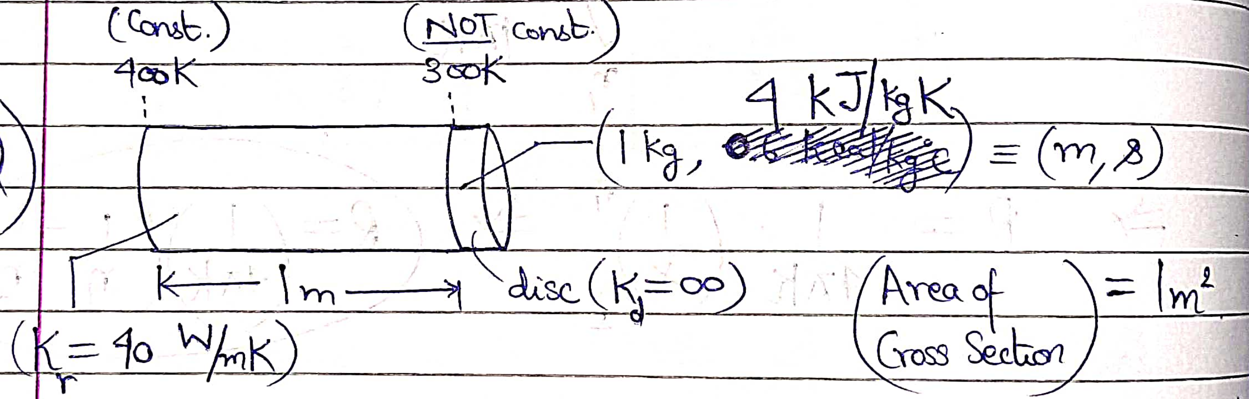
A) Consider a cylindrical shell with radius 'r' & length 'l'.

A) $dR = \dfrac{dr}{K(2\pi r L)} \Rightarrow R = \displaystyle\int_{r_1}^{r_2} \dfrac{dr}{(2\pi K L)\, r}$

$$\Rightarrow R = \left(\frac{1}{2\pi K L}\right) \ln\left(\frac{r_2}{r_1}\right)$$

Hence, $\boxed{\left(\dfrac{Q}{t}\right) = \left[\dfrac{(T_1 - T_2)(2\pi K L)}{\ln(r_2) - \ln(r_1)}\right]}$

Q)



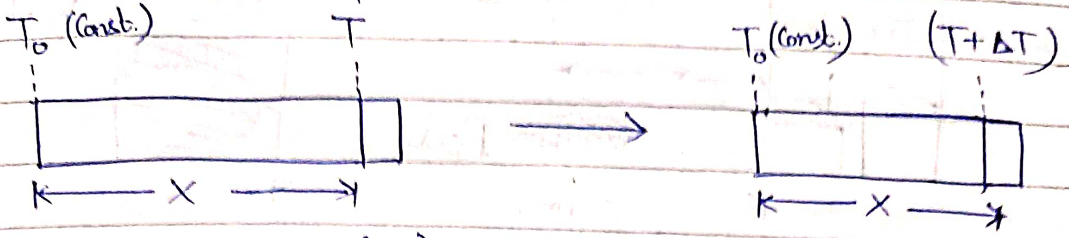
Find time taken ~~by~~ to inc. disc's temp by 50°C. Init temp of disc = 300K.

A) $K_{disc} = \infty \Rightarrow$ ~~Alwa~~ (Allows heat to pass easily).

$\Rightarrow (dT/dx) = 0 \Rightarrow$ (Same temp in whole body).

$\left\{ (dT/dx) = \dfrac{(Q/t)}{K \cdot A} \right\}$

Consider (Δt) time period,



Now,
$$Q = \underbrace{(\Delta t)\left(K_r A \cdot \frac{(T - T_0)}{x}\right)}_{\text{Heat transferred in rod}} = \underbrace{ms(\Delta T)}_{\text{Heat taken by disc.}}$$

$$\Rightarrow \left(\frac{\Delta T}{\Delta t}\right) = \left(\frac{K_r A}{x\, ms}\right)(T - T_0)(-1)$$

$$\Rightarrow \left(\frac{dT}{dt}\right) = \left(\frac{K_r A}{x\, ms}\right)(T - T_0)(-1)$$

$$\Rightarrow \left(-\int_{T_1}^{T_2} \frac{dT}{T - T_0}\right) = \int_0^t \left(\frac{K_r A}{x ms}\right) dt$$

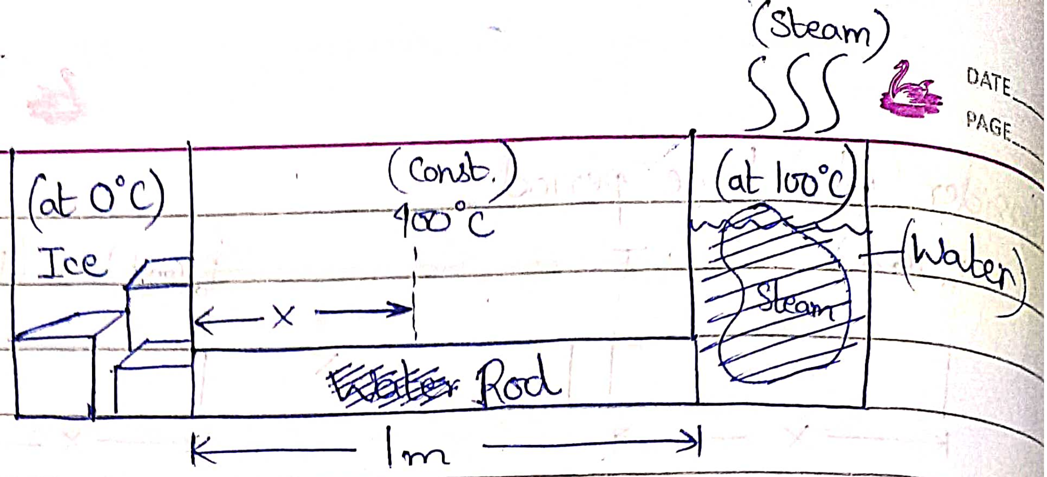
$$\Rightarrow \boxed{\ln\left(\frac{T_1 - T_0}{T_2 - T_0}\right) = \left(\frac{K_r A}{x\, ms}\right) t}$$

In this Q, $T_0 = 400\,K$, $T_1 = 300\,K$, $T_2 = 350\,K$,
$K_r = 40 \frac{W}{mK}$, $x = 1\,m$, $A = 1\,m^2$,
$m = 1\,kg$ $\quad s = 4\,kJ/kg.K$

$$\Rightarrow \left(-\ln\left(\frac{350 - 400}{300 - 400}\right)\right) = \left(\frac{40 \cdot 1}{1 \cdot 1 \cdot 4 \times 10^3}\right) t$$

$$\Rightarrow \boxed{t = 69.8}$$

Q)



Given, $\begin{pmatrix}\text{Rate of Melting} \\ \text{of Ice}\end{pmatrix} = \begin{pmatrix}\text{Rate of forming} \\ \text{of Steam}\end{pmatrix}$

find 'x'. $\left(L_f = 80 \text{ cal/g},\ L_v = 540 \text{ cal/g}\right)$

A) $\left(\dfrac{Q_{ice}}{t}\right) = L_f\left(\dfrac{m_{ice\ formed}}{t}\right) = \dfrac{(KA)(400)}{x}$

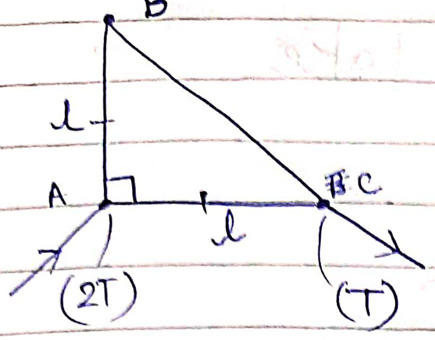
$\left(\dfrac{Q_{water}}{t}\right) = L_v\left(\dfrac{m_{steam\ formed}}{t}\right) = \dfrac{(KA)(300)}{(1-x)}$

Given, $\left(\dfrac{m_{ice\ formed}}{t}\right) = \left(\dfrac{m_{steam\ formed}}{t}\right)$

$\Rightarrow \dfrac{(KA)(400)}{x \cdot L_f} = \dfrac{(KA)(300)}{(1-x)\ L_v}$

$\Rightarrow \left(\dfrac{x}{1-x}\right) = \left(\dfrac{400}{300}\right)\left(\dfrac{540}{80}\right) = 9$

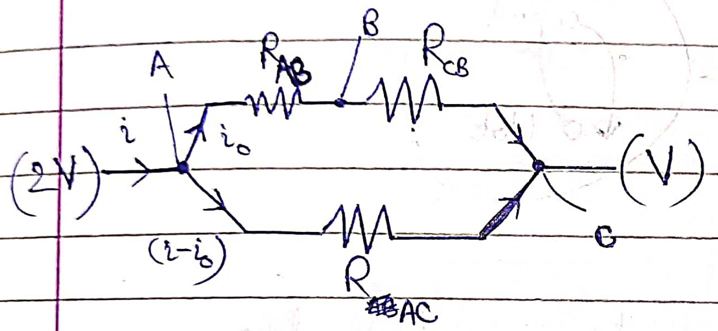
$\Rightarrow \boxed{x = 9/10}$

Q) 

find T_B.

(All rods of same material)

A) $R_{AC} = \frac{l}{AK}$, $R_{BC} = \frac{l\sqrt{2}}{AK}$, $R_{AB} = \frac{l}{AK}$



$R_{eq.} = \frac{1}{\sqrt{2}} \cdot \frac{l}{AK}$

$\Rightarrow$ $i = \left(\frac{VAK}{l}\right)\sqrt{2}$

By K.V.L., $2V - \left(\frac{l}{AK}\right)(\sqrt{2}+1) \cdot i_0 = V$

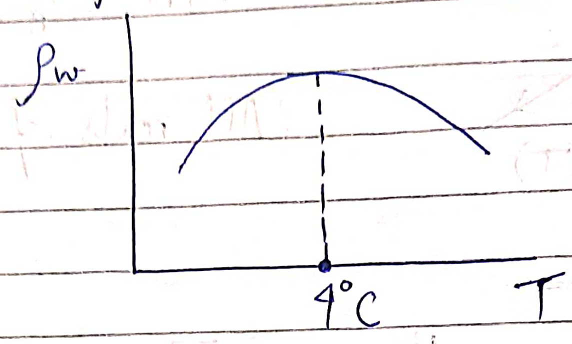
$\Rightarrow$ $i_0 = \left(\frac{VAK}{l}\right)(\sqrt{2}-1)$

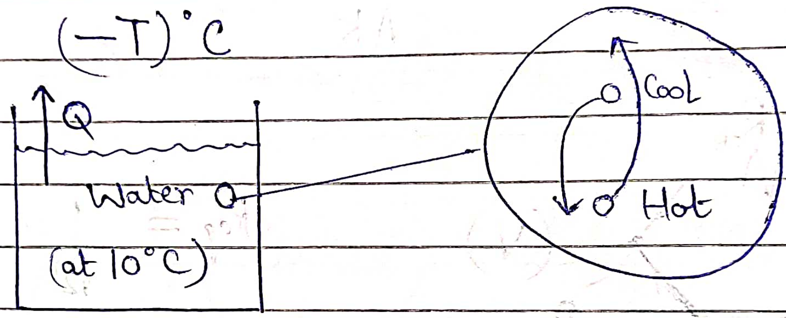
Now, $2V - i_0\left(\frac{l}{AK}\right) = V_B \Rightarrow V_B = 2V - V(\sqrt{2}-1)$

$\Rightarrow$ $V_B = (3-\sqrt{2})V$

Hence, $\boxed{T_B = (3-\sqrt{2})T}$

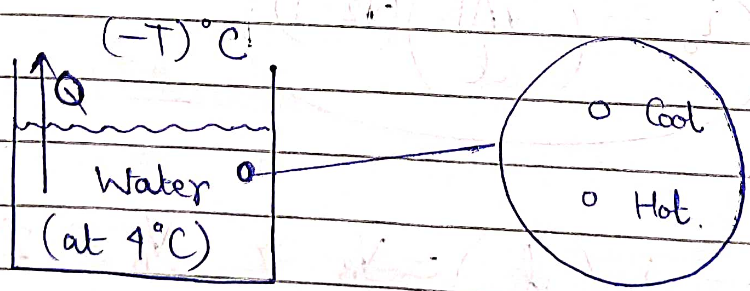
Growth of Ice in a Lake





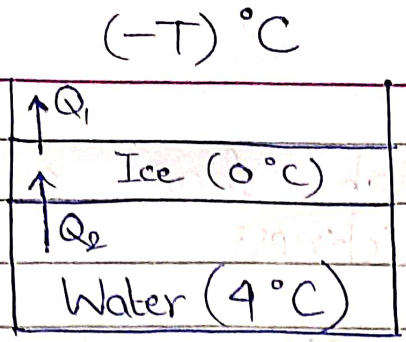
Cool more dense particles come down hot less dense particles come up. ⇒ (Uniform Cooling.)

