Kinematics

Position : Location of Particle

Diplacement : Change in posⁿ.

$$\boxed{\Delta\vec{x} = (\vec{x}_2 - \vec{x}_1)} ; \quad \vec{x}_1 = \text{Initial Posⁿ.}, \quad \vec{x}_2 = \text{Final Posⁿ.}$$

Disp. ← $\Delta\vec{x}$

Displacement is shortest path b/w the pts.

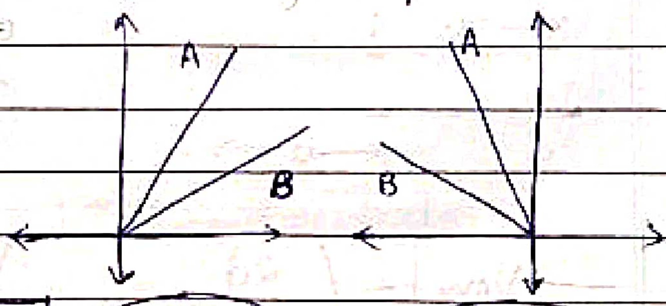
☆ $\boxed{|\text{Dist.}| \geq |\text{Disp.}|}$ ['=' if velocity does NOT change dirxⁿ]

Motion in 1D —

1) Uniform :

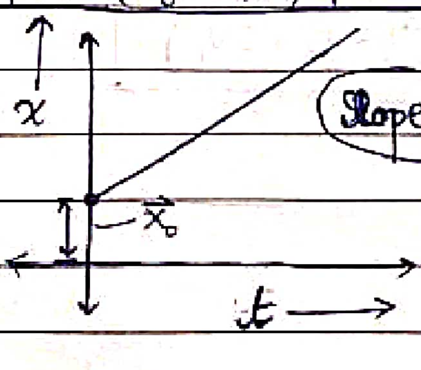
$\vec{v}$ = Const.
$\vec{a} = 0$

~~2) Non-Uniform~~

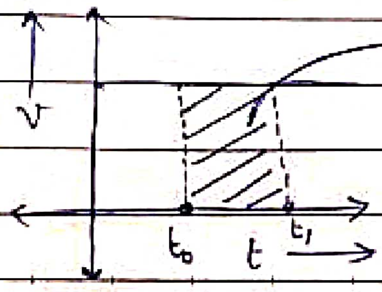


$V_A > V_B$ $\quad$ $V_A > V_B$

$\boxed{\vec{x} = (\vec{x}_0 + \vec{v}t)}$



Slope = $\vec{v}$



disp. b/w $t = t_0$ & $t = t_1$

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Average Velocity & Speed —

$$\boxed{\vec{v}_{Avg} = \left(\frac{\text{Total disp.}}{\text{Total time}}\right)} \; ; \; \boxed{\text{Speed}_{Avg} = \left(\frac{\text{Total dist.}}{\text{Total time}}\right)}$$

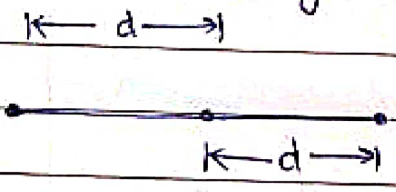
Q) A body covers s_1 dist. with v_1 velocity and s_2 dist. with v_2 velocity. find $|\vec{v}_{avg}|$

A) $t_1 = \left(\frac{s_1}{v_1}\right)$; $t_2 = \left(\frac{s_2}{v_2}\right)$

$$\boxed{v_{Avg}} = \left(\frac{s_1+s_2}{t_1+t_2}\right) = \left(\frac{s_1+s_2}{s_1/v_1 + s_2/v_2}\right) = \boxed{\frac{(v_1 v_2)(s_1+s_2)}{(s_1 v_2 + s_2 v_1)}}$$

Q) A body covers half dist. with velocity v_1 and 2nd half in 2 equal time intervals with velocity v_2 and v_3. find v_{avg}

A)



$d = v_1 t_1$

$d = (v_2 + v_3) t_2$

$$\boxed{v_{Avg}} = \left(\frac{2d}{d/v_1 + 2d/(v_2+v_3)}\right) = \boxed{\frac{2(v_2+v_3)(v_1)}{(v_2+v_3+2v_1)}}$$

Q) $x = t^2 + 4t + 5$. find avg. velocity of first 2s

A) $\left(\dfrac{x_2 - x_0}{2-0}\right) = \dfrac{(4+8+5-0-0-5)}{(2-0)} = \left(\dfrac{12}{2}\right) = \boxed{6 = v_{Avg}}$

Q) $x = t^2 - 2t + 4$. find avg velocity & avg. speed b/w $t = 0s$ and $t = 3s$.

A) $\left(\dfrac{x_3 - x_0}{3-0}\right) = \left(\dfrac{9-6+4-0+0-4}{3}\right) = \boxed{1 = v_{Avg}}$

At ($t=1$); $v(1) = 2\cdot1 - 2\cdot1 = 0$; The body changes dirx^n. So,

$\dfrac{(x_3 - x_1) + (x_0 - x_1)}{(3-0)} = \dfrac{9-6+4+0-0+4-2(1-2+4)}{(3-0)}$

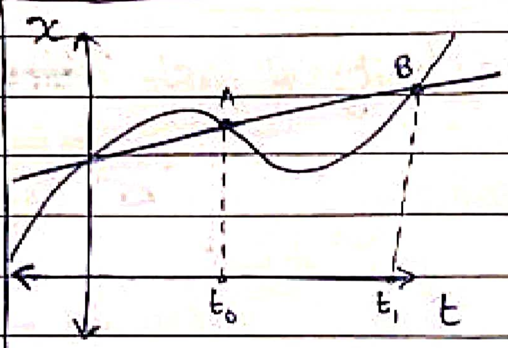
$= \left(\dfrac{3+8-2\cdot3}{3}\right) = \boxed{\left(\dfrac{5}{3}\right) = \text{Speed}_{Avg.}}$

Q) $x = t^2 - 4t + 5$. find avg. speed b/w $t = 0s$ & $t = 3s$.

A) At ($t = 2$), $v(2) = 2\cdot2 - 4 = 0$; the body changes dirx^n. So,

$\dfrac{(x_3 - x_2) + (x_0 - x_2)}{(3-0)} = \left(\dfrac{9-12+5+0+0+5-2(4-8+5)}{3}\right)$

$= \left(\dfrac{7-2\cdot1}{3}\right) = \boxed{\left(\dfrac{5}{3}\right) = \text{Speed}_{Avg.}}$



Avg. velocity from 't_0' to 't_1' is Slope of $\overrightarrow{AB}$

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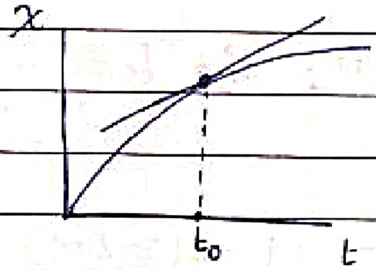
Instantaneous Velocity —



$$\boxed{v = \left(\frac{dx}{dt}\right)\Bigg|_{t=t_0}}$$

Motion in 1D (Contd.) —

✓ Non - Uniform —

1) Constant Acceleration :

$\vec{a} = \text{Const.}$ $\Leftrightarrow$ $\left(\frac{d\vec{v}}{dt}\right) = \vec{a}$

$\Rightarrow$

$$\boxed{\begin{array}{ll} \vec{v} = \vec{u} + \vec{a}t \quad ; & v^2 = u^2 + 2as \\ \vec{x} = \vec{x}_0 + \vec{u}t + \frac{1}{2}at^2 \quad ; & \vec{s} = \left(\frac{\vec{u}+\vec{v}}{2}\right)t \end{array}}$$

Disp in 'nth' second –

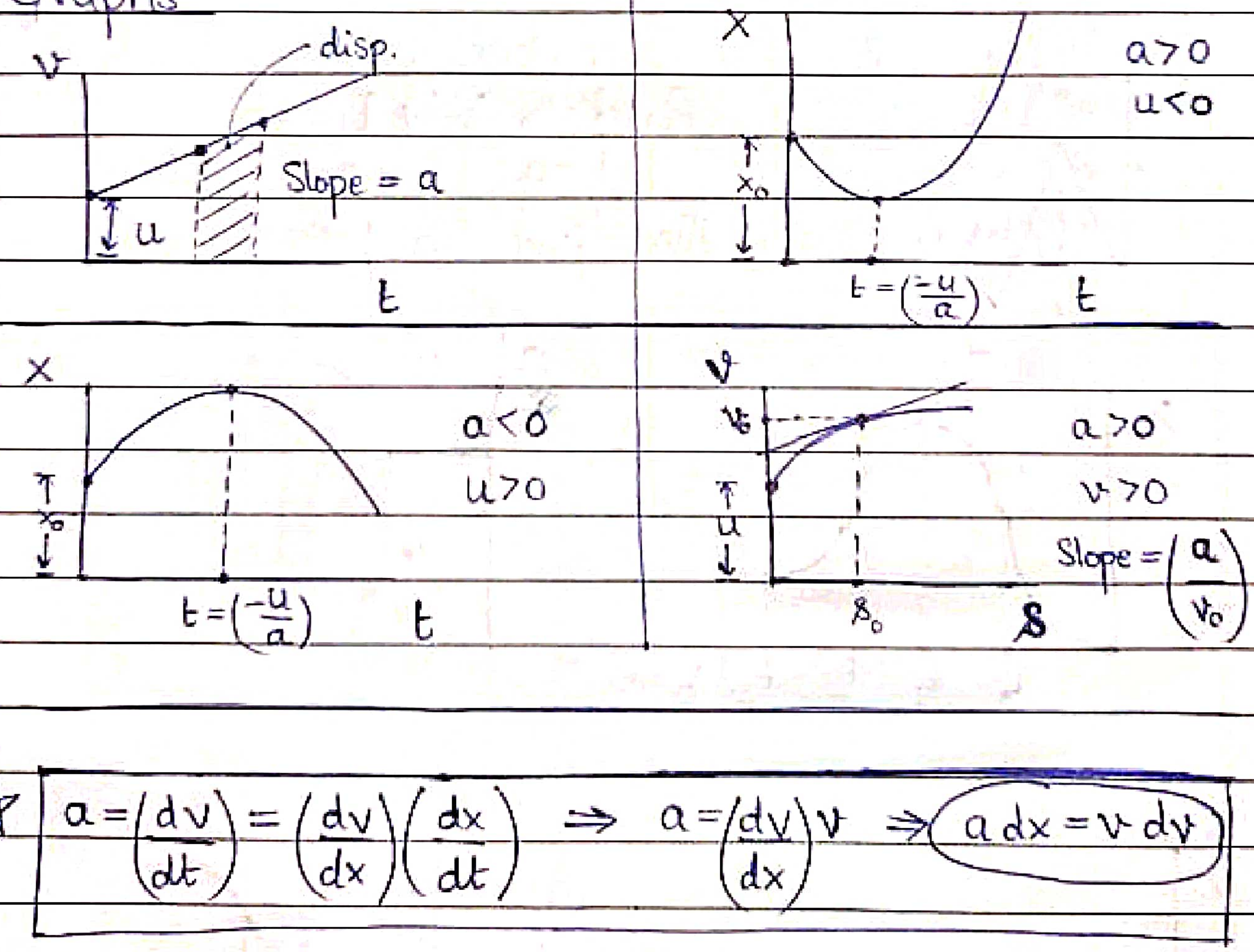
This is disp. b/w $t = (n-1)$ and $t = n$

