Momentum

$$\vec{F} = \frac{d\vec{p}}{dt} \Rightarrow \int_{p_0}^{p_1} d\vec{p} = \int_0^t \vec{F}\,dt$$

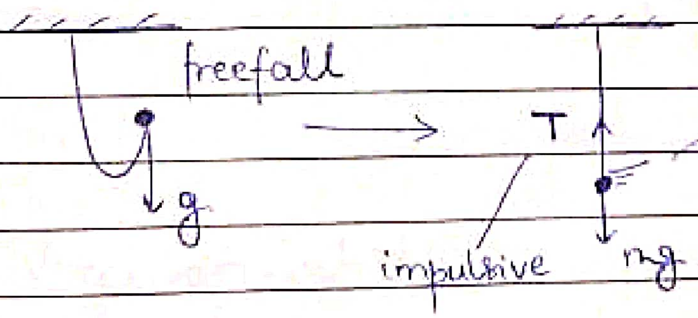
$$\Rightarrow \boxed{\vec{p_1} - \vec{p_0} = \Delta\vec{p} = \int_0^t \vec{F}\,dt}$$

* If F large and t very small

$\Rightarrow$ $\vec{F}$ is Impulsive force.

* Change in momentum due to Impulsive force is called Impulse.

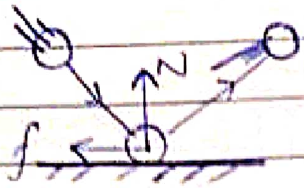
Eg:



(At this inst.)
$T >> mg$

Suddenly Tension appears.

☆ Make FBDs of normal forces & impulsive forces seperately.

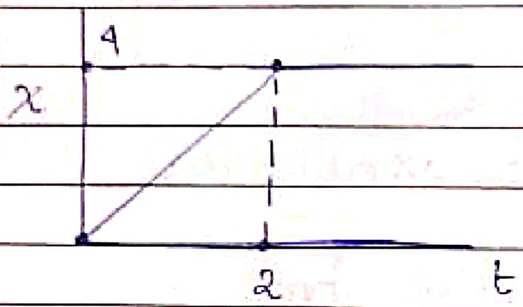
Eg: 

f suddenly develops

$\underbrace{f = \mu N}_{\text{Impulsive}}$ but $N \neq mg$

$N >> mg$

(At this inst)

Q) 

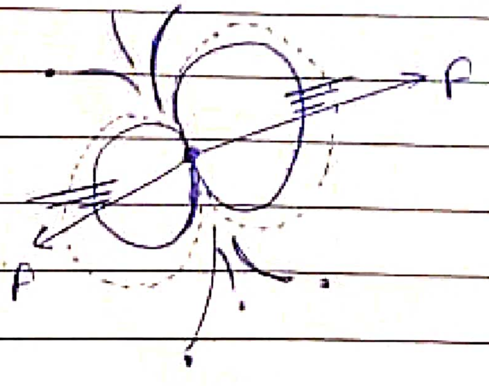
If mass of body = 2kg

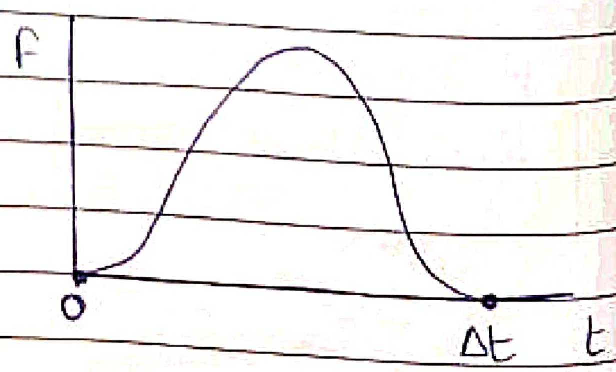
find impulse.

A) Let A at $t = 2 - \delta$, B at $t = 2 + \delta$.

$v_A = 2$, $v_B = 0$ $\Rightarrow$ $\boxed{\Delta p = (-4)}$

When 2 bodies collide & rebound,

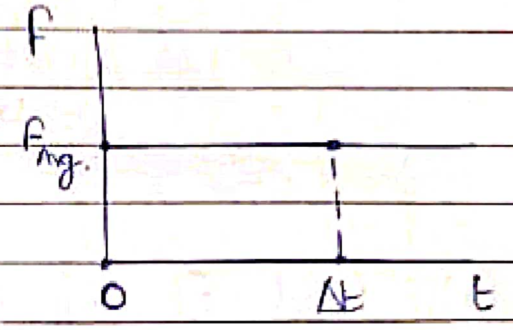




Since difficult to calc. area under curve, hence impulse also diff to calc.