This happens till all water at 4°C.



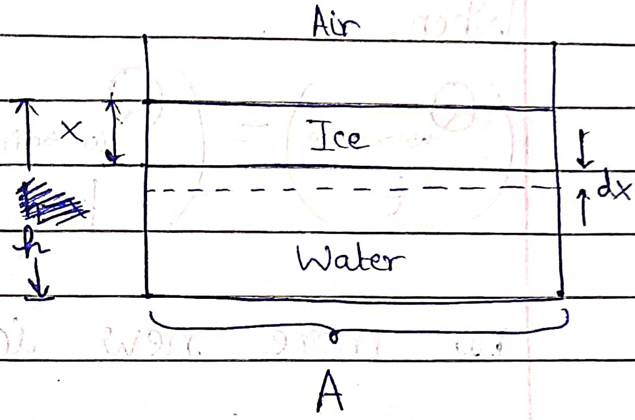
After reaching 4°C temp., cooler molecules are less dense & remain at top.

Hence, top layer first reaches 0°C.



Ice conducts heat from water to air.

Let us find thickness of ice slab. Consider a slab of thickness 'x' is already present.



$$(M_{new\ ice}) = \rho_{ice} \cdot A \cdot dx$$

Now, heat absorbed by new ice

$$dQ = (M_{new\ ice}) L_f$$

$$= \rho_{ice} \cdot A \cdot L_f \cdot dx$$

$$\Rightarrow \left(\frac{dQ}{dt}\right) = \rho_{ice} \cdot A \cdot L_f \left(\frac{dx}{dt}\right)$$

By heat conduction thru ice, $\left(\frac{dQ}{dt}\right) = \frac{\boxed{0-(-T)}}{x}(K_{ice} A)$

$$\Rightarrow \left(\frac{dQ}{dt}\right) = (K_{ice} A)\left(\frac{T}{x}\right)$$

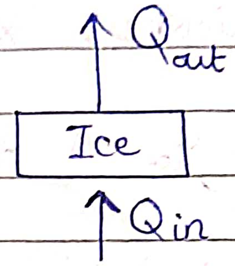
Equating, $\rho_{ice} \cdot \cancel{A} \cdot L_f \left(\frac{dx}{dt}\right) = K_{ice} \cdot \cancel{A} \cdot \frac{T}{x}$

$$\Rightarrow \int_{x_i}^{x_f} x \, dx = \int_0^t \left(\frac{K_{ice} \cdot T}{\rho_{ice} \cdot L_f}\right) dt$$

$$\Rightarrow \boxed{t \propto (x_f^2 - x_i^2)}$$

Now, the slab has a max possible thickness.

Since from 0°C – 4°C, water transfers heat via conduction, we have a water slab.



When,

$$\left(\frac{Q_{water \to ice}}{t}\right) = \left(\frac{Q_{ice \to air}}{t}\right)$$

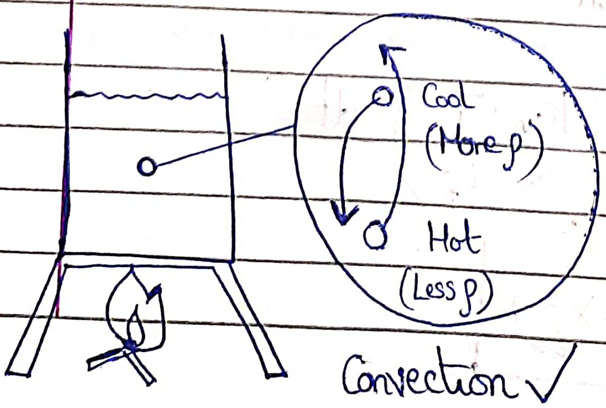
no more new ice forms.

$$\Rightarrow (K_w \cancel{A})\left(\frac{4-0}{h-x}\right) = (K_{ice}\cancel{A})\left(\frac{0-(-T)}{x}\right)$$

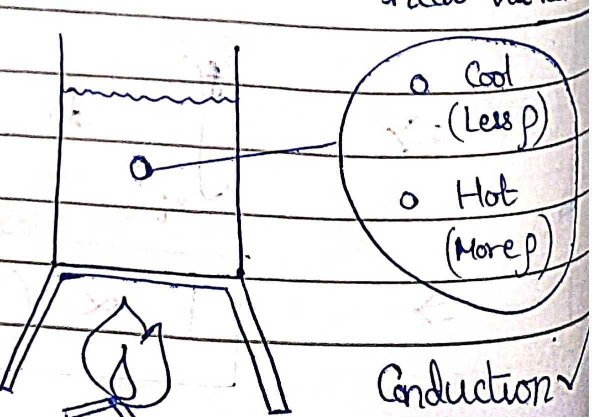
$$\Rightarrow \boxed{x_{max.} = \left(\frac{T K_{ice} \cdot h}{T K_{ice} + 4 K_w}\right)}$$

Convection

Happens in a fluid whose 'ρ' DEC. on INC. T
If a fluid's 'ρ' INC. with INC. T, then it tra
heat via condu



Convection ✓



Conduction ✓

Radiation

Black Body Radiation

1) Stefan's Law — $\boxed{E = \sigma A T^4}$

E = (Emissive Power) = (Energy given off per sec.)

A = (Surface Area)

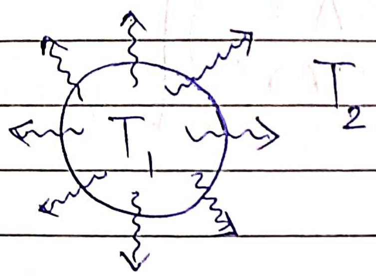
T = (Abs. temp.)

σ = Const.

$$\boxed{\sigma = (\text{Stefan Boltzmann's Const.}) = (5.67 \times 10^{-8}\ W/m^2K^4)}$$

For a body with surface area A at temp. T.

Eg:



if $T_1 < T_2$,

then ALSO body radiate heat.

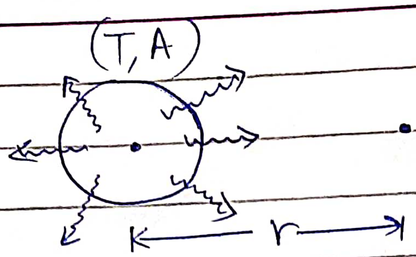
It just happens that it absorbs more than it radiates.

$Q_{radiated}$ depends on T_{body}

$Q_{absorbed}$ depends on T_{surr}

★ Every body above 0°K radiates energy

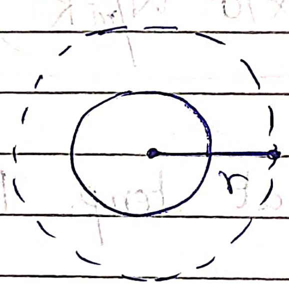
117

Q) 

find intensity at P.

A) $\left(\text{Intensity}\right) = \left(\begin{array}{c}\text{Energy per Area}\\ \text{per sec.}\end{array}\right)$

Radiation spreads out in a Sphere.



$\left(\begin{array}{c}\text{Surface Area}\\ \text{of Spreading}\end{array}\right) = 4\pi r^2$

$\Rightarrow \boxed{I = \left(\dfrac{\sigma A T^4}{4\pi r^2}\right)}$

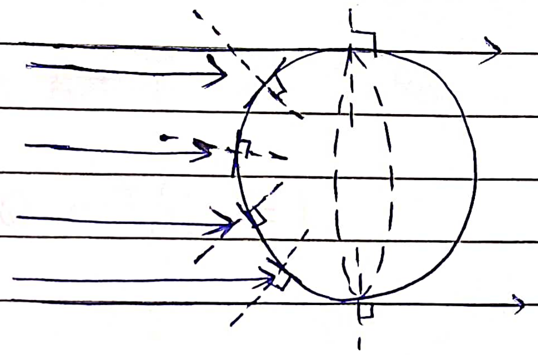
☆

for pt / spherical ~~body~~ source, $\boxed{I \propto 1/r^2}$

for cylindrical source, $\boxed{I \propto 1/r}$

for plane source, $\boxed{I \propto r^0}$

Since $r >> R$, we can assume rays are coming $\parallel$ & dist. of every pt. on Earth from Sun $= r$.



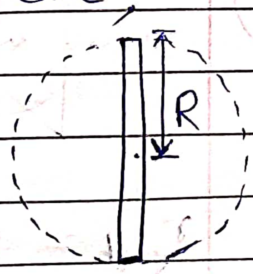
Now, ~~since~~ only half receive & at every pt. ray are NOT normal to surface.

$\boxed{\text{Only Normal component of rays is absorbed !} \quad \text{(i.e. comp. of rays normal to surface)}}$

$$\boxed{(\text{Rate of Absorption}) = (\text{Intensity})\left(\text{Effective Absorption Area}\right)}$$

By integration we get in above case,

$$\boxed{\left(\begin{array}{c}\text{Effective Area}\\ \text{of Absorption}\end{array}\right) = \pi R^2}$$



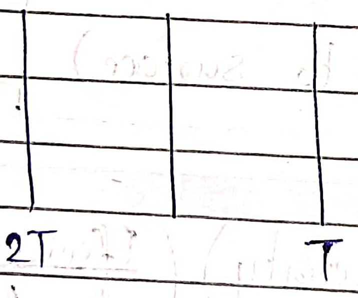
i.e. area of disc thru centre of earth normal to line joining centres of Earth & Sun.

$$\left(\text{Rate of Absorption}\right) = \left(\frac{\sigma A T^4}{4\pi r^2}\right)\left(\pi R^2\right)$$

Earth absorbs & emits heat simultaneously.

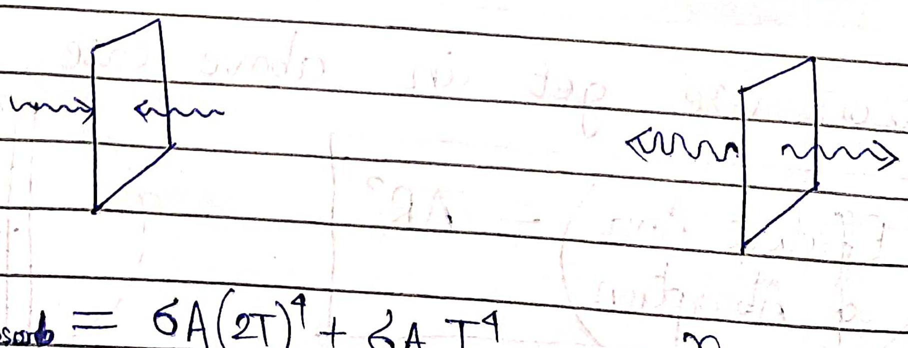
Now, $$\left(\text{Rate of Emission}\right) = \sigma\left(4\pi R^2\right)\left(T^4\right).$$

Q) 3 identical plane sources.



Find temp. of middle plate.