$$\vec{S}_{n-1} = \vec{u}(n-1) + \left(\frac{1}{2}\right)\vec{a}(n-1)^2$$

$$\vec{S}_{n} = \vec{u}(n) + \left(\frac{1}{2}\right)\vec{a}(n^2)$$

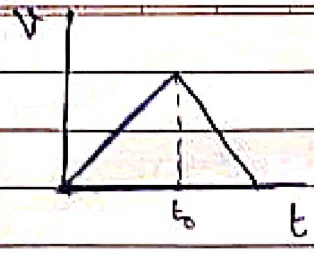
$$\vec{S}_n - \vec{S}_{n-1} = \boxed{\vec{S}_{nth} = \vec{u} + \left(\frac{1}{2}\right)(\vec{a})(2n-1)}$$

Graphs –



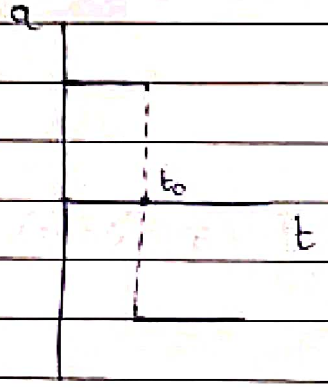
$$\boxed{a = \left(\frac{dv}{dt}\right) = \left(\frac{dv}{dx}\right)\left(\frac{dx}{dt}\right) \Rightarrow a = \left(\frac{dv}{dx}\right)v \Rightarrow a\,dx = v\,dv}$$

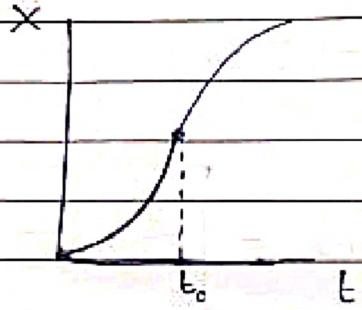
Q)



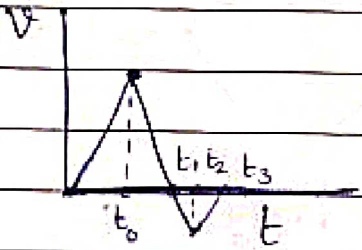
find a-t, x-t
at t=0, x=0.

A)



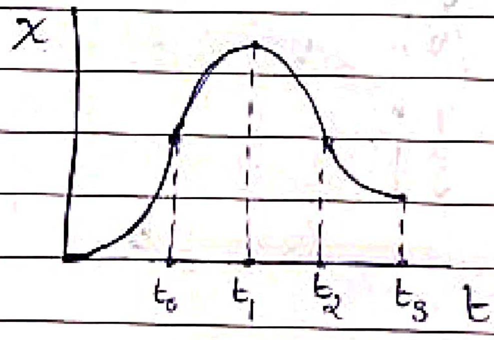


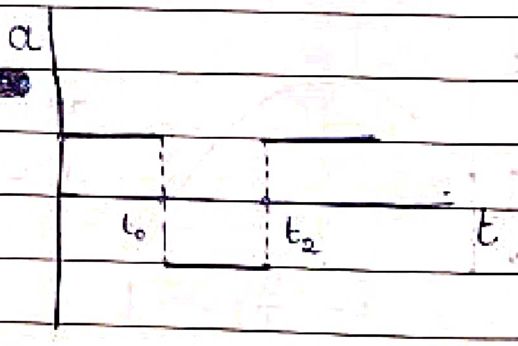
Q)



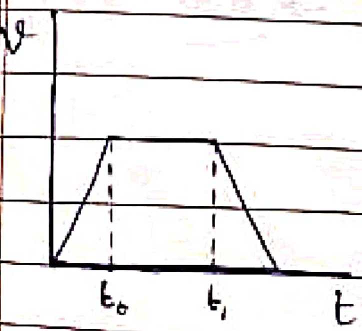
find x-t s.t. at
t=0, x=0.
Also find a-t.

A)



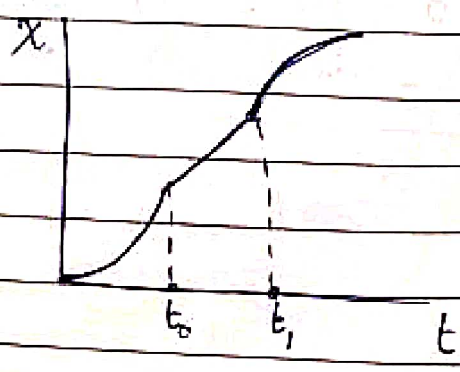


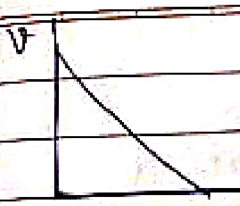
Q)



Find xt. if x=0 at t=0

A)



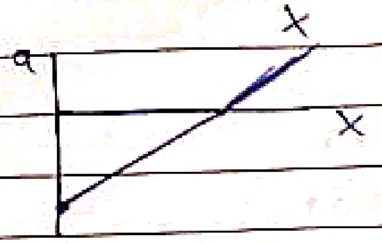
Q) 

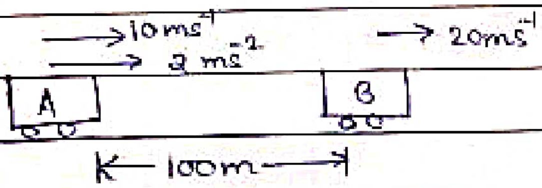
find a-x graph.

A) Slope $= \left(\frac{dv}{dx}\right) = -\text{Const.}$

$$a = v\left(\frac{dv}{dx}\right) = -(\text{Const})v = -(\text{Const})(C_0 - x)$$

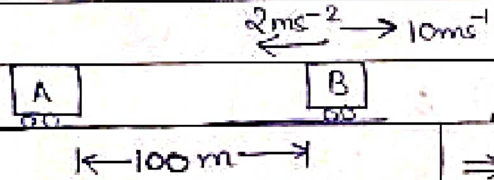
$$\Rightarrow \boxed{a = (C_1 x - C_1 C_0)}$$



Q) 

find time after which car A catches car B.

A) Consider relative motion w.r.t. A.

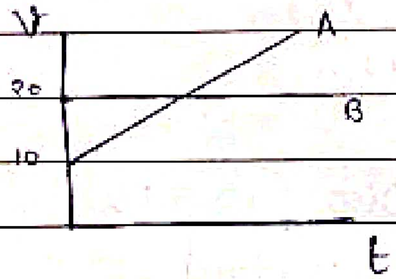


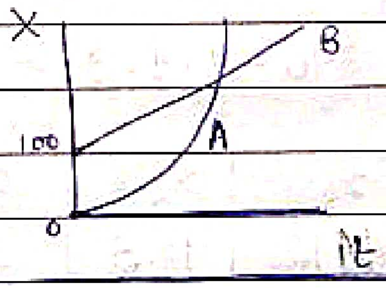
$v^2 = u^2 + 2as$ ✗ (Don't Apply as no time)

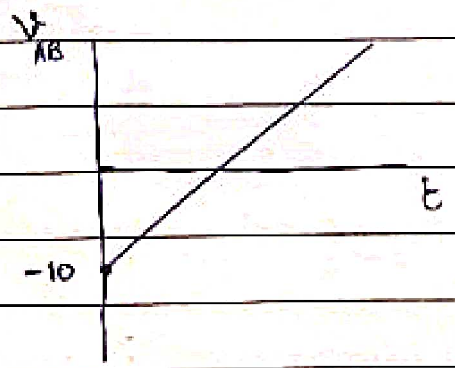
$\Rightarrow v^2 = 100 + 2\cdot 2\cdot 100 = 5\cdot 100$

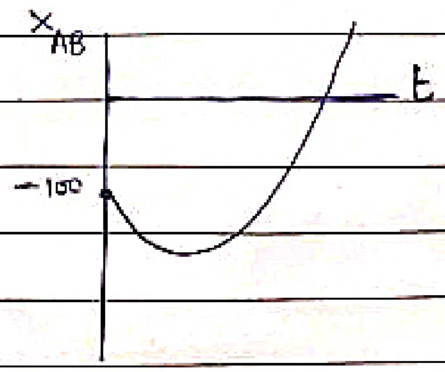
$$\vec{s} = ut + \frac{1}{2}at^2 \Rightarrow -100 = 10t - t^2 \Rightarrow t^2 - 10t - 100 = 0$$

$$\Rightarrow \boxed{t = (5 + 5\sqrt{5})\,s}$$









Q) A body moves 200 m in first 2 sec. In next 4 s, it moves 220 m. Find velocity at end of 7th sec if motion is uniformly accelerated.