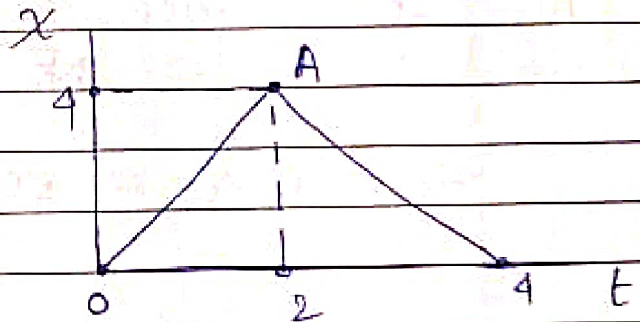
So we take F_{avg}.

$$\boxed{\text{Impulse} = F_{Avg} \cdot \Delta t}$$

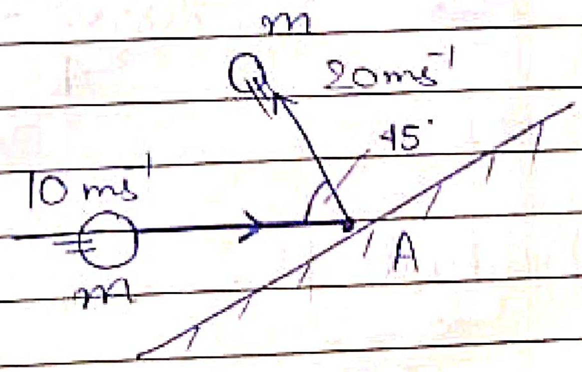


(We take F_{avg} in such a way that area under curve in both same)

Q) Find magnitude of impulse.
Mass of obj 2 kg.



A) $v_{A^+} = -2$
$v_{A^-} = 2$ $\Rightarrow$ $\boxed{\Delta p = (-8)}$



Q) Find horizontal and vertical force by surface.

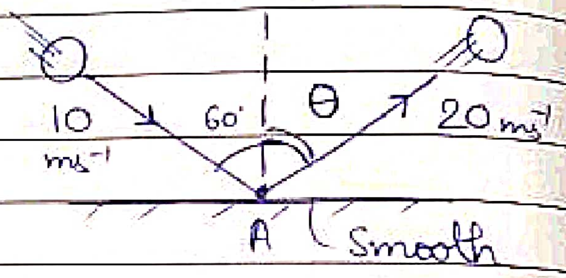
$\Delta t = 0.01\,s$, $m = 1\,kg$

A) $\vec{v}_{A^-} = \langle 10, 0 \rangle$, $\vec{v}_{A^+} = \langle -10\sqrt{2}, 10\sqrt{2} \rangle$

$$\Delta \vec{p} = \langle -10(\sqrt{2}+1), \; 10\sqrt{2} \rangle$$

$$\Rightarrow \boxed{\vec{f}_{Avg} = \langle \underbrace{(-10^3)(\sqrt{2}+1)}_{\text{Horiz.}}, \; \underbrace{10^3\sqrt{2}}_{\text{Vertical}} \rangle}$$

Q) find θ, if surface smooth.
Also find f if $\Delta t = 0.01\,s$
$m = 1\,kg$.



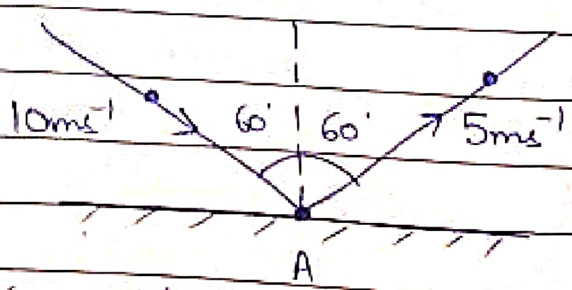
A) If surface smooth $\Rightarrow$ No friction $\Rightarrow$ $V_{\text{along surface}}$ Same

$$\Rightarrow 10\, s_{60°} = 20\, s_{\theta} \Rightarrow \boxed{\theta = s^{-1}\frac{\sqrt{3}}{4}}$$

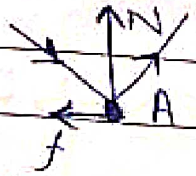
$$|\Delta \vec{p}| = \Delta p_y \quad (\text{As } p_x = \text{Const.})$$

$$= (1)\left(20 \cdot \frac{\sqrt{13}}{4} - \left(-10 \cdot \frac{1}{2}\right)\right) \Rightarrow \boxed{f_{Avg} = 500(1+\sqrt{13})}$$