A)



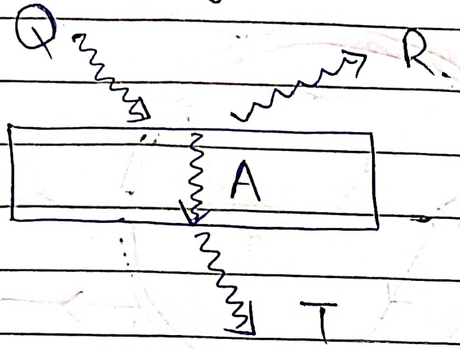
final temp

$$r_{absorb} = \sigma A(2T)^4 + \sigma A T^4 \qquad r_{radiate} = \sigma(2A)\left(T'\right)^4$$

emit from both side

$$r_{absorb} = r_{radiate} \Rightarrow \boxed{T' = T\left(\frac{17}{2}\right)^{1/4}}$$

When heat is given to a body,



It can be — reflected (R), absorbed (A) or transmitted (T)

Now, $Q = R + A + T$

$\Rightarrow \quad R/Q + A/Q + T/Q = 1$

$\Rightarrow \quad \boxed{r + a + t = 1}$

reflecivity — r; absorptivity — a; transmitivity — t

If body is Opaque $\Longleftrightarrow$ $\boxed{t = 0}$

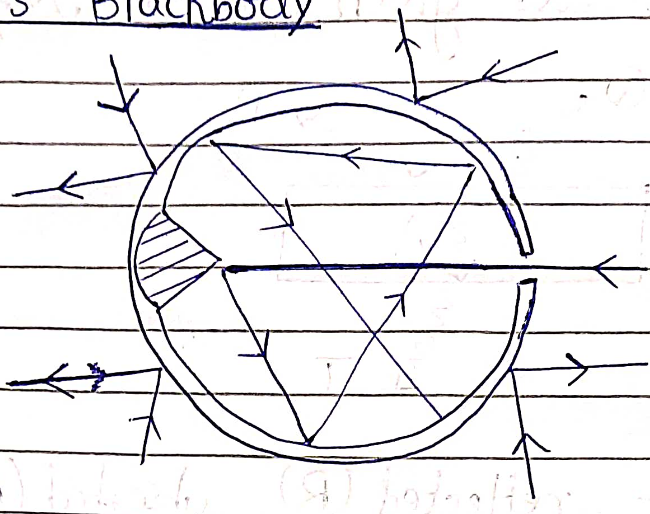
If body is Blackbody $\Longleftrightarrow$ $r = 0,\ t = 0$

$\Rightarrow \boxed{a = 1}$

☆ **Transmission ≠ Radiation ≠ Reflection.**

1) Transmission — ~~Radiation~~ Rays passes w/o affecting body (passes right thru)
2) Radiation — Remitting of rays after absorbing.
3) Reflection — Rays jis medium se ati hain, usi main chali jati hain!

Ferry's Blackbody



1) Blackened inside walls
2) Small enterance hole.
3) Reflector

Any light comes thru opening, ~~the~~ gets refleced to sides & remains inside. The sides absorb the light, one after another until whole light absorbed.

The opening behaves like a Blackbody!

The other surface are NOT blackbody as they may reflect light bouncing on them.

$$\boxed{\text{Emissivity } (\epsilon) = \left(\frac{\text{Emissive power of body}}{\text{Emissive power of body, if it were blackbody}} \right)}$$

Blackbody $\Longleftrightarrow$ $\epsilon = 1$

Whitebody $\Longleftrightarrow$ $\epsilon = 0$

Graybody $\Longleftrightarrow$ $0 < \epsilon < 1$

for a graybody with temp (T) & surface area (A),

$$\boxed{\left(\text{Energy given out per sec}\right) = \epsilon \cdot \sigma A T^4}$$

If surr. at 'T_0' temp, $\boxed{\left(\text{Energy absorbed/sec}\right) = \epsilon \cdot \sigma A T_0^4}$

$$\boxed{\left(\text{Net energy given out per sec}\right) = \epsilon \cdot \sigma A \left(T^4 - T_0^4\right)}$$

Kirchoff's Law

$$\boxed{\left(\frac{\text{Emissive Power}}{\text{Absoptive Power}}\right) = \text{Const.}}$$

Since this is true for all objs,

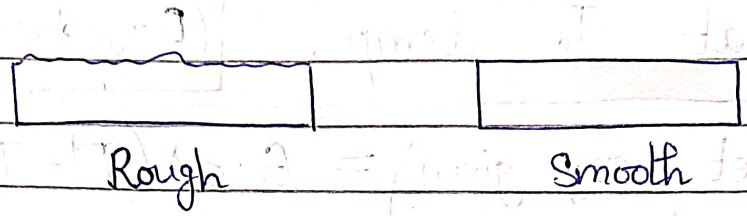
$$\left(\frac{\text{Emissive Power}}{\text{Absorptive Power}}\right)_{\text{Body}} = \left(\frac{\text{Emissive Power}}{\text{Absorptive Power}}\right)_{\left(\text{Body, if it were blackbody}\right)} = \left(\text{Emissive Power}\right)_{\left(\text{if body were blackbody}\right)}$$

$$\Rightarrow \left(\frac{\epsilon \cdot \sigma A T^4}{a}\right) = \left(\sigma A T^4\right) \Rightarrow \boxed{\epsilon = a}$$

$$\Rightarrow \boxed{\text{Emitivity} = \text{Absorptivity}}$$

$\therefore$ Good absorbers are Good emitters

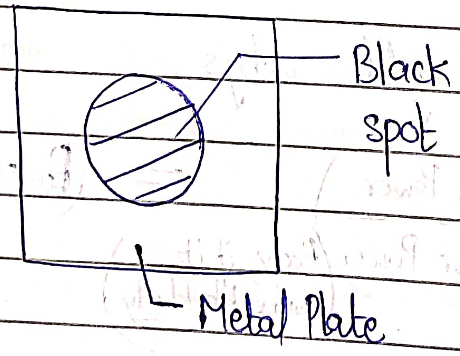
Application —

1) 

Since, $r_{rough} < r_{smooth}$ & $a + r = 1$
(Opaque body)

$\Rightarrow \quad a_{rough} > a_{smooth}$

$\Rightarrow$ Rough body emits more than smooth body

2) 

It is first heated in furnace, then taken in a dark room.

Then, Black spot will glow the brightest.

3) 

Red glass plate

It is heated in furnace & then taken in a dark room.

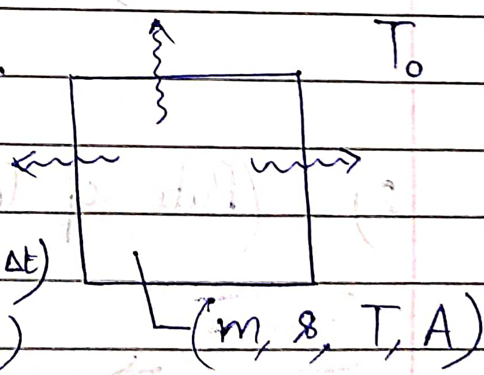
It appears Green.

Reason: Obj. looking red ⇒ It reflects Red most.
⇒ It absorbs Green most.

When kept in dark room, it reflects no light.
Only light ~~for~~ from emmission only ⇒ Green (Antired) most detected

Cooling

If $T > T_0$, it emits radiation.



$$\begin{pmatrix}\text{Heat}\\ \text{Loss}\end{pmatrix} = ms\,(-\Delta T)$$

(if temp. of body red. by $|\Delta T|$ in time Δt)

(as temp. dec)

$$\Rightarrow \begin{pmatrix}\text{Rate of}\\ \text{Heat Loss}\end{pmatrix} = ms\left(-\frac{\Delta T}{\Delta t}\right)$$

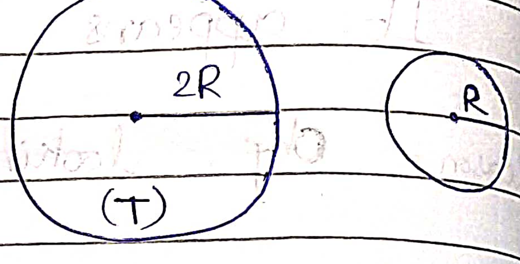
$$= \epsilon \cdot A \sigma \cancel{A} \left(T^4 - T_0^4\right)$$

$$\Rightarrow \boxed{-ms\left(\frac{dT}{dt}\right) = \epsilon \sigma A\left(T^4 - T_0^4\right)}$$

(Diff. Eq^n of Cooling)

Observe, $\left(\text{Rate of Cooling}\right) \propto \left(\frac{A}{m}\right)$ $\left(\text{Rate of Heat Loss}\right) \propto A$

Q) 2 solid spheres are made of same material & have ~~item~~ identical surface finish.



1) find ratio of rate of heat loss.

2) find ratio of rate of ~~heat~~ temp. loss (= cooling)

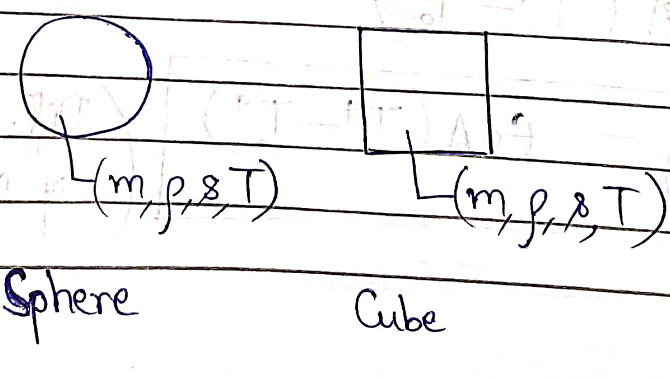
A) 1) $\left(\text{Rate of Heat Loss}\right) \propto A \propto R^2$

$\Rightarrow \boxed{4:1}$

2) $\left(\text{Rate of Cooling}\right) \propto \left(\frac{A}{m}\right) \propto \left(\frac{R^2}{\rho R^3}\right) \propto \left(\frac{1}{R}\right)$

$\Rightarrow \boxed{1:2}$

Q)



Same surface finish.

find ratio of rate of temp. loss.

A) m same, ρ same $\Rightarrow$ V same

$$\Rightarrow \frac{4}{3}\pi R^3 = L^3$$

Now, $\left(\text{Rate of Cooling}\right) \propto \left(\frac{A}{m}\right) = \left(\frac{A}{\rho V}\right)$

$$\Rightarrow \left(\frac{r_1}{r_2}\right) = \left(\frac{A_1}{A_2}\right)\left(\frac{V_2}{V_1}\right) = \left(\frac{4\pi R^2}{6L^2}\right)\cancel{\frac{4}{3}\pi R^3}\left(\frac{L^3}{\frac{4}{3}\pi R^3}\right)$$

$$\Rightarrow \left(\frac{r_1}{r_2}\right) = \left(\frac{L}{2R}\right) = \left(\frac{1}{2}\right)\left(\frac{4\pi}{3}\right)^{1/3}$$

$$\Rightarrow \boxed{\left(r_1/r_2\right) = \left(\pi/6\right)^{1/3}}$$

Stefan's Law of Cooling — $\boxed{\left(\frac{dT}{dt}\right) \propto \left(T^4 - T_0^4\right)}$

Newton's Law of Cooling — If temp. diff. small,

$(T+T_0) \approx 2T_0$ & $(T^2+T_0^2) \approx 2T_0^2$

$$\Rightarrow \left(\frac{dT}{dt}\right) \propto (T-T_0)(T+T_0)(T^2+T_0^2) \approx (T-T_0)(2T_0)(2T_0^2)$$

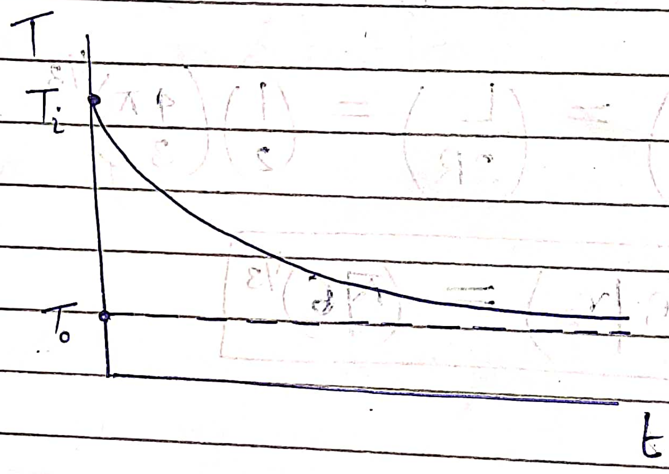
$$\Rightarrow \left(\frac{dT}{dt}\right) \propto (T-T_0) \Rightarrow \boxed{\left(\frac{-dT}{dt}\right) = k\,(T-T_0)}$$