A) $200 = 2u + 2a \Rightarrow 100 = u + a$

$420 = 6u + 18a \Rightarrow 70 = u + 3a$

$a = (-15); \ u = 115$

$v(10) = u + \cancel{10a} = 115 - 10 \cdot 15 = 10\,ms^{-1}$

Q) $u = 10\,ms^{-1}$, $a = -4\,ms^{-2}$. Find dist. covered by body in 3rd sec.

A) $v = 0$ at $t = (2.5)\,s$.

$x_2 = 2 \cdot 10 - \frac{1}{2} \cdot 4 \cdot 2^2 = 20 - 8 = 12\,m$

$x_{2.5} = \frac{5}{2} \cdot 10 - \frac{1}{2} \cdot 4 \cdot \frac{25}{4} = 25/2\,m = 12.5\,m$

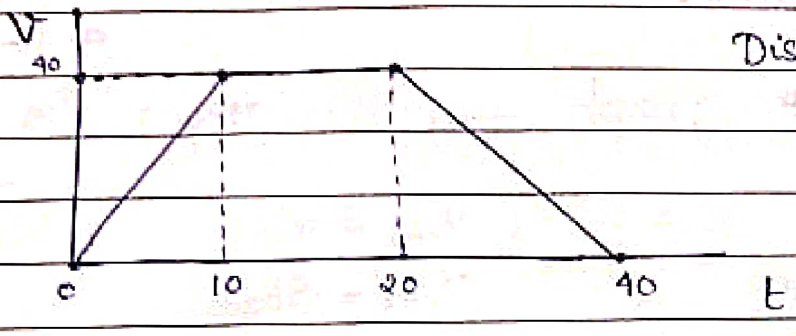
$x_3 = 3 \cdot 10 - \frac{1}{2} \cdot 4 \cdot 9 = 30 - 18 = 12\,m$

x_2	$x_{2.5}$	x_3
12	12.5	12

Dist. $= (0.5 + 0.5)\,m$

$= \boxed{1m}$

Q) Body starts from rest and moves with following $\vec{a}(t) = \begin{cases} 4\ ms^{-2}, & t \le 10 \\ 0, & 10 \le t \le 20 \\ -2\ ms^{-2}, & 20 \le t \end{cases}$. Until it returns back to original posi. find total dist. covered.

A) 

$\text{Dist.} = \tfrac{1}{2}\cdot 10\cdot 40 + 40\cdot 10 + \tfrac{1}{2}\cdot 20\cdot 40$

$= 200 + 400 + 400$

$= 1000\ m$ for half the journey

$\Rightarrow$ Total Dist. $= \boxed{2000\ m}$

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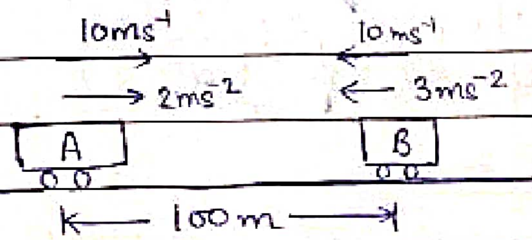
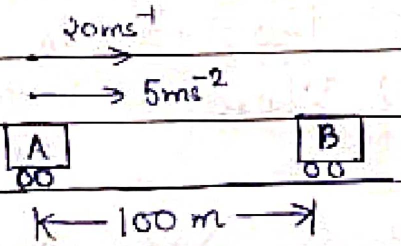
Q) 

find meeting time.

A) Consider relative motion w.r.t. B.



$s = ut + \tfrac{1}{2}at^2$

$\Rightarrow 100 = 20\cdot t + (5/2)t^2$

$\Rightarrow 5t^2 + 40t - 200 = 0$

$\Rightarrow t^2 + 8t - 40 = 0$

$\Rightarrow t = \left(\dfrac{-8 + \sqrt{224}}{2}\right) s \Rightarrow \boxed{t = (-4 + 2\sqrt{14})\ s}$

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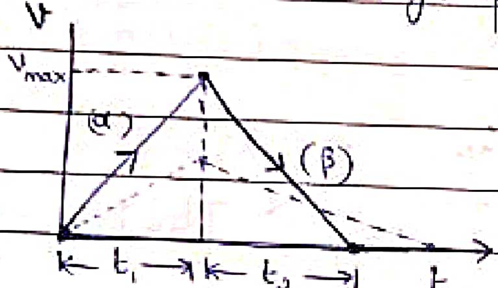
★ Q) A car moves from A to B (dist. = s). It can accelerate max. at $\alpha\, ms^{-2}$ for some time, then can retards max. with $\beta\, ms^{-2}$. Initial and final velocity is 0. Find min. time. ~~if the car attain max. velocity possible.~~

A) Consider v-t graph, any other graphs takes more time, or less dist.



$$v_{max} = \alpha t_1$$

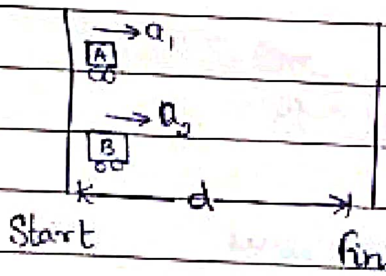
$$v_{max} = \beta t_2$$

$$s = \left(\frac{1}{2}\right)(t_1 + t_2)(v_{max})$$

$$\Rightarrow \quad s = \left(\frac{1}{2}\right)\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)(v^2_{max})$$

Now $t_{min} = t_1 + t_2 = (v_{max})\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = \boxed{\sqrt{2s\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)}}$

Q)



($v = ?$) Car A crosses finish line 't's before B. Also, ~~at that pt.~~ $v_A = v_B + v$. Where (v_B = velocity at finish line)

A) Let 't_1' be time taken by A and 't_2' be time taken by B.

$$(t_2 - t_1) = \sqrt{\frac{2d}{a_2}} - \sqrt{\frac{2d}{a_1}} = (\sqrt{2d})\left(\frac{\sqrt{a_1} - \sqrt{a_2}}{\sqrt{a_1 a_2}}\right) = t$$

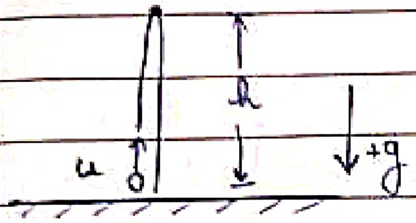
$$(v_A - v_B) = \sqrt{2da_1} - \sqrt{2da_2} = (\sqrt{2d})(\sqrt{a_1} - \sqrt{a_2}) = v$$

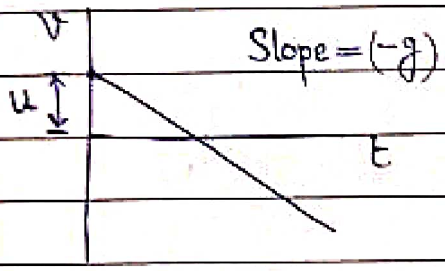
$$\Rightarrow \boxed{v = t\sqrt{q_1 q_2}}$$

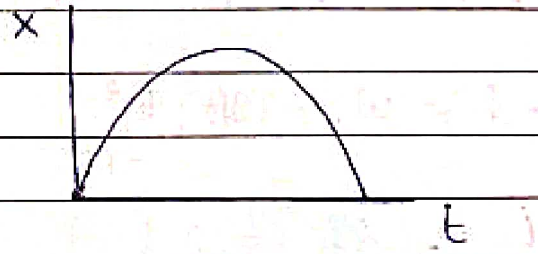
✓ Motion Under Gravity (1D) —

$\boxed{v = u - gt}$, $\boxed{h = ut - \left(\frac{1}{2}\right)gt^2}$

$\boxed{t_{flight} = \left(\frac{2u}{g}\right)}$ $\boxed{h_{max} = \left(\frac{u^2}{2g}\right)}$







Q1) The body is at same height after time 't_1' and 't_2'. Find —

(i) time of flight ii) height at $t = t_1$

A) i) $h = ut - \frac{1}{2}gt^2 \Rightarrow gt^2 - 2ut + 2h = 0 \begin{smallmatrix} t_1 \\ t_2 \end{smallmatrix}$

$$\Rightarrow t_1 + t_2 = \left(\frac{2u}{g}\right) \Rightarrow \boxed{t_{flight} = (t_1 + t_2)}$$

ii) $h = ut_1 - \frac{1}{2}gt_1^2 = \left(\frac{g}{2}\right)(t_1)(t_1+t_2) - \left(\frac{g}{2}\right)t_1^2$

$$\Rightarrow h = \left(\frac{g}{2}\right)(\cancel{t_1^2} + t_1 t_2 - \cancel{t_1^2}) \Rightarrow \boxed{h\Big|_{t=t_1} = \frac{1}{2}gt_1 t_2}$$

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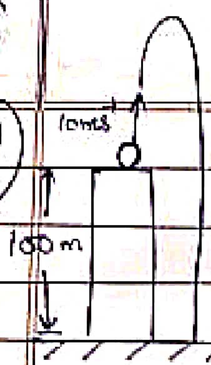
Q) 

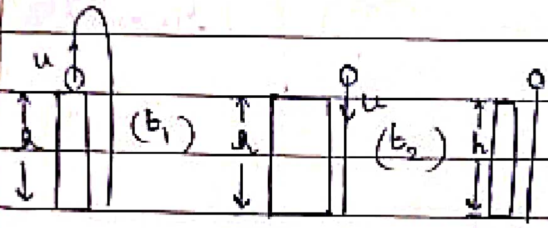
Find 't$_{flight}$'

A) $h = ut - \frac{1}{2} g t^2$

$\Rightarrow -100 = 10t - (5)t^2$

$\Rightarrow t^2 - 2t - 20 = 0$

$\Rightarrow \boxed{t = (1+\sqrt{21})\ s}$

Q) 

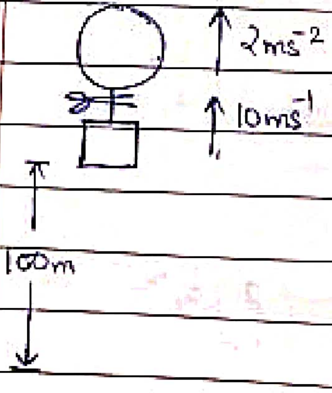
Find 't_3'.

A) $(-h) = ut_1 - (g/2)\,t_1^2$ —(i) ; $(-h) = -ut_2 - (g/2)\,t_2^2$ —(ii)

$t_3 = \sqrt{\frac{2h}{g}} = ?$

(i)t_2 + (ii)t_1 $\Rightarrow$ $\cancel{(t_1+t_2)}\,h = (+g/2)[t_1t_2]\cancel{(t_1+t_2)}$

$\Rightarrow t_1t_2 = \left(\frac{2h}{g}\right) \Rightarrow \boxed{t_3 = \sqrt{t_1t_2}}$

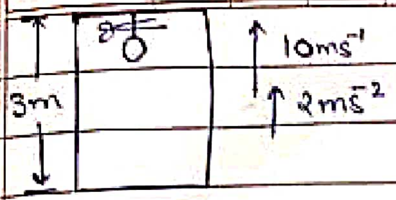
Q) 

Find time taken by mass to fall.

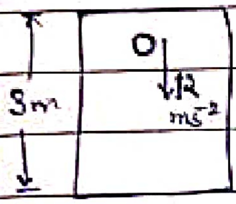
A) At ~~a~~ time of seperation, objects only possess same velocity

Hence, $\boxed{t_{fall} = (1+\sqrt{21})\ s}$

Q) Find time taken by mass to reach bottom of lift.



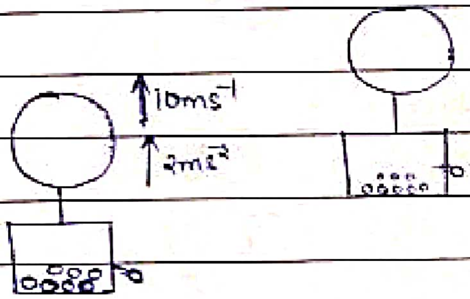
A) Take in reference frame of lift.



$$3 = \frac{1}{2}\cdot 12\cdot t^2 \Rightarrow \boxed{t = 1/\sqrt{2}\ s}$$

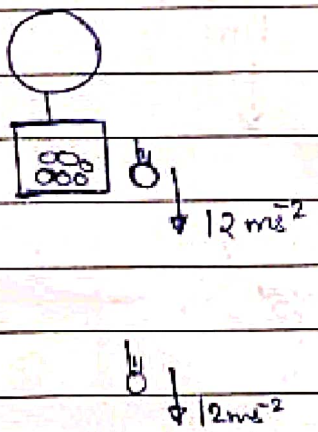
$$s_{ML} = \frac{1}{2}\cdot a_{ML}\cdot t^2$$

Q) Find seperation b/w stones at t=8s.



$t=0s$ $t=3s$

A) Take in reference frame of balloon



(Disp. of Stone 1 at $t=8s$)

$$= -\frac{1}{2}\cdot 12\cdot 64 = -384\ m$$

(Disp. of Stone 2 at $t=8s$)

$$= -\frac{1}{2}\cdot 12\cdot 25 = -150\ m$$

$$\Rightarrow \text{Seperation} = (-150) + 384\ m$$
$$= \boxed{234\ m}$$

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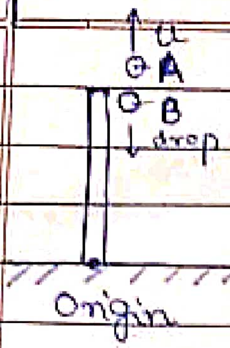
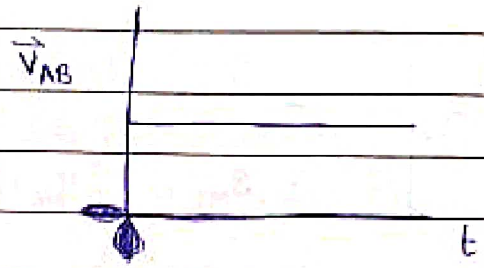
Q) 

Draw $v_{AB}-t$, $x_{AB}-t$

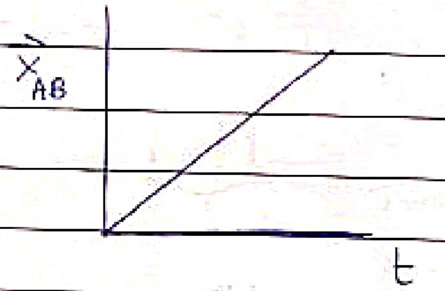
A) $\vec{v}_A = \vec{u} - gt$

$\vec{v}_B = -gt$

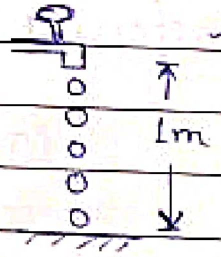
$\vec{v}_{AB} = \vec{u}$



$\vec{x}_{AB} = \vec{u}t$



Q) Find height of 2nd & 3rd drop, if the 5th drop is about to touch the ground & each drop at fixed interval.



A) Let this instant be at $t = t_0$.

Then, $-1\text{ m} = -\frac{1}{2} \cdot 10 \cdot t_0^2 \Rightarrow t_0 = \frac{1}{\sqrt{5}}\text{ s}$

$\Rightarrow$ (Time Interval b/w each drop) $= \left(\frac{1}{4\sqrt{5}}\right)\text{s}$

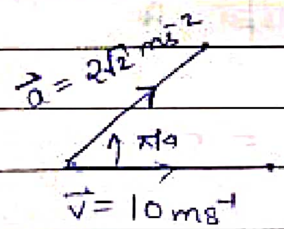
$s_2 = -\frac{1}{2} \cdot 10 \cdot \frac{1}{16 \cdot 5} = \left(\frac{-1}{16}\right) \Rightarrow$ Height from ground $= \boxed{(15/16)\text{m}}$

$s_3 = -\frac{1}{2} \cdot 10 \cdot \frac{1}{4 \cdot 5} = \left(-\frac{1}{4}\right) \Rightarrow$ Height from ground $= \boxed{(3/4)\text{m}}$

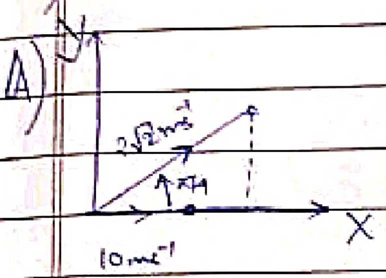
Motion in 2D

☆ Consider motion in 2 ⊥ dirxⁿs and solve seperately.

Q) find disp. of body in 2 s



A)



$\vec{a} = \langle 2, 2 \rangle$

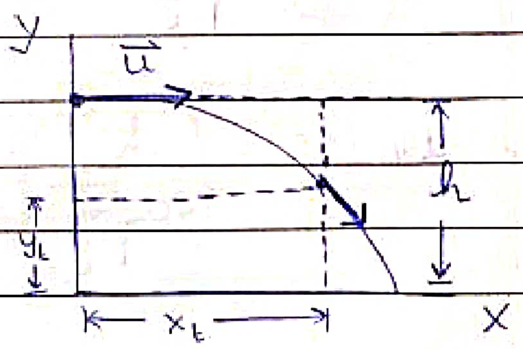
$\vec{v} = \langle 10, 0 \rangle$

$$\vec{s} = ut + \frac{1}{2}at^2 = 2\langle 10,0\rangle + \left(\frac{1}{2}\right)\langle 2,2\rangle \cdot 4$$

$$\Rightarrow \vec{s} = \langle 20, 0\rangle + \langle 4, 4\rangle \Rightarrow \boxed{\vec{s} = \langle 24, 4\rangle}$$

Projectile Motion

1) Horizontal :



	$\vec{v}_0$	$\vec{a}$	$\vec{x}_0$	$\vec{x}_t$	$\vec{v}_t$
x	u	0	0	ut	u
y	0	$-g$	h	$\left(h - \frac{1}{2}gt^2\right)$	$(-gt)$

✓ Eqⁿ of Trajectory : $\boxed{y = h - \left(\frac{gx^2}{2u^2}\right)}$

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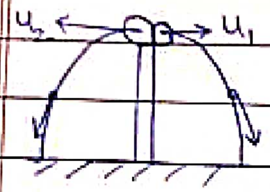
$$\boxed{t_{flight} = \sqrt{\frac{2h}{g}}}$$

Q)

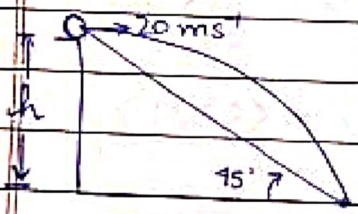


Find time after which their velocities are $\perp$.