This is Newton's Law of Cooling

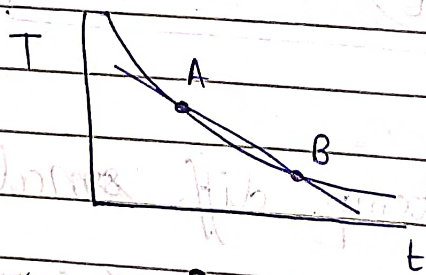
$$\Rightarrow \quad -\int_{T_i}^{T_f} \frac{dT}{T-T_0} = K\int_0^t dt$$

$$\Rightarrow \quad \ln\left(\frac{T_i - T_0}{T_f - T_0}\right) = Kt$$

$$\Rightarrow \quad \boxed{(T_f - T_0) = (T_i - T_0)\,e^{-Kt}}$$



for solving Q,



$$-\left(\frac{dT}{dt}\right) \approx \left(\frac{T_A - T_B}{t}\right)$$

&

$$\left\{ T_m = \left(\frac{T_A + T_B}{2}\right) \right\} \qquad -\left(\frac{dT}{dt}\right) = k\,(\underset{\text{mean.}}{T_m} - \underset{\text{surr.}}{T_0})$$

$$\Rightarrow \quad \boxed{\left(\frac{T_A - T_B}{t}\right) = k\,(T_m - T_0)}$$

Q) $70^\circ C \rightarrow 60^\circ C \Rightarrow t = 10$ min.
$60^\circ C \rightarrow 50^\circ C \Rightarrow t = ?$

$T_{surr} = 25^\circ C$

A) M1 :

$$(T_f - T_o) = (T_i - T_o)\, e^{-kt}$$

$$\Rightarrow \left(\frac{45}{35}\right) = (e^k)^{(10)}$$

$$\& \quad \left(\frac{35}{25}\right) = (e^k)^{t}$$

$$\Rightarrow \frac{t}{10} = \left(\frac{\ln(35) - \ln(25)}{\ln(45) - \ln(35)}\right) \Rightarrow \boxed{t \approx 13.38 \text{ min}}$$

M2 :

$$\left(\frac{T_A - T_B}{t}\right) = k\,(T_m - T_o)$$

$$\Rightarrow \left(\frac{10}{10}\right) = k\,(65 - 25) \quad \& \quad \left(\frac{10}{t}\right) = k\,(55-25)$$

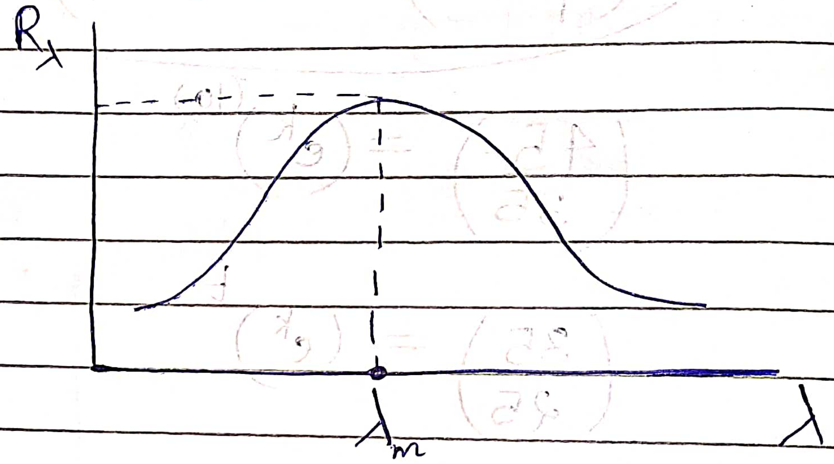
$$\Rightarrow \left(\frac{t}{10}\right) = \left(\frac{65-25}{55-25}\right) \Rightarrow \boxed{t = 40/3 \text{ min}}$$

DATE
PAGE

Plank's Distribution of Energy

An obj. above 0K, emits raditions with all possible λ.

Some emitted in more qty, some in le



R_λ – Radiation emitted at a specific λ

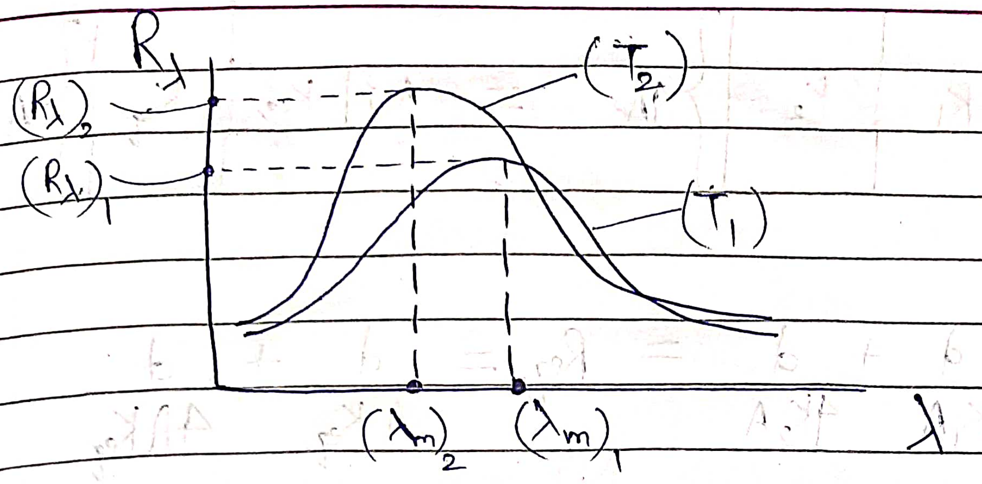
λ_m – λ ~~at~~ with which R_λ max.

☆ Area ~~are~~ under $R_\lambda - \lambda$ graph gives $\left(\frac{\text{Energy emitted}}{\text{sec.}}\right)$ by the body.

Wein's Disp. Law

$$\boxed{T \cdot \lambda_m = b}$$

Wien's Const – $b = 2.9 \times 10^{-4}$ K·m



$(R_\lambda)_2 > (R_\lambda)_1 \Rightarrow$ Higher Peak

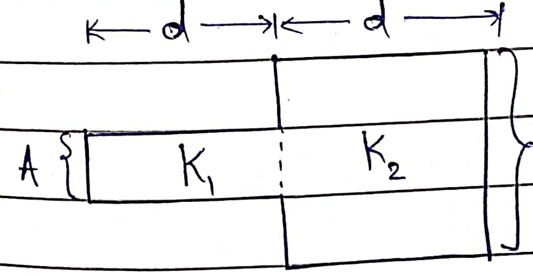
$(\lambda_m)_2 < (\lambda_m)_1 \Rightarrow$ Peak comes Earlies

$T_2 > T_1 \Rightarrow$ Temp. Higher.

Now, $E \propto T^4$

$$\Rightarrow \boxed{(\text{Area under curve}) \propto T^4}$$

1) ☆



find K_{eq}

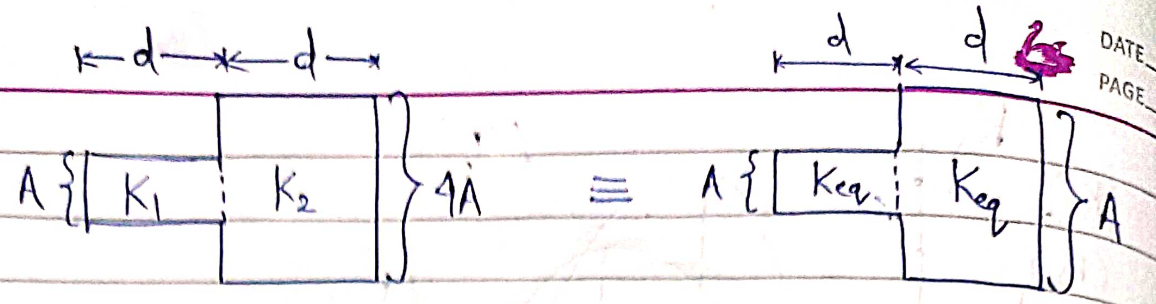
A) We will have problem if we directly apply R_{eq} as we would need A_{eq}.

Instead use defn of K_{eq}. ☆ $\boxed{\text{Replace } K_i \text{ (with) } K_{eq} \text{ everywhere!}}$

DATE
PAGE

3

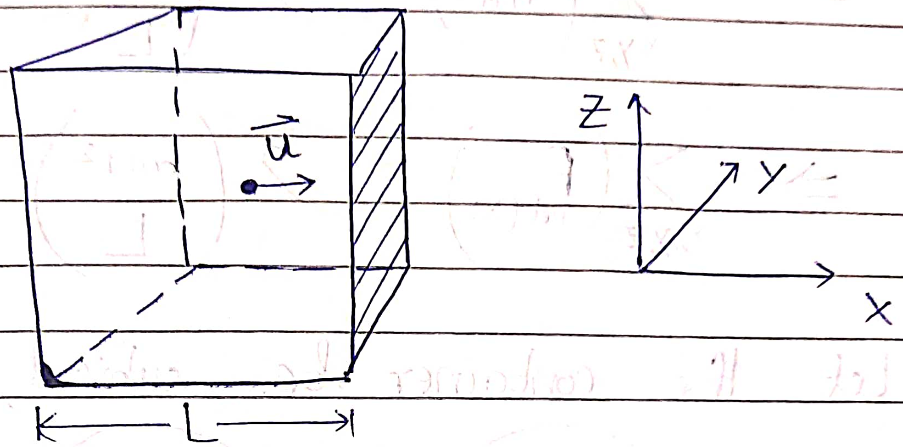
A)



$$\frac{d}{K_1 A} + \frac{d}{4K_2 A} = R_{eq} = \frac{d}{A K_{eq}} + \frac{d}{4 A K_{eq}}$$

$$\Rightarrow \quad \boxed{K_{eq} = \left(\frac{5}{4\left(1/K_1 + 1/4K_2 \right)} \right)}$$

K.T.G



We assume all ~~xxx~~ collisions are perfectly elastic.

Now, $\Delta p_x = 2mu_x$

[Time b/w 2 collisions (succesive) with the same wall] $= \left(\frac{2L}{u_x}\right)$

$$\left(\text{Rate of } \Delta p_x \text{ w.r.t. Time}\right) = \frac{\Delta p_x}{(\text{Time b/w 2 collisions})}$$

$$\Rightarrow \left(\text{Force By wall ON particle } (X)\right) = \frac{(2mu_x)}{(2L/u_x)}$$

$$\Rightarrow \left(\text{Force ON wall By particle } (X)\right) = \left(\frac{mu_x^2}{L}\right)$$

$$\Rightarrow \boxed{f_x = \left(\frac{mu_x^2}{L}\right)}$$

(by 1 particle)

$$f_{net(x)} = \sum\left(\frac{m u_x^2}{L}\right)$$

$$\Rightarrow \sum_{x,y,z}\left(f_{net(x)}\right) = \sum\left(\frac{m}{L}\left(u_x^2+u_y^2+u_z^2\right)\right)$$

$$\Rightarrow \sum_{x,y,z}\left(f_{net(x)}\right) = \sum\left(\frac{mu^2}{L}\right)$$

Let the container be cubical,

$$\Rightarrow V = L^3 \qquad A = L^2$$

$$\Rightarrow \sum_{x,y,z}\left(\frac{f_{net(x)}}{L^2}\right) = \sum\left(\frac{mu^2}{L^3}\right)$$

$$\Rightarrow \sum_{x,y,z}\left(P_x\right) = \left(\frac{m\cdot n}{L^3}\right)\sum\left(\frac{u^2}{n}\right)$$

$$\Rightarrow \left(P_x+P_y+P_z\right) = \left(\frac{m_T}{V}\right)\left(\overline{u^2}\right)$$

(Total mass of gas)