A) $\vec{v_1} = \langle u_1, -gt \rangle$ and $\vec{v_2} = \langle -u_2, -gt \rangle$

$\vec{v_1} \cdot \vec{v_2} = (gt)^2 - u_1u_2 = 0 \Rightarrow \boxed{t = \left(\frac{\sqrt{u_1u_2}}{g}\right)}$

Q)



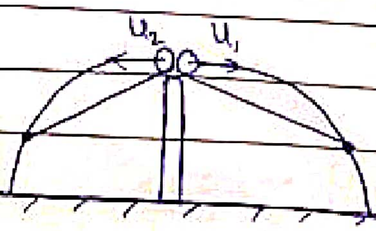
If angle made by line joining pt. of projection and pt. of hitting is 45°. Find 'h'.

A) $t_{flight} = \sqrt{\frac{2h}{g}} \Rightarrow \vec{x}_{x final} = 20\sqrt{\frac{2h}{g}}$

$\tan(45^\circ) = \left(\frac{h}{20\sqrt{\frac{2h}{g}}}\right) = 1 \Rightarrow h^2 = 400 \cdot \frac{2h}{g}$

$\Rightarrow \boxed{h = 80\,m}$

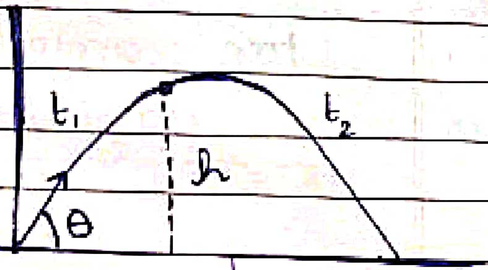
Q)



Find time after which $\angle$ b/w lines joining pts. of proj. & ball is 90°.

A) (Similar to 'velocity' Q)

2) Oblique:



	$\vec{x_0}$	$\vec{v_0}$	$\vec{a}$	$\vec{v_t}$	$\vec{x_t}$
x	0	uc_θ	0	$(uc_\theta)\cancel{t}$	$(uc_\theta)t$
y	0	us_θ	$-g$	$(us_\theta)-gt$	$(us_\theta)t-\frac{1}{2}gt^2$

$$\boxed{t_{flight} = \left(\frac{2u\,s_\theta}{g}\right) = \left(\frac{2u_y}{g}\right)} \; ; \; \boxed{Range = \left(\frac{u^2 s_{2\theta}}{g}\right) = \left(\frac{2u_x u_y}{g}\right)}$$

$$\boxed{h_{max} = \left(\frac{u^2 s_\theta^2}{2g}\right)} \; ; \; \boxed{\frac{R}{h} = 4\cot(\theta)} \; ; \; \boxed{h = \frac{1}{2}g\,t_1 t_2}$$

✓ Eq^n of Trajectory: $\boxed{y = x t_\theta - \left(\frac{gx^2}{2u^2 c_\theta^2}\right)}$

$$\Rightarrow \boxed{y = (x t_\theta)\left(1 - \frac{x}{R}\right)}$$

✓ Max. Range Condt:

$$\boxed{R_{\underset{condt}{max}} = \left(\frac{u^2}{g}\right)} \; ; \; \boxed{h_{\underset{condt.}{max}} = \left(\frac{u^2}{4g}\right)} \; ; \; \boxed{\theta = 45^\circ}$$

$$\Rightarrow \boxed{\left(\frac{R_{max}}{h_{\underset{condt}{max}}}\right) = 4}$$

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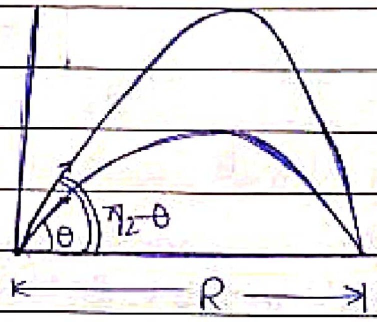
Same Range Condt :

1) for angles θ and $(\pi/2 - \theta)$, range of the projectile is same.



2) If proj. with θ angle takes T_1 time, and that with $(\pi/2-\theta)$ takes T_2 time then,

$$\boxed{\left(\frac{T_1 T_2}{2}\right)(g) = R} \qquad \left(R = \frac{1}{2}g\,T_1 T_2\right)$$

3) If proj. with θ angle attains H_1 max. height and that with $(\pi/2-\theta)$ attains H_2 max height then,

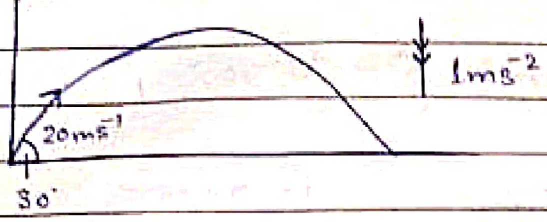
$$\boxed{(H_1 H_2) = \left(\frac{R^2}{16}\right)} \qquad \left(R = 4\sqrt{H_1 H_2}\right)$$

☆ (Q) $x = (10\sqrt{3})t$; $y = 10t - \left(\frac{1}{2}\right)t^2$.

Find —

i) time of flight. ii) range

iii) h_{max} ~~iv) Range if $y = (ax - bx^2)$~~

A) 

☆ Just use value of given acc. insted of 'g' in formulae

i) $t_{flight} = \left(\frac{2 \cdot 20\, s_{30^\circ}}{1}\right) s = 20\, s$

ii) $R = \left(\frac{20^2 \cdot s_{60^\circ}}{1}\right) m = 200\sqrt{3}\ m$

iii) $h_{max} = \left(\frac{20^2\, s^2_{30^\circ}}{2 \cdot 1}\right) m = 50\ m$

~~iv)~~ $\cancel{\frac{y}{?} = \left(\frac{x}{\sqrt{3}}\right)\left(1 - \frac{x}{R}\right) = (x)\left(\frac{1}{\sqrt{3}}\right) - (x^2)\left(\frac{1}{R}\right) = ax - bx^2}$

☆ Q) If $y = ax - bx^2$. find –

i) Range   ii) h_{max}

(Best Method is to compare $(ax - bx^2) \longleftrightarrow \left(x t_\theta - \frac{x^2 t_\theta}{R}\right)$)

A) ~~M-1 : Standard Way~~

i) $(x = R)$ at $(y = 0)$ $\Rightarrow$ $bR^2 = aR \Rightarrow R = \left(\frac{a}{b}\right)$

ii) $(x = R/2)$ at $(y = h_{max})$ $\Rightarrow$ $h_{max} = \left(\frac{aR}{2}\right) - b\left(\frac{R^2}{4}\right)$

$= \left(\frac{a^2}{2b}\right) - \left(\frac{b}{4}\right)\left(\frac{a^2}{b^2}\right) \Rightarrow h_{max} = \left(\frac{a^2}{4b}\right)$

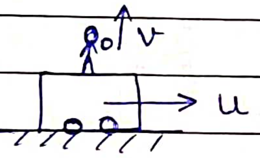
Q) If $\vec{u} = \langle 3, 4 \rangle$ and $\vec{g} = \langle 0, -10 \rangle$ find –

i) t_{flight} ii) Range iii) $h_{max.}$

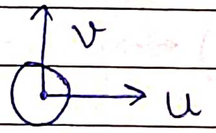
A) i) $t_{flight} = \left(\dfrac{2u_y}{g}\right) = (0.8)\ s$

ii) $R = \left(\dfrac{2u_x u_y}{g}\right) = \left(\dfrac{2 \cdot 3 \cdot 4}{10}\right) = (2.4)\ m$

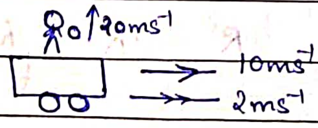
iii) $h_{max.} = \left(\dfrac{u_y^2}{2g}\right) = \left(\dfrac{16}{2 \cdot 10}\right) = (0.8)\ m$

Q) 

A person on car throws up a ball with 'v' velocity. Find range of ball.

A) 

$R = \left(\dfrac{2u_x u_y}{g}\right) \Rightarrow \boxed{R = \left(\dfrac{2uv}{g}\right)}$

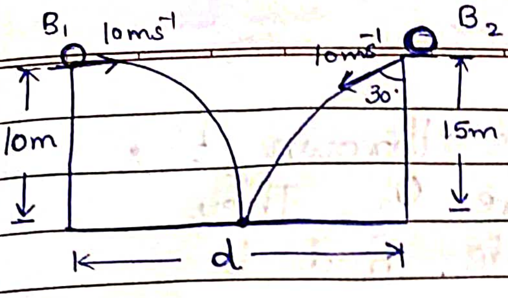
Q) 

Find dist. b/w ball & car when it hits the ground.

A) $\vec{u}_{BC_x} = 0$; $\vec{a}_{BC_x} = -2ms^{-1}$

Dist. $= \left|\left(\dfrac{1}{2}\right)(-2) \cdot \left(\dfrac{20 \cdot 2}{10}\right)^2\right| \Rightarrow$ Dist. $= 16m$

Q)



find 'd'

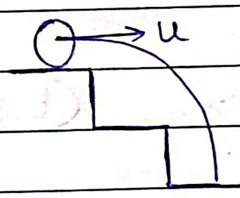
A) $\vec{u}_{B_1} = \langle 10, 0 \rangle$; $\vec{u}_{B_2} = \langle -5, -5\sqrt{3} \rangle$

$\vec{x}_{B_1} = \vec{x}_{B_2} \Rightarrow \langle 0, 10 \rangle + \langle 10, 0 \rangle t + \langle 0, -10 \rangle t^2$
$= \langle d, 15 \rangle - \langle 5, 5\sqrt{3} \rangle t + \langle 0, -10 \rangle t^2$

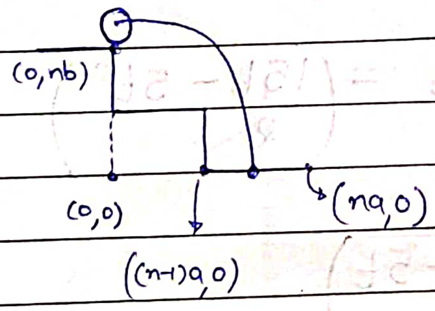
$\Rightarrow \langle 15, 5\sqrt{3} \rangle t = \langle d, 5 \rangle$

$\Rightarrow t = \left(\frac{1}{\sqrt{3}}\right) \Rightarrow \boxed{d = 5\sqrt{3}\ m}$

Q) Which step will the ball hit?