In absence of gravity,

$$P_x = P_y = P_z = P$$

(called Total P)

$$\Rightarrow \quad 3P = \rho\,(\overline{u^2})$$

$$\Rightarrow \quad \boxed{P = \frac{1}{3}\rho\,(\overline{u^2})}$$

Also, $$PV = \frac{1}{3} m_T (\overline{u^2})$$

$$= \frac{2}{3} \sum \left(\frac{1}{2} m u^2\right)$$

$$\Rightarrow \quad \boxed{PV = \frac{2}{3}(KE_{gas})}$$

Now, we know $\dfrac{P}{\rho} = \dfrac{RT}{M}$ ← (Molar mass of gas)

$$\Rightarrow \quad \left(\frac{RT}{M}\right) = \frac{1}{3}(\overline{u^2})$$

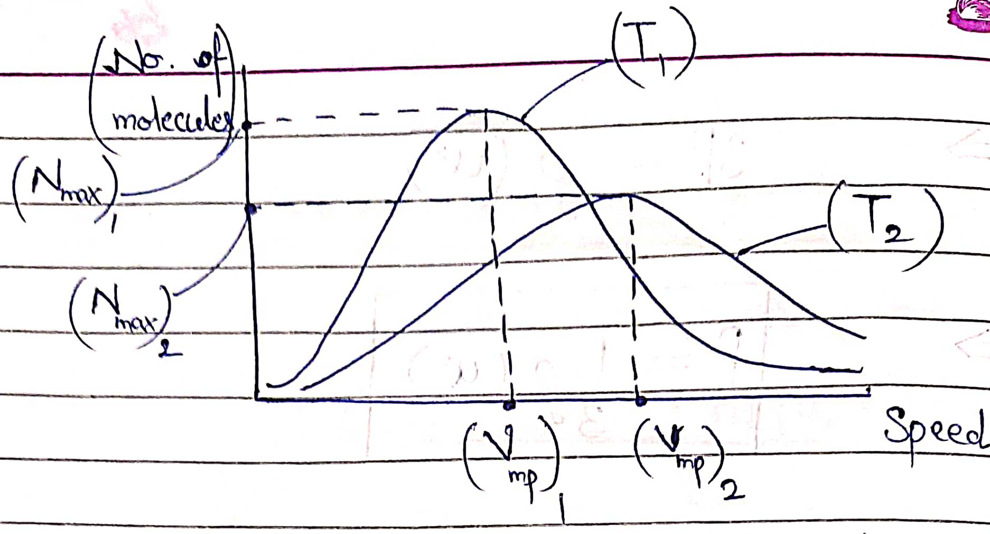
$$\Rightarrow \quad \boxed{v_{rms} = \sqrt{\frac{\sum u^2}{n}} = \sqrt{\frac{3RT}{M}}}$$

$\boxed{\text{Avg. vel.} = 0}$

$\boxed{\text{Avg. speed} = \sqrt{\dfrac{8RT}{\pi M}}}$

$\boxed{(\text{Most Probable Speed}) = \sqrt{\dfrac{2RT}{M}}}$

DATE
PAGE



$$\boxed{N_{max} \propto \left(\frac{1}{T}\right)}$$ $\{T_1 < T_2\}$

Degree of freedom

No. of independent ways in which a molecule can have energy

Translational ⟶ Max. 3

Rotational ⟶ Max. 3

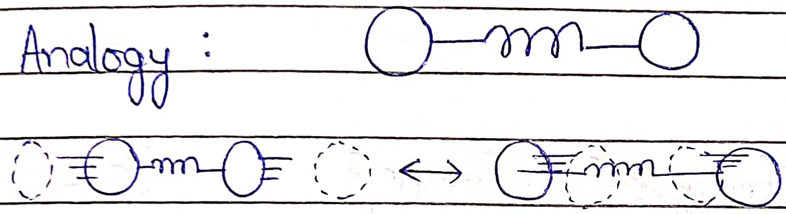
Vibrational ⟶ Max. 2

★ We see rotational degree of freedom about axis, about which MoI of system is NON ZERO.

Atomicity	Trans.	Rot.	Dof
Mono.	3	0	3
Di.	3	2	5
Tri (Linear)	3	2	5
Tri (Non-Linear)	3	3	6

Vibrational Energy –

Analogy :



2 masses attached with a spring to each other

It is generally NOT comparable to other energies

Obiviously, for monoatomic molecule it doesn't exist.

~~Eos~~ Consider vibrational energy in Dof only when temp. high.

And ALWAYS, Dof += 2

if vib. dof. are inc.

Law of Equipartition of Energy

Energy associated with each dof is

$$\frac{1}{2}k_bT$$

k_b = Boltzmann Const. $= \left(\frac{R}{N_A}\right)$

~~Atomicity~~	~~Energy of 1 molecule~~	~~Energy of 1 mol (Internal Energy U_{gas})~~	~~$C_{v,m} = \left(\frac{dU}{dt}\right)$~~
~~Mono.~~	~~$\frac{3}{2}k_bT$~~	~~$\frac{3}{2}RT$~~	~~$\frac{3}{2}R$~~

Now,

(KE of gas = Int. Energy) $\boxed{\frac{U_{gas}}{mol} = (f)\left(\frac{1}{2}RT\right)}$

☆ Use Chem. wale formula. Just add ② to Dof. when high temp is there. Study from Chem. notes.

When ~~mix~~ mix. of gases, $\boxed{U_{mix.} = \sum(U_{gas})}$

Q) ☆ 3 mol O_2 + 2 mol He. Find ratio of avg. KE.

A) $U_{O_2} = (5)\left(\frac{1}{2}RT\right)(3)$ ✗

☆ When avg. mentioned, we need $\left(\frac{\text{Energy}}{\text{Mol.}}\right)$

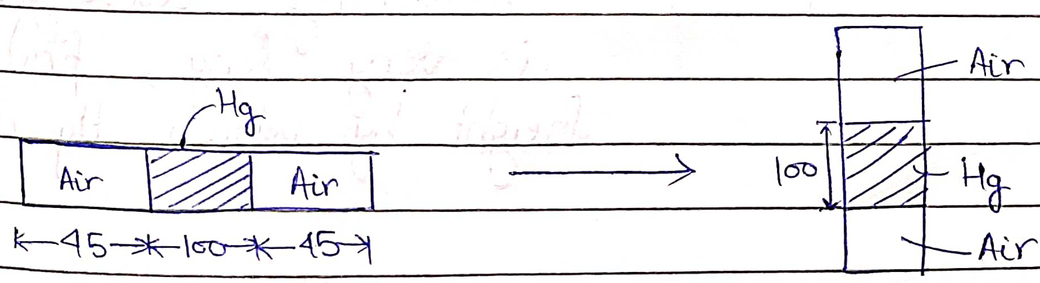
$U_{O_2}/\text{mol} = (5)\left(\frac{1}{2}RT\right)$ $\quad U_{He}/\text{mol} = (3)\left(\frac{1}{2}RT\right)$

$\Rightarrow \boxed{\text{Req.} = 5/3}$

Ideal Gas Eqn

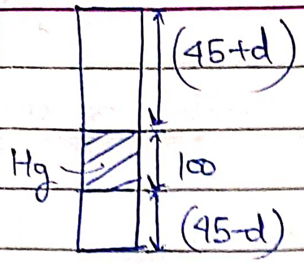
$$\boxed{PV = nRT}$$

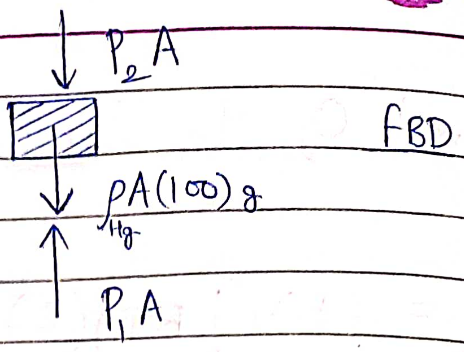
Q)



Air init. at P_0. Find dist. moved by Hg after tube is made vertical.

A)





(PV = Const.)

$$P_0 \cdot (45) = P_1 \cdot (45-d) = P_2 \cdot (45+d)$$

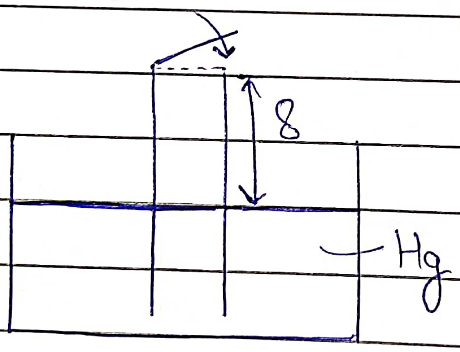
Now,

$$P_2 A + \rho_{Hg} A(100) g = P_1 A$$

$$\Rightarrow \boxed{\rho_{Hg} g(100) + \left(\frac{45 P_0}{45+d}\right) = \left(\frac{45 P_0}{45-d}\right)}$$

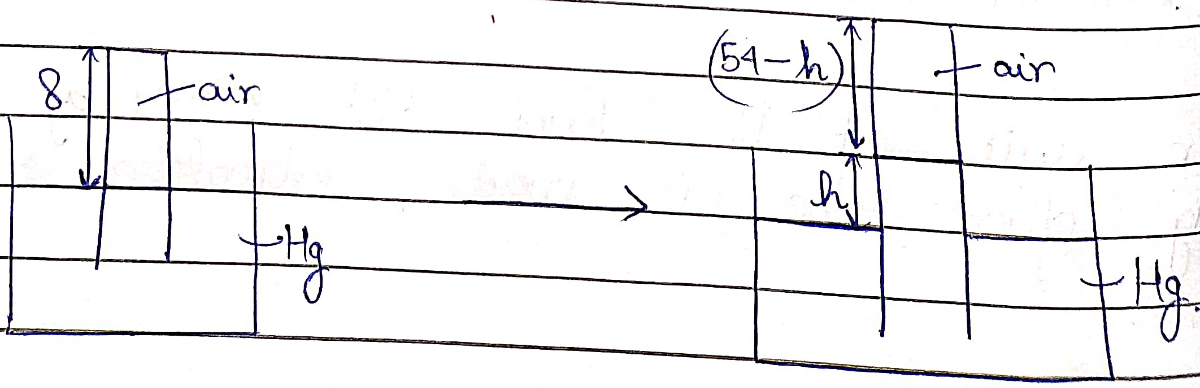
Now solve to find 'd'.

1)



Top of tube is closed & tube is raised further by 46 cm. Assuming tube is very long, find height by which Hg rises.

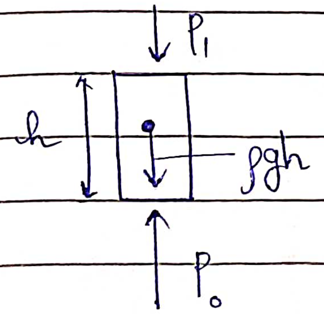
A)



Since, at first, vol. of air $\uparrow \Rightarrow P \downarrow$ (less than P_0)

$\Rightarrow$ Due to P diff, Hg rises up.

Now,



$$P_1 + \rho g h = P_0$$

~~At~~ Also, $P_1 \cdot (54-h) = P_0 \cdot 8 \Rightarrow P_1 = \left(\dfrac{8P_0}{54-h}\right)$

$$\Rightarrow \left(\frac{8P_0}{54-h}\right) + \rho g h = P_0$$

In terms of Hg cm, $\left(\dfrac{8 \cdot 76}{54-h}\right) + h = 76$

$$\Rightarrow 8 \cdot 76 + (54h - h^2) = (76 \cdot 54 - 76h)$$

$$\Rightarrow h^2 - 130h + \cancel{76 \cdot 54} = 0$$

$$76 \cdot 46$$

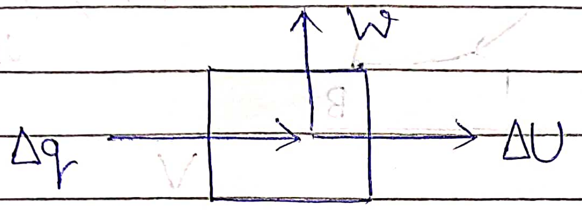
$$\Rightarrow h^2 - 130h + 38 \cdot 92 = 0$$

$$\Rightarrow (h-38)(h-92) = 0$$

$$\Rightarrow \boxed{h = 38 \text{ cm}}$$

Thermodynamics

1st Law of Therm.



$$\boxed{\Delta q = \Delta U + W}$$

☆ To remember use:

$$\boxed{\Delta U = q + (-W)}$$

Sign Convention:

(Heat given To system) $\Longleftrightarrow$ $\Delta q > 0$

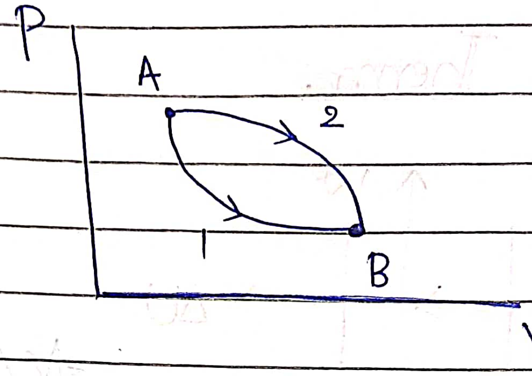
(Heat given By system) $\Longleftrightarrow$ $\Delta q < 0$

(Expansion) (Work done By Gas) $\Longleftrightarrow$ $W > 0$

(Compression) (Work done ON Gas) $\Longleftrightarrow$ $W < 0$

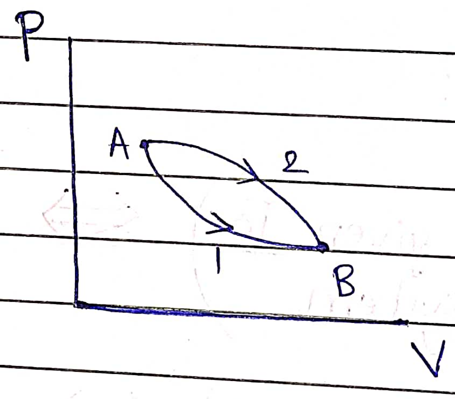
☆ Study therm. chap. (before Carnot stuff) from Chem. notes & put $W \rightarrow (-W)$ in all formulae.

State fx^n : Value depends only on initial & final pt. Doesn't depend on ~~path~~ path.
($P, V, T, \Delta U, \ldots$)



($P, V, T, \Delta U$) same in 1 & 2.

Path fx^n : Value depends on path taken
($W, q, \ldots$)



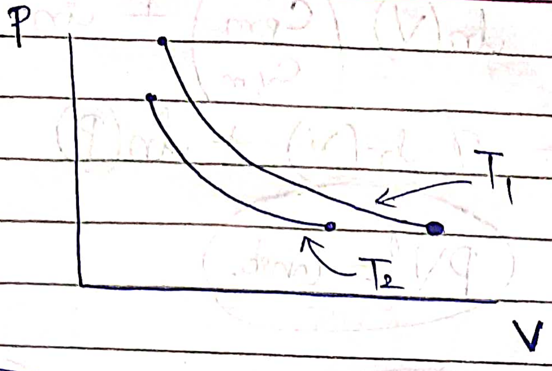
$(W, q)_2 > (W, q)_1$

Various Processes

1) Isothermal (iT) — T is Const.

$\Rightarrow \Delta U = 0$

$\Rightarrow W = q \Rightarrow PV = \text{Const}$



$(T_1 > T_2)$

$$W = (+nRT)\ln\left(\frac{V_2}{V_1}\right)$$

2) Adiabatic (AdB) — $q = 0$

$\Rightarrow W + \Delta U = 0$

Now, $\boxed{PV^{\gamma} = \text{Const.}}$ (only if $C_{v,m}$ independent of temp.)

Derivation: $q = W + \Delta U$

$$\Rightarrow 0 = nC_{v,m}\, dT + PdV$$

$$\Rightarrow 0 = nC_{v,m}\left(\frac{PdV + VdP}{nR}\right) + PdV$$

$$\Rightarrow 0 = P\left(\frac{C_{v,m}}{R} + 1\right)dV + V\left(\frac{C_{v,m}}{R}\right)dP$$

$$\Rightarrow 0 = \left(\frac{1}{V}\right)\left(\frac{C_{p,m}}{R}\right)dV + \left(\frac{1}{P}\right)\left(\frac{C_{v,m}}{R}\right)dP$$

$$\Rightarrow \text{Const.} = \left(\frac{C_{p,m}}{R}\right)\ln(V) + \left(\frac{C_{v,m}}{R}\right)\ln(P)$$

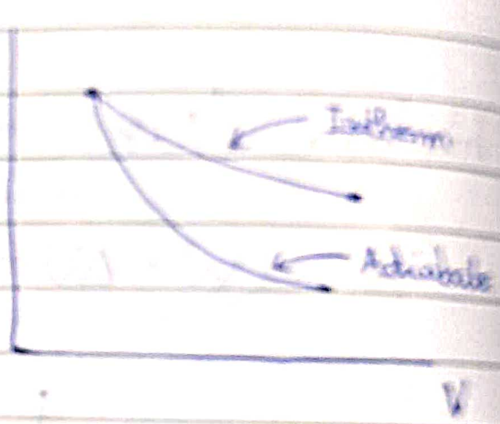
$\Rightarrow$ $\text{Const} = \ln(V)\left(\frac{C_{p,m}}{C_{v,m}}\right) + \ln(P)$

$\Rightarrow$ $\text{Const} = \gamma \ln(V) + \ln(P)$

$\Rightarrow$ $PV^{\gamma} = \text{Const}$

Now, $TV^{(\gamma-1)} = \text{Const}$ & $P^{(1-\gamma)}T^{\gamma} = \text{Const}$

$\text{Slope} = \left(\frac{dP}{dV}\right) = (-\gamma)\left(\frac{P}{V}\right)$



$$\boxed{\left(\frac{\text{Slope of Adiabate}}{\text{Slope of Isotherm}}\right) = \gamma}$$

$W = \int P\,dV = \int_{V_1}^{V_2} V^{-\gamma} \cdot PV^{\gamma}\,dV$

$= \left(\int_{V_1}^{V_2} V^{-\gamma}\,dV\right)\left(PV^{\gamma}\right)$ ← Const

$= \left(PV^{\gamma}\right)\left(\frac{V^{1-\gamma}}{1-\gamma}\right)_{V_1}^{V_2}$ $\Rightarrow$ $\boxed{W = (-1)\left(\frac{P_2V_2 - P_1V_1}{\gamma - 1}\right)}$

$\Rightarrow$ $\boxed{W = (-1)\left(\frac{nR}{\gamma-1}\right)\left(T_2 - T_1\right)}$

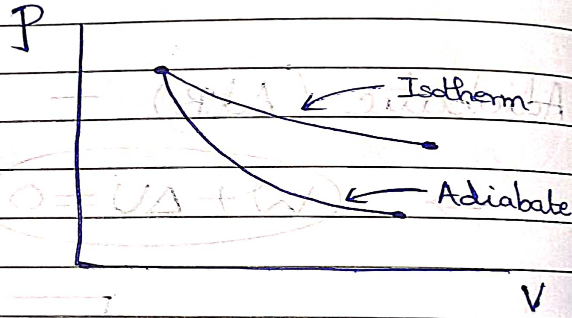
$$\Rightarrow \quad Const. = \ln(V)\left(\frac{C_{p,m}}{C_{v,m}}\right) + \ln(P)$$

$$\Rightarrow \quad Const. = \gamma \ln(V) + \ln(P)$$

$$\Rightarrow \quad PV^{\gamma} = Const.$$

Now, $TV^{(\gamma-1)} = Const.$ & $P^{(1-\gamma)}T^{\gamma} = Const.$

$$Slope = \left(\frac{dP}{dV}\right) = (-\gamma)\left(\frac{P}{V}\right)$$



$$\boxed{\left(\frac{\text{Slope of Adiabate}}{\text{Slope of Isotherm}}\right) = \gamma}$$

$$W = \int P\,dV = \int_{V_1}^{V_2} V^{-\gamma} \cdot PV^{\gamma}\,dV$$

$$= \left(\int_{V_1}^{V_2} V^{-\gamma}\,dV\right)\left(PV^{\gamma}\right) \leftarrow \text{Const.}$$

$$= \left(PV^{\gamma}\right)\left(\frac{V^{1-\gamma}}{1-\gamma}\right)_{V_1}^{V_2} \Rightarrow \boxed{W = (-1)\left(\frac{P_2V_2 - P_1V_1}{\gamma - 1}\right)}$$

$$\Rightarrow \quad W = (-1)\left(\frac{nR}{\gamma-1}\right)(T_2 - T_1)$$

Q) ~~An~~ 2 mol ideal gas obeys ~~P~~ $VT^{1/2} = \text{Const.}$ find ~~word~~ work done to inc. its temp by 300K

A) $VT^{1/2} = \text{Const.} \Rightarrow PV^3 = \text{Const.} \Rightarrow \gamma = 3$

$$(-W) = \left(\frac{nR}{\gamma-1}\right)(T_2 - T_1) = \frac{\cancel{2}\cdot 25/3 \cdot 300}{\cancel{2}}$$

$$\Rightarrow \boxed{W = (-2500)\ J}$$

Q) Isobaric (iP) — $P = \text{Const.}$

$\Rightarrow q = nC_{p,m}(\Delta T)$ $\Rightarrow W = P(\Delta V)$

$\Rightarrow W = nR(\Delta T)$

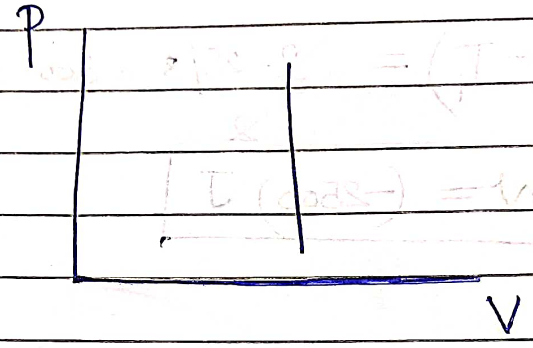
Q) 14 J heat supplied to diatomic gas at const. P. find work done.

A)
$$\left.\begin{aligned} q &= nC_{p,m}(\Delta T) \\ W &= nR(\Delta T) \end{aligned}\right\} \Rightarrow W = \left(\frac{R}{C_{p,m}}\right) q = \left(\frac{\gamma-1}{\gamma}\right) q$$

$$\Rightarrow \boxed{W = 4\ J}$$

4) Isochoric (iV) — $V = \text{Const.}$

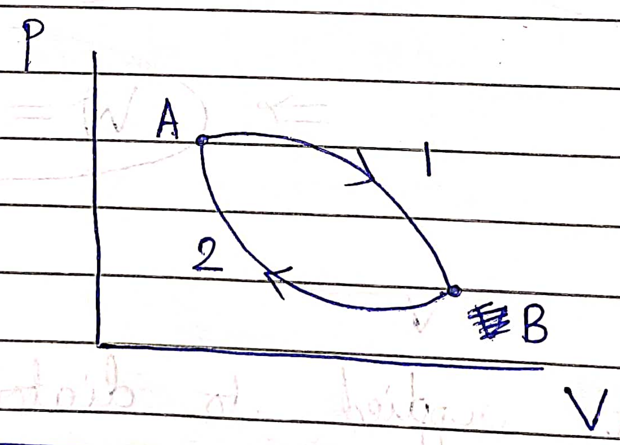
$\Rightarrow W = 0 \Rightarrow q = \Delta U$



(see Pg 149)

5) Polytropic — $PV^x = \text{Const.} \Rightarrow \boxed{C = C_{v,m} + \frac{R}{(1-x)}}$

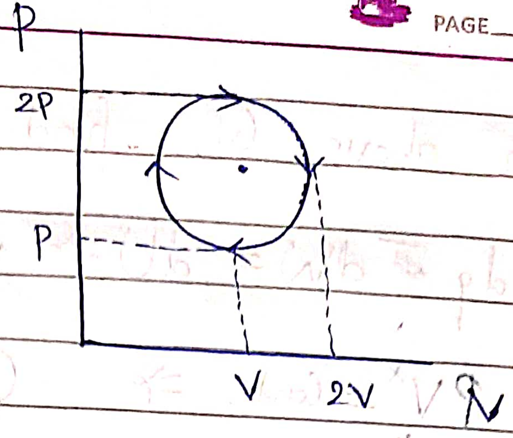
Cyclic Process



$\boxed{W = +(\text{Area of loop})}$

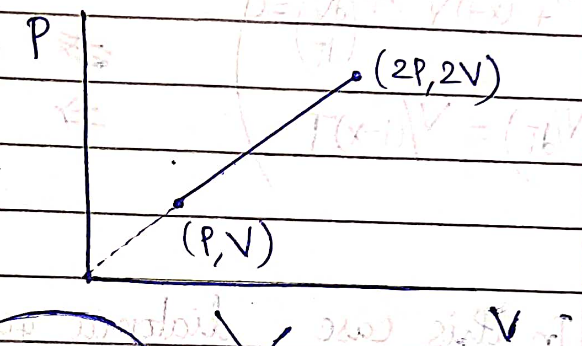
if ↻, use (−) if ↺

Q) Find work done.