A)



$y_{final} = 0$

$y = \left(nb - \frac{gx^2}{2u^2}\right)$

$\Rightarrow x^2_{final} = \left(\frac{2u^2nb}{g}\right)$

For hitting the step, $x_{final} \leq na$

$\Rightarrow \left(\frac{2u^2nb}{g}\right) \leq n^2a^2 \Rightarrow n \geq \left(\frac{2u^2b}{ga^2}\right)$ (Smallest 'n' value satisfying the inequality)

$\Rightarrow \boxed{n = \left\lceil \frac{2u^2b}{ga^2} \right\rceil}$ → Ceiling fx^n

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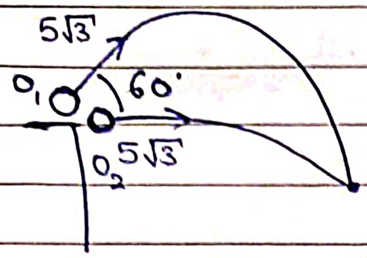
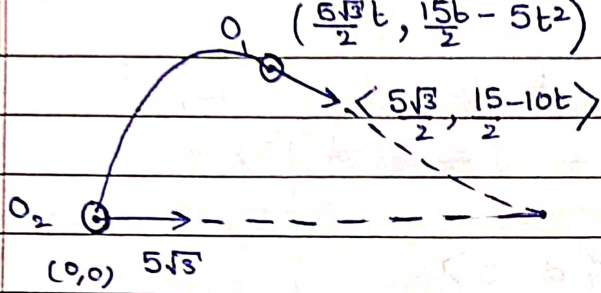
Q) 

O_1 is thrown 't' s before O_2. They collide. Find t.

A) 

From frame of observer in freefall.

We have,

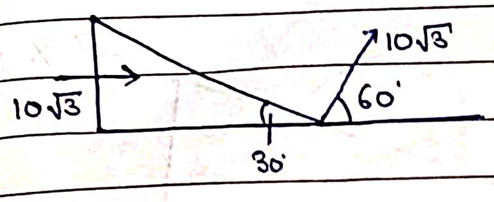
$$\left(\frac{5\sqrt{3}t}{2}\right) = t_{meet}\left(5\sqrt{3} - \frac{5\sqrt{3}}{2}\right) = t_{meet} \cdot \frac{5\sqrt{3}}{2}$$

$$\Rightarrow \quad t = t_{meet}$$

We have, $-\left(\frac{15-10t}{2}\right)t_{meet} = \left(\frac{15t - 5t^2}{2}\right)$

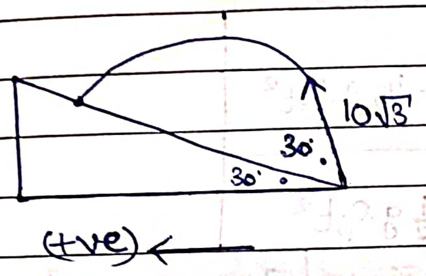
$$\Rightarrow \quad \left(\frac{10t - 15}{2}\right) = \left(\frac{15\cancel{t} - 5t}{2}\right)$$

$$\Rightarrow \quad \boxed{t = 1\,s}$$

Q) 

Find t_{meet}.

A) From wedge's frame.

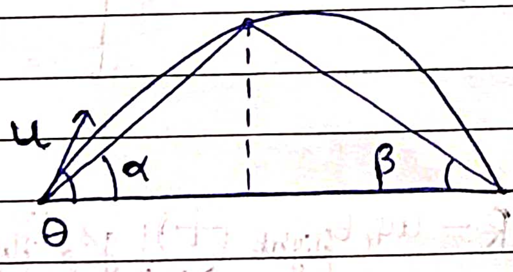


$$y = x\sqrt{3} - \frac{x^2}{15}$$

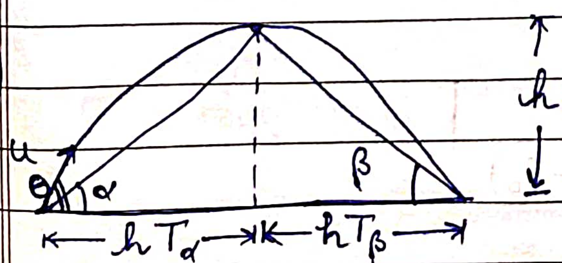
$$y = \frac{x}{\sqrt{3}}$$

Equating, $\frac{x^2}{15} = \left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)x \Rightarrow x = \frac{30}{\sqrt{3}}$

$$t_{meet} = \left(\frac{30/\sqrt{3}}{5\sqrt{3}}\right) \Rightarrow \boxed{t_{meet} = 2s}$$

☆ Q) 

Find relⁿ b/w α, β and θ

A) 

$$y = (x t_\theta)\left(1 - \frac{x}{R}\right)$$

$$\Rightarrow h = (h T_\alpha t_\theta)\left(1 - \frac{h T_\alpha}{h(T_\alpha + T_\beta)}\right)$$

$$\Rightarrow \boxed{t_\theta = t_\alpha + t_\beta}$$

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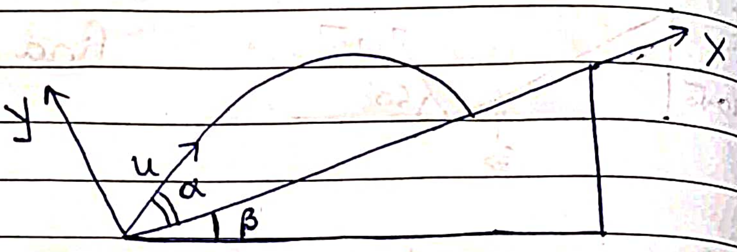
3) Incline:

i) Up:

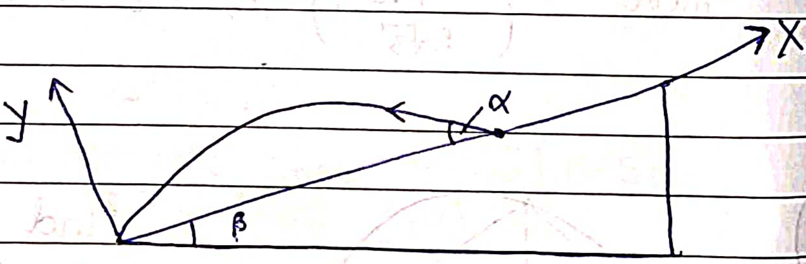


	$\vec{u}$	$\vec{a}$	$\vec{s}$
X	$u c_\alpha$	$-g s_\beta$	$u c_\alpha t - \frac{1}{2} g s_\beta t^2$
y	$u s_\alpha$	$-g c_\beta$	$u s_\alpha t - \frac{1}{2} g c_\beta t^2$

$\left(2\alpha+\beta=\frac{\pi}{2}\right) \rightarrow \boxed{R_{max} = \frac{u^2}{g(1+s_\beta)}}$

$\boxed{t_{flight} = \left(\frac{2u s_\alpha}{g c_\beta}\right)}$; $\boxed{R = u c_\alpha t_{flight} - \frac{1}{2} g s_\beta t_{flight}^2}$

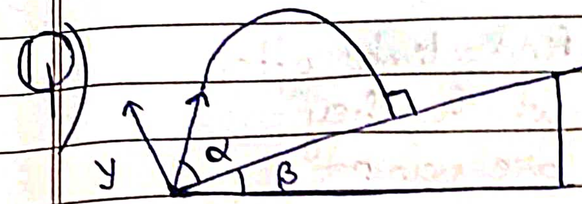
ii) Down:



$\boxed{t_{flight} = \left(\frac{2u s_\alpha}{g c_\beta}\right)}$ $\boxed{R = u c_\alpha t_{flight} + \frac{1}{2} g s_\beta t_{flight}^2}$

$\left(2\alpha+\beta=\frac{\pi}{2}\right) \rightarrow \boxed{R_{max} = \frac{u^2}{g(1-s_\beta)}}$

Q)



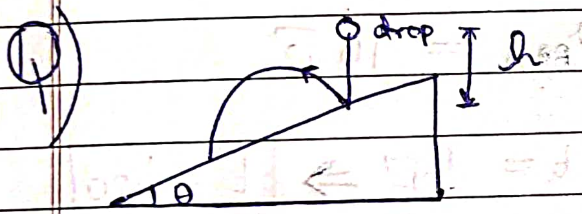
find relⁿ b/w α and β.

A) At $t = t_{flight}$, $v_x = 0$

$$\Rightarrow \quad u c_\alpha = (g s_\beta) t_{flight} = (g s_\beta)\left(\frac{2u s_\alpha}{g c_\beta}\right)$$

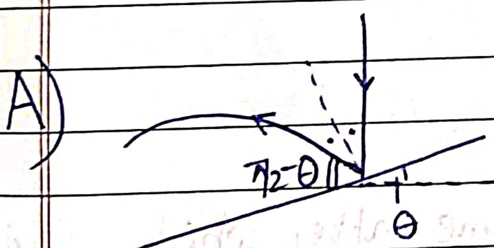
$$\Rightarrow \quad \boxed{t_\alpha t_\beta = 1/2}$$

Q)



find R. Assume collision perfectly elastic.

A)

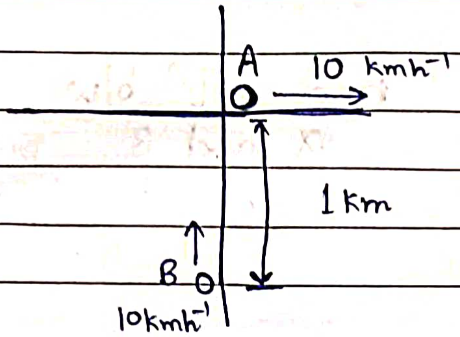


We have $\alpha = \pi/2 - \theta$ and $u = \sqrt{2gh}$

$$t_{flight} = \left(\frac{2u s_\alpha}{g c_\beta}\right) = \frac{2\sqrt{2gh}\, c_\theta}{g c_\theta} \Rightarrow t_{flight} = \frac{2\sqrt{2gh}}{g}$$

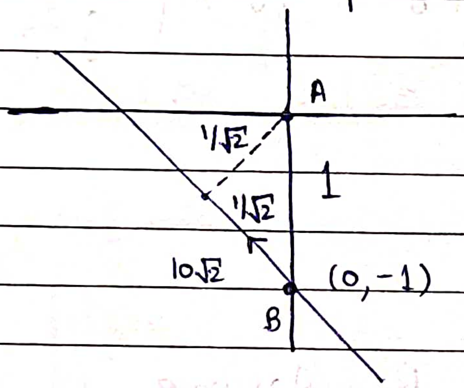
$$R = \overset{(\sqrt{2gh})}{\cancel{u}}\, s_\theta\left(\frac{2\sqrt{2gh}}{g}\right) + \frac{1}{2} g s_\theta\left(\frac{2\sqrt{2gh}}{g}\right)^2 \Rightarrow \boxed{R = 8h s_\theta}$$

☆ Q)



Find time after which they are closest.

A) In A's frame,



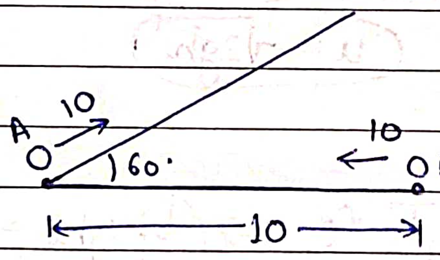
Path $= x + y + 1 = 0$

Short dist. $= 1/\sqrt{2}$

$v_{B,A} = 10\sqrt{2}$

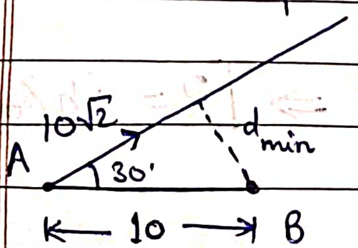
$\Rightarrow \quad t = \dfrac{1/\sqrt{2}}{10\sqrt{2}} \Rightarrow \boxed{t = 1/20}$

☆ Q)



Find time after which they are closest.

A) In B's frame,



$d_{min} = 10\, s_{30} \Rightarrow d_{min} = 5$

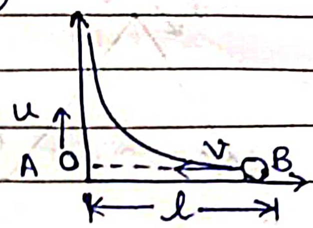
Dist. Covered $= 5\sqrt{3}$

$t = \dfrac{5\sqrt{3}}{10\sqrt{3}} \Rightarrow \boxed{t = 1/2}$

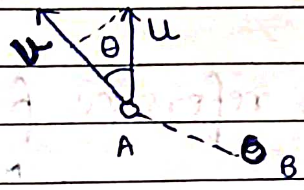
$v_{A,B} = 10\sqrt{3}$

(Chasing Problems)

Q) B is always directed towards A. Find t_{meet} if B catches A.



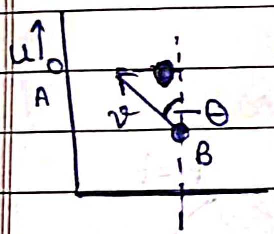
A) At any time 't',



$$v_{Approach} = v_{\underset{\text{along AB}}{B,A}} = v - u c_\theta$$

$$\Rightarrow \quad -\int_l^0 dl = \int_0^T (v - u c_\theta)\, dt = vT - u\int_0^T c_\theta\, dt$$

$$\Rightarrow \quad \boxed{l = vT - u\int_0^T c_\theta\, dt} \quad \text{— (i)}$$



Since B catches A,

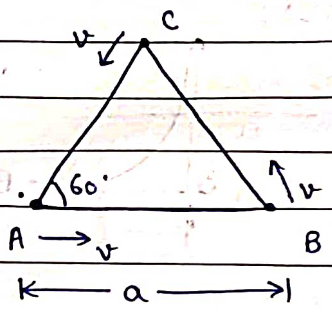
$$uT = \int_0^T v c_\theta = v\int_0^T c_\theta\, dt$$

$$\Rightarrow \quad \boxed{\int_0^T c_\theta\, dt = \frac{uT}{v}} \quad \text{— (ii)}$$

Using in (i), $\quad l = vT - u\left(\dfrac{uT}{v}\right)$

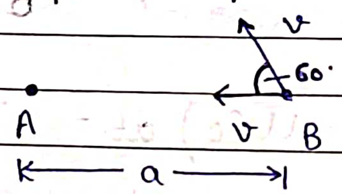
$$\Rightarrow \quad \boxed{T = \left(\frac{vl}{v^2 - u^2}\right)}$$

Q)



A → B , B → C , C → A always.
find t_{meet}.

A) In reference frame of A,



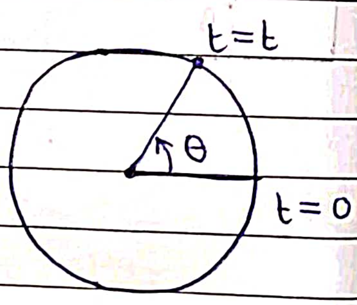
$v_{B,A} = \frac{3v}{2}$ $s_{B,A} = a$

$\Rightarrow \boxed{t_{meet} = \frac{2a}{3v}}$

Circular Motion

Angular Disp. $= \theta$ (rad)

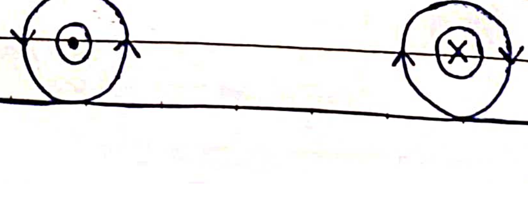
Angular Vel. $= \omega$ (rad/s)



$$\boxed{\omega = \frac{d\theta}{dt}}$$

☆ Note : ω is an Axial Vector, dirx^n by Right Hand Thumb Rule

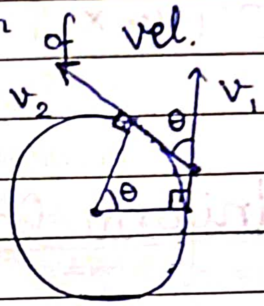
Axial Vectors are vectors with no endpt./pt. of application.
It is used to describe rotation.



Angular Acc. $= \alpha\ (rad/s^2)$ $\quad \boxed{\alpha = \dfrac{d\omega}{dt}}$

☆ **Note:** If $\omega \uparrow$, $\alpha \parallel$ to ω
If $\omega \downarrow$, α anti$\parallel$ to ω

Centripetal acc. : Changes dirx^n of vel.



$\Delta v = \sqrt{v_1^2 + v_2^2 - 2v_1v_2 c_\theta}$

$\Rightarrow \Delta v = \sqrt{2v^2(1 - c_\theta)}$

$\Rightarrow \Delta v = 2v\,|s_{\theta/2}| \approx 2v\left(\frac{\theta}{2}\right)$ (for small θ)

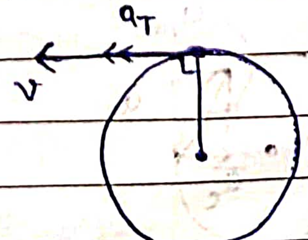
$\Rightarrow \Delta v = v\theta \Rightarrow dv = v\,d\theta$

$\Rightarrow \boxed{\dfrac{d\hat{v}_T}{dt} = a_c = \omega v = \omega^2 R = \dfrac{v^2}{R}}$ (Dirx^n always towards centre)

☆ $\boxed{ds = R\,d\theta \Rightarrow \boxed{v = R\omega}}$ (Only for Const. R)

Tangential acc. : Changes mag. of vel.

$\boxed{a_T = \dfrac{dv_T}{dt}}$ mag.



(Dirx^n always along vel.)