A) $W = +PV \cdot \frac{\pi}{4}$

Q) For diatomic gas, find q.



A) $\cancel{PV^{1} = \text{Const.} \Rightarrow \gamma = (-1)}$

$\cancel{W = (-1)\left(\frac{P_2V_2 - P_1V_1}{\gamma - 1}\right) = \left(\frac{1}{2}\right)(3PV)}$

$\cancel{\Delta U = nC_{v,m}(\Delta T) = \left(\frac{1}{\gamma - 1}\right)(P_2V_2 - P_1V_1) =}$

A) $W = \int P\,dV = \int_{V}^{2V} V\,dV = \left(\frac{V^2}{2}\right)_{V}^{2V} = \frac{3V^2}{2}$

$\Delta U = nC_{v,m}(\Delta T) = \left(\frac{1}{\gamma - 1}\right)(P_2V_2 - P_1V_1) = \left(\frac{5}{2}\right)(3PV) = \frac{15V^2}{2}$

Now, $\Delta U = q + (-W) \Rightarrow \boxed{q = 9PV}$

Q) In above Q, find C.

A) $dq - dW = dU \Rightarrow nC\,dT - P\,dV = nC_{v,m}\,dT$

If $PV^x = \text{Const.} \Rightarrow C = C_{v,m} + \left(\frac{P}{n}\right)\left(\frac{dV}{dT}\right)$

$\Downarrow$

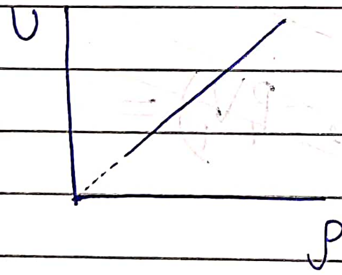
$TV^{(x-1)} = \text{Const.} \Rightarrow C = C_{v,m} + \left(\frac{RT}{V}\right)\left(\frac{V}{T(1-x)}\right)$

$$\left(\begin{array}{l} \Rightarrow V^{(x-1)} + (x-1)V^{(x-2)}T\left(\frac{dV}{dT}\right) = 0 \\ \Rightarrow (dV/dT) = V/(1-x)T \end{array}\right)$$

$\Rightarrow \boxed{C = C_{v,m} + \frac{R}{(1-x)}}$

In this case, diatomic gas & $x = (-1)$

$\Rightarrow C = 3R$

Q) 

Diatomic gas.

Find C.

A) $C = C_{v,m} + \frac{R}{(1-x)} = \frac{5}{2}R - R \Rightarrow C = \frac{3R}{2}$

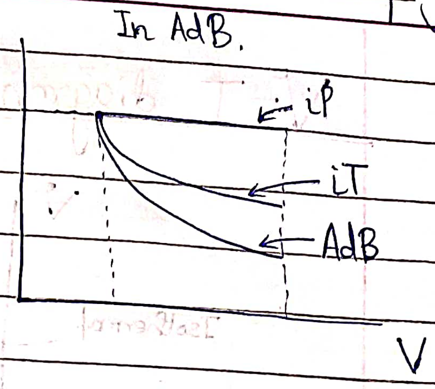
$U/\rho = \text{Const.} \Rightarrow TV = \text{Const.} \Rightarrow PV^2 = \text{Const.}$

$\Rightarrow x = 2$

Q) A gas expands from vol. V to 2V by iP, iT & AdB processes.

Which has min. P at end?

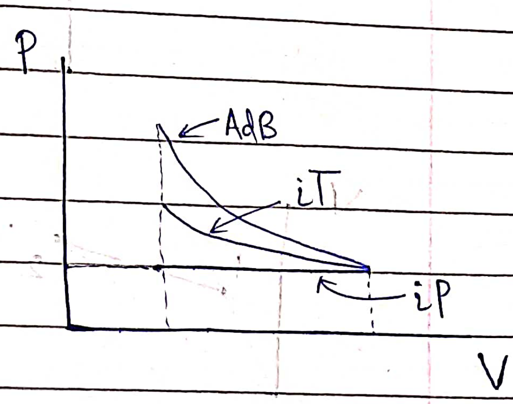
A) In AdB.



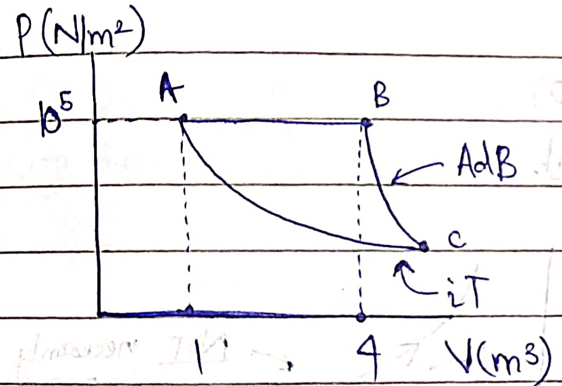
Q) A gas is compressed from vol. 2V to V by iP, iT & AdB processes.

Where is |W| max.?

A) In AdB.



Q)



$\gamma = 1.5$

find $(P, V)_C$.

Also find W.

A) iT: $P_A V_A = P_C V_C \Rightarrow P_C V_C = 10^5$

AdB: $P_B V_B^{\gamma} = P_C V_C^{\gamma} \Rightarrow P_C V_C^{3/2} = 8\cdot 10^5$

$P_C = 10^5/64$

$V_C = 64$

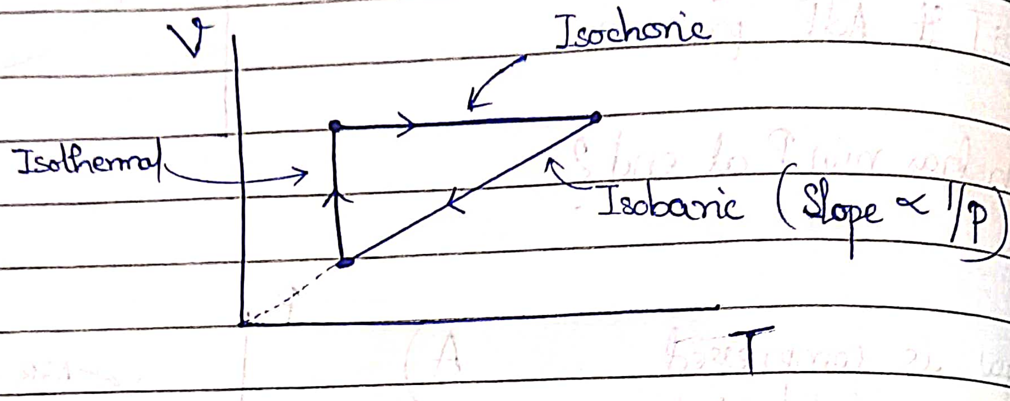
$$W = W_{AB} + W_{BC} + W_{CA} = (10^5)(3) + \left(\frac{-1}{0.5}\right)(10^5 - 4\cdot 10^5) + (10^5)\ln\left(\frac{1}{4}\right)$$

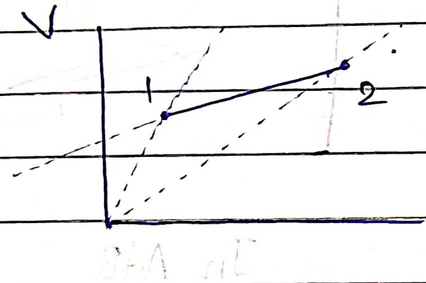
$$= \left(3\cdot 10^5 + 6\cdot 10^5 - \ln(2)\cdot 10^5\right) = \left(9 - \ln(2)\right)\cdot 10^5$$

$$\Rightarrow \boxed{W \approx (8.3 \times 10^5)\ J}$$

DATE
PAGE

V-T diagram

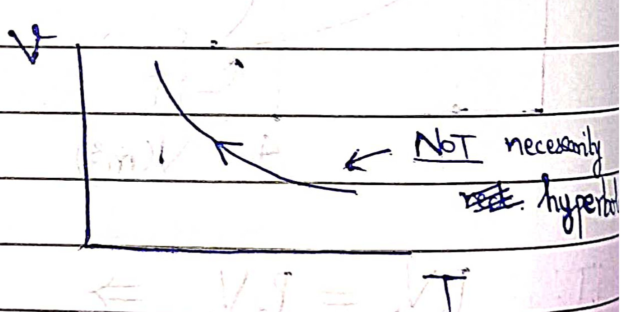




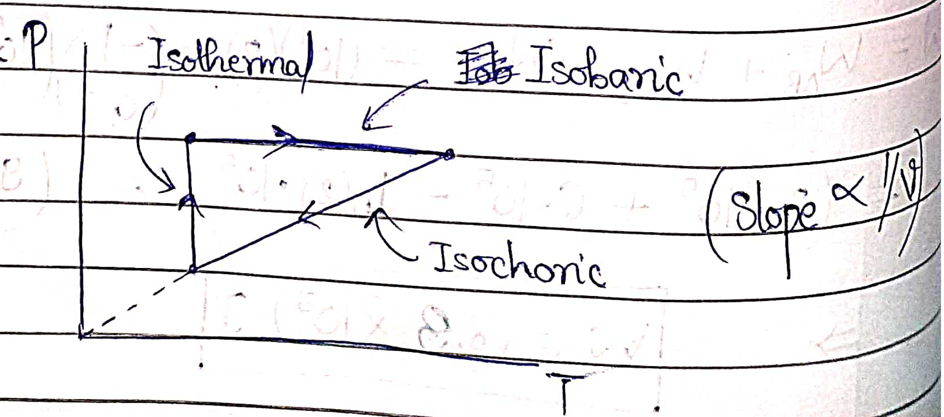
$P_2 > P_1$

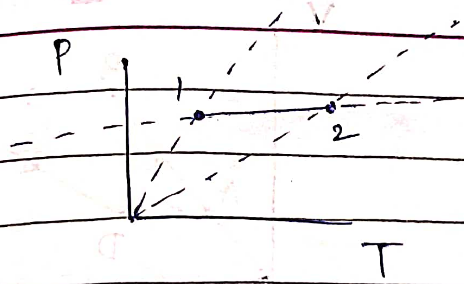
for AdB, $TV^{\gamma-1} = \text{Const.}$ ($\gamma - 1$ (≥ 0))

$\Rightarrow$



P-T diagram

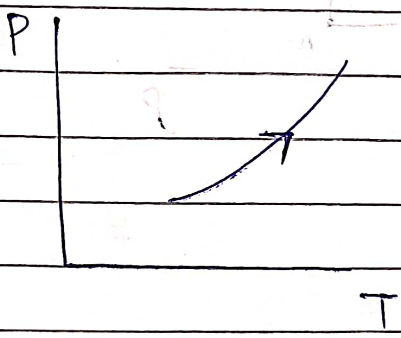




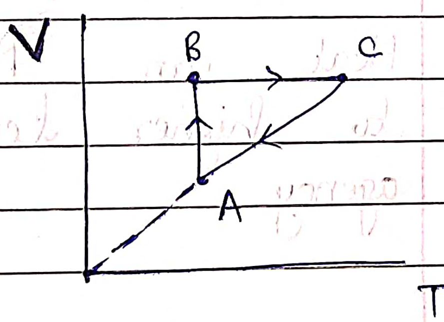
$V_2 > V_1$

for AdB, $TP^{\left(\frac{1}{\gamma}-1\right)} = \text{Const.}$ (<0)

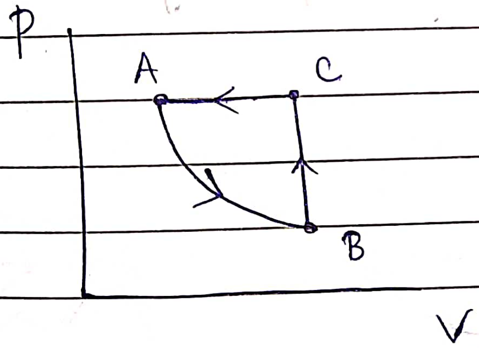
$\Rightarrow$

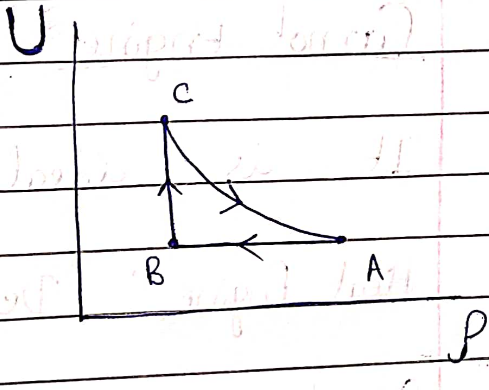


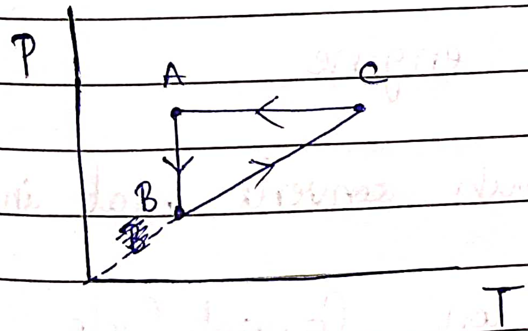
Q) Convert to P–V diagram, U–ρ diagram & P–T diagram