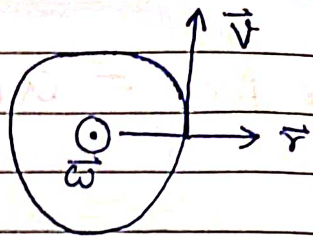
☆ $\boxed{\left(\dfrac{dv}{dt}\right) = R\left(\dfrac{d\omega}{dt}\right) \Rightarrow \boxed{a_T = R\alpha}}$ (Only for Const. R)

(Only for Const. R)

☆

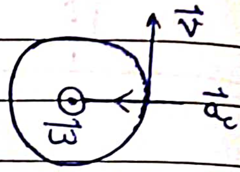
$$\boxed{\vec{V} = \vec{\omega} \times \vec{r}}$$

$(\odot \times \rightarrow \; = \; \uparrow)$



$$\boxed{\vec{a}_c = \vec{\omega} \times (\vec{\omega} \times \vec{r}) = \vec{\omega} \times \vec{v}}$$

$(\odot \times \uparrow \; = \; \leftarrow)$

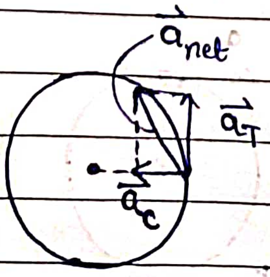


Uniform Circular Motion —

Const.	Not Const.
Speed	Velocity
K.E.	$\vec{a}_c$
a_c	Momentum

Non-Uniform Circular Motion —

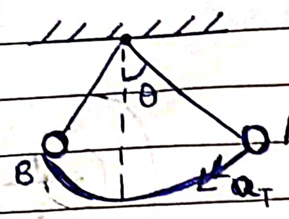
Speed is NOT const.



$$\boxed{a_{net} = \sqrt{a_c^2 + a_T^2}}$$

Pendulum :

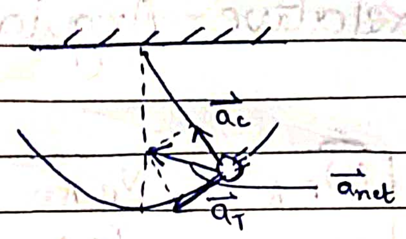
Extreme



$a_c = 0$ (vel. $= 0$)

$a_T = g\sin\theta$

At some Pt.

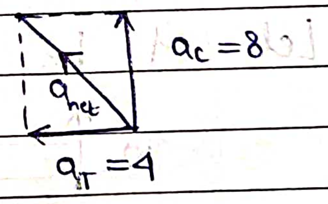


$a_c \neq 0$

Q) A body is moving is a circle of radius 2m. Its speed is dec. at $4\ ms^{-2}$. Find net. acc. of body when its speed is $4\ ms^{-1}$.

A) $a_c = \frac{v^2}{R} = \frac{16}{2} \Rightarrow \boxed{a_c = 8\ ms^{-2}}$

$a_T = \frac{dv}{dt} = 4 \Rightarrow \boxed{a_T = 4\ ms^{-2}}$



$a_{net} = \sqrt{8^2 + 4^2}$

$\Rightarrow \boxed{a_{net} = 4\sqrt{5}}$

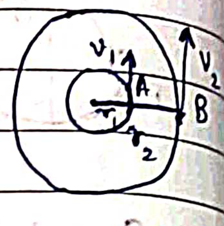
Eqⁿs of Motion (for U.C.M. & N.U.C.M with const. α)

$\boxed{\omega = \omega_0 + \alpha t}$; $\boxed{\theta = \omega_0 t + \frac{1}{2}\alpha t^2}$; $\boxed{\theta = \left(\frac{\omega + \omega_0}{2}\right)t}$

Date: ______ Page: ______

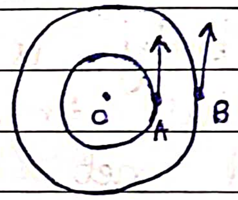
Relative Angular Vel. —

$$\boxed{\omega_{B,A} = \frac{v_{B,A}}{r_{B,A}} = \left(\frac{v_2 - v_1}{r_2 - r_1}\right)}$$



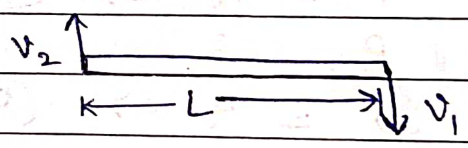
☆ Observer is at A.

$$\boxed{\omega_{B,A\ \text{w.r.t}\ O} = \omega_B - \omega_A}$$

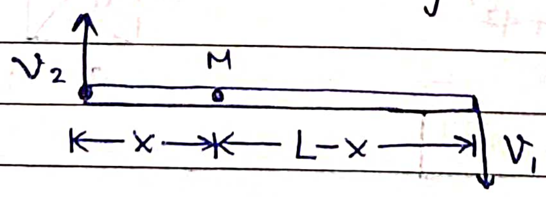


☆ Observer at centre O

☆ Q) Find ω.



A) Let M be centre of rotation.

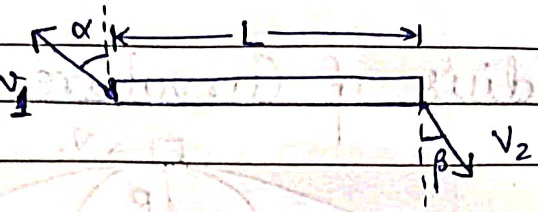


$\omega x = v_2$; $\omega(L-x) = v_1$

$$\Rightarrow \boxed{\omega = \left(\frac{v_1 + v_2}{L}\right)}$$

$$\Rightarrow \boxed{\omega = \frac{(v_{rel})_{\perp \text{ to rod}}}{L}}$$

Q) Find ω.

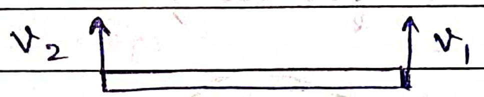


A) ☆ $v_{\text{along rod}}$ is same for all pts.

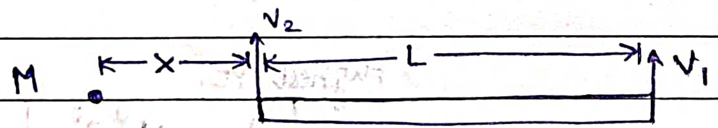
$$\Rightarrow \quad v_1 s_\alpha = v_2 s_\beta \quad \Rightarrow \quad s_\beta = \left(\frac{v_1}{v_2}\right) s_\alpha$$

Now, $$\omega = \frac{(v_{rel})_\perp}{L} = \left(\frac{v_1 c_\alpha + v_2 c_\beta}{L}\right)$$

Q) Find ω $(v_1 > v_2)$



A) ☆ Centre of rotation outside rod.

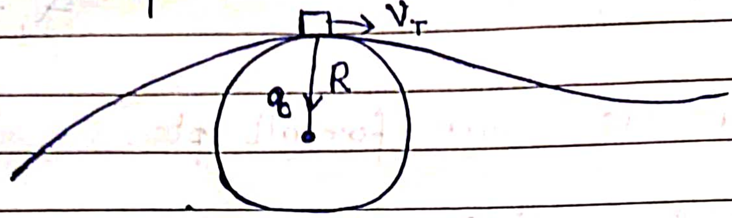


$$\omega x = v_2 \quad ; \quad \omega(L+x) = v_1$$

$$\Rightarrow \quad \boxed{\omega = \left(\frac{v_1 - v_2}{L}\right)}$$

Date:

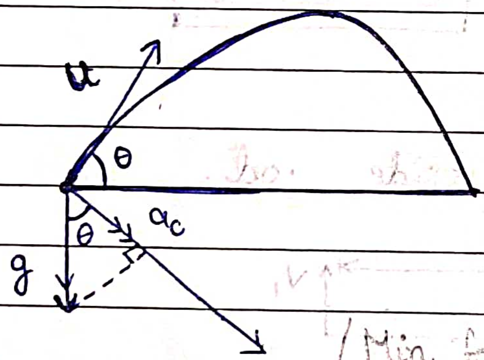
Radius of Curvature –



$$\boxed{a_c = \frac{v_T^2}{R}}$$

(Remember $a_c \perp v_T$ and v_T tangent to curve)

For Projectile,



Start

$$\boxed{R = \frac{u^2}{g\cos\theta}}$$

Highest Pt.

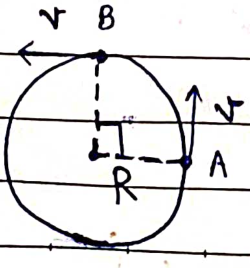
(Min. for any pt. on proj.)

$$\boxed{R = \frac{u^2\cos^2\theta}{g}}$$

Average Acceleration –

$$\boxed{a_{Avg.} = \left|\frac{\Delta\vec{v}}{\Delta t}\right| = \left|\frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}\right|}$$

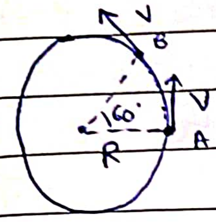
Q)



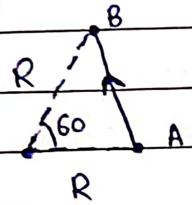
Find a_{Avg} and v_{Avg}.

A) $\Delta s = R\sqrt{2}$, $\Delta \vec{v} = v\sqrt{2}$, $\Delta t = \frac{\pi}{2} \cdot \frac{R}{v}$

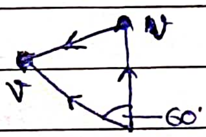
$\boxed{a_{Avg} = \left(\frac{2\sqrt{2}\, v^2}{\pi R}\right)}$, $\boxed{v_{Avg} = \left(\frac{2\sqrt{2}\, v}{\pi}\right)}$

Q) 

Find a_{Avg} and v_{Avg}.

A) 

$\Delta s = R$



$\Delta v = v$

$\Delta t = \pi/3 \cdot R/v$

$\boxed{a_{Avg} = \left(\frac{3v^2}{\pi R}\right)}$, $\boxed{v_{Avg} = \left(\frac{3v}{\pi}\right)}$

Some Tricks —

1) If 2 objs reach a place at same time, set their times equal.

(See Cengage, Kinematics I, CAE 4.4, Q11)

2) Oblique proj. with initial vel. u = Horiz. proj. with initial vel. u_x from h_{max}.

(See Kinematics, CPP-7, Q 12, 13, 14, 15 & CPP-8, Level 2, Q 3)