A)



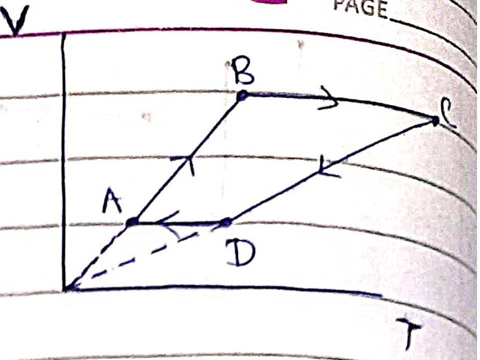




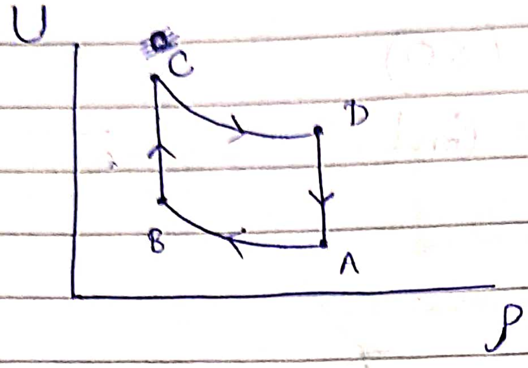
155

U–P

Q) Convert into ~~other~~ graphs.



A)



2nd Law of Therm.

Heat can NOT flow from lower temp. to higher temp. w/o help of external agency.

Carnot Engine

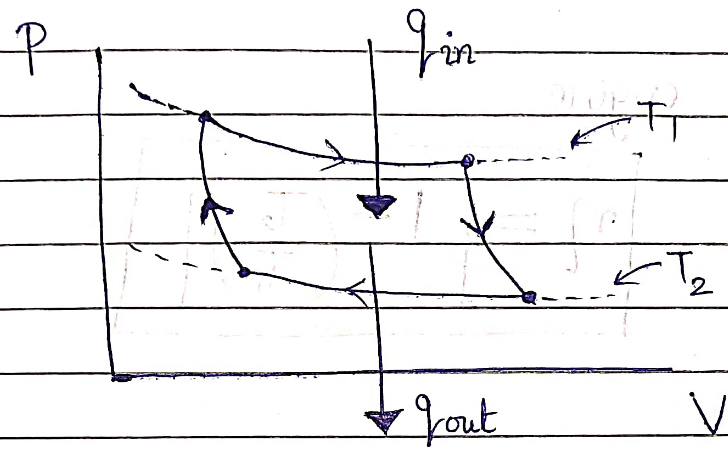
It is ideal heat engine

Heat Engine : Device which converts heat into wo

Carnot engine is based on Carnot Cycle

Carnot Cycle: A cyclic process consisting of –

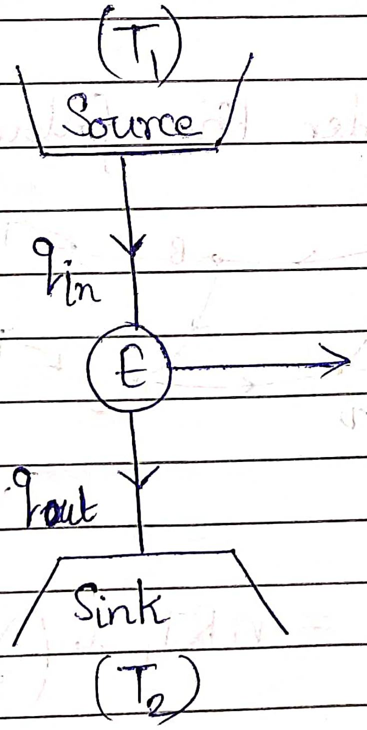
1) Isothermal Exp^n
2) Adiabatic Exp^n
3) Isothermal Comp^n
4) Adiabatic Comp^n



During iT exp^n, q in; during iT comp^n, q out

☆ $T_1 > T_2$

☆ ~~$T_1 > T_2$~~



☆ Since time to complete VERY large (as processes are reversible) $\Rightarrow$ $P = 0$
$\Rightarrow$ Power of Carnot Engine is Zero.

☆ q_{in} & q_{out} are MAGNITUDE of heat entering & exiting resp.

$$q_{in} = q_{out} + W$$

For any engine,

$$\boxed{(\text{Efficiency}) = \left(\frac{W}{q_{in}}\right)}$$

$$\Rightarrow \boxed{\eta = \left[1 - \left(\frac{q_{out}}{q_{in}}\right)\right]}$$

★ If $\eta_{engine} > 1$

⇒ It doesn't exist!

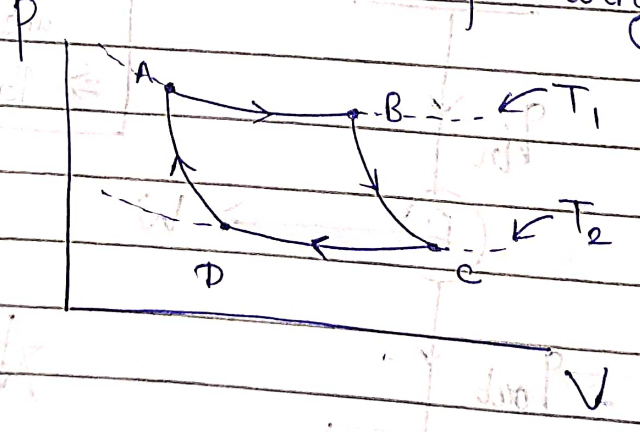
For Carnot engine,

$$\boxed{\eta = \left[1 - \left(\frac{T_2}{T_1}\right)\right]}$$

Combining gives,

$$\boxed{\left(\frac{q_{in}}{T_1}\right) = \left(\frac{q_{out}}{T_2}\right)}$$

Derivation: Consider the following process.



$$q_{in} = q_{AB} = W_{AB} = nRT_1 \ln\left(\frac{V_B}{V_A}\right)$$

$$q_{out} = (-q_{CD}) = (-W_{CD}) = (-nRT_2) \ln\left(\frac{V_D}{V_C}\right)$$

(as $q_{CD} < 0$)

By AdB, $T_1 V_B^{(\gamma-1)} = T_2 V_C^{(\gamma-1)}$

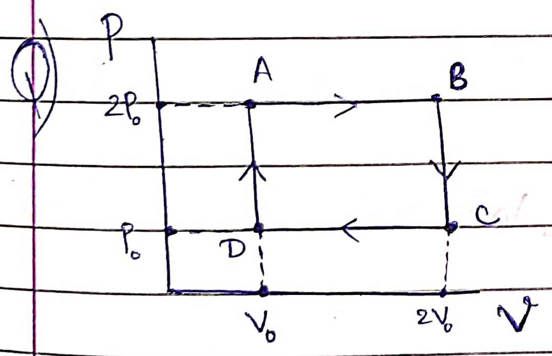
& $T_1 V_A^{(\gamma-1)} = T_2 V_D^{(\gamma-1)}$

$\Rightarrow \left(\frac{V_B}{V_A}\right) = \left(\frac{V_C}{V_D}\right)$

Now, $\left(\frac{q_{in}}{q_{out}}\right) = \frac{nRT_1 \ln(V_B/V_A)}{(-nRT_2)\ln(V_D/V_C)} = \left(\frac{\cancel{-}\,T_1}{+\,T_2}\right)$

$\Rightarrow \left[1 - \left(\frac{q_{out}}{q_{in}}\right)\right] = \left[1 \cancel{+} \left(\frac{T_2}{T_1}\right)\right]$

$\Rightarrow \eta = \left[1 - \left(\frac{T_2}{T_1}\right)\right]$

Q) 

Diatomic gas.

Find efficiency.

A) $W_{AB} = 2P_0V_0$, $W_{BC} = 0$, $W_{CD} = (-P_0V_0)$, $W_{DA} = 0$

$q_{AB} = n\left(\frac{7R}{2}\right)(\Delta T)$, $q_{BC} = (-ve)$, $q_{DC} = (-ve)$, $q_{DA} = (\Delta U)_{DA}$

$= 7P_0V_0$ $\qquad q_{out}$ $\qquad = n\left(\frac{5}{2}R\right)(\Delta T)$

$= \frac{5P_0V_0}{2}$

$\Rightarrow W = P_0V_0$, $q_{in} = \frac{19P_0V_0}{2} \Rightarrow \boxed{\eta = 2/19}$

DATE

PAGE

Q) Carnot engine. Efficiency $= 1/6$. If temp. of sink reduced by $50°C$, efficiency gets doubled find init. temp. of source & sink

A) $\cancel{\frac{T_{source}}{T_{sink}}}$ $\left(1 - \frac{T_2}{T_1}\right) = \frac{1}{6} \Rightarrow \left(\frac{T_2}{T_1}\right) = \frac{5}{6}$

$$\left(1 - \left(\frac{T_2 - 50}{T_1}\right)\right) = \frac{1}{3} \Rightarrow \left(\frac{T_2 - 50}{T_1}\right) = \frac{2}{3}$$

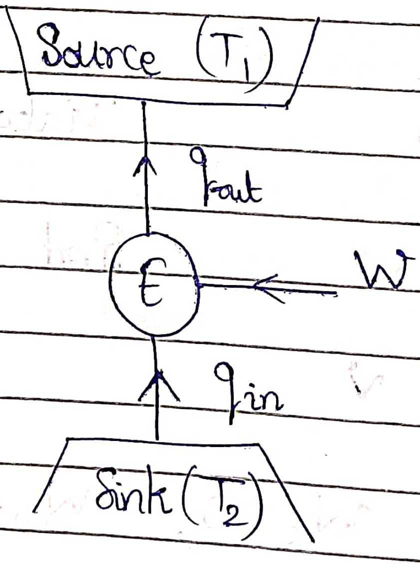
$$\Rightarrow \quad T_2 = 250 \; , \quad T_1 = 300$$

Sink Source

Refrigirator (Heat Pump)

☆ $T_1 > T_2$

~~☆ $T_1 > T_2$~~



Here,

Source = Room

Sink = Fridge

$$q_{out} = q_{in} + W$$

For any heat pump,

$$\boxed{(\text{Coeff. of Performance}) = \left(\frac{q_{in}}{W}\right)}$$

$$\Rightarrow \quad \boxed{\beta = \left(\frac{q_{in}}{q_{out} - q_{in}}\right)}$$

For Carnot pump (Carnot cycle ulti chala do),

$$\boxed{\beta = \left(\frac{T_2}{T_1 - T_2}\right).}$$

Q) Temp. of freezer = $\cancel{-20}$ (−23) °C
Room temp = 27 °C

10 g water at 0°C ⟶ 10 g ice at 0°C

Find work done by compressor

A) $\beta = \left(\frac{q_{in}}{W}\right) = \left(\frac{T_2}{T_1 - T_2}\right) \Rightarrow \frac{(10)(80)\ \text{cal}}{W} = \frac{250}{(300-250)} = 5$

$$\Rightarrow \quad \boxed{W = 160\ \text{cal}}$$

Carnot Theorem : No engine is more efficient than carnot engine.

Q) Find efficiency of a ~~Non~~ non-carnot engine

$T_1 = 500\,K$, $T_2 = 300\,K$

A) $$\eta_{carnot} = \left[1 - \left(\frac{T_2}{T_1}\right)\right] = \left(1 - \frac{3}{5}\right) = 0.4$$

$$\Rightarrow \boxed{\eta_{\text{(Non-Carnot Engine)}} < 0.4}$$