Q) find μ.
$m = 1\,kg$



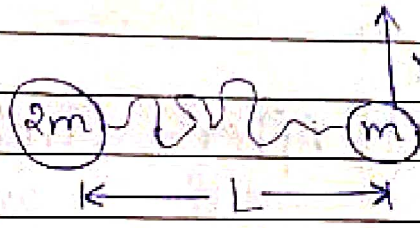
A) $X: \; 5\sqrt{3}/2 - 10\sqrt{3}/2 = (-\mu N)(\Delta t)$
$y: \; 5/2 - (-10/2) = N(\Delta t)$



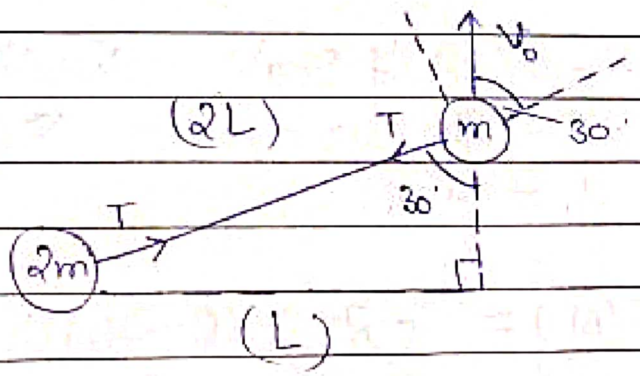
On dividing eq^{n}s, $\boxed{\mu = 1/\sqrt{3}}$

Q) Length of String = 2L.
Objs on frictionless surface.
☆ Find vel. of 2m and m when string becomes taut.

Top view



A)



☆ Impulse will change vel. of objs. ALONG string.

$^{m}v_{along} = v_0\sqrt{3}/2$, $^{m}v_{\perp} = v_0/2$

After effect of impulse, $^{m}v_{\perp}' = {}^{m}v_{\perp} = v_0/2$.

Now, 2m: $T(\Delta t) = (2m)(^{2m}v'_{along})$
m: $-T(\Delta t) = (m)(^{m}v'_{along} - {}^{m}v_{along})$

$$\Rightarrow 2(^{2m}v'_{along}) + \left(^{m}v'_{along} - \frac{v_0\sqrt{3}}{2}\right) = 0$$

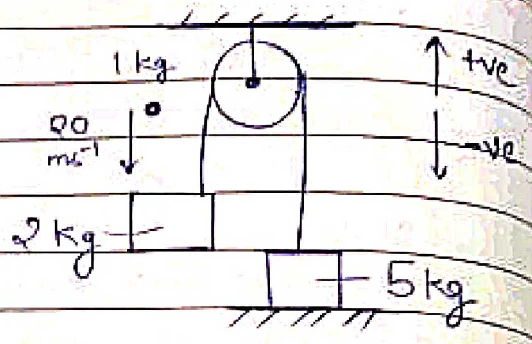
Since string taut $\Rightarrow$ String Constraint $\Rightarrow$ $^{2m}v'_{along} = {}^{m}v'_{along}$

$\Rightarrow$ $^{m}v'_{along} = {}^{2m}v'_{along} = v_0/2\sqrt{3}$

Finally, $\boxed{v_{2m} = \frac{v_0}{2\sqrt{3}}}$, $\boxed{v_m = \frac{v_0}{\sqrt{3}}}$

(If we didn't take 1 & 2 as system, we would have to include N b/w 1 & 2 in calc., as it is also impulsive.)

Q) Find speed of masses just after 1 kg sticks to block.



A) After sticking 1 kg, 2 kg move with same velocity as a system

$u_5 = 0$, $v_5 = v' =$ (Vel. of 5 kg after collision), $v_2 = v_1 = v =$ (Vel. of system after collision)

$u_2 = 0$, $u_1 = +20$.

(2+1) : $T(\Delta t) = (-2v_2) - (-2u_2) + (-v_1) - (-u_1)$

$\Rightarrow -T(\Delta t) = 3v - 20$

(5): $T(\Delta t) = 5v_5 - 5u_5 \Rightarrow T(\Delta t) = 5v'$

By String Constraint, $v = v'$ $(v_5 = v_2 = v_1)$

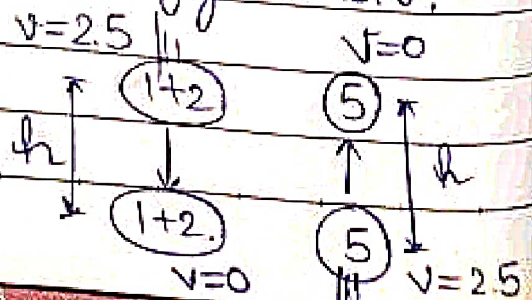
$\Rightarrow$ $\boxed{v = 2.5}$

Q) Find max. height attained by 5 kg block in prev. Q.

A) Let it be 'h'. By Energy Consrv.

$\frac{1}{2} \cdot 3 \cdot \left(\frac{5}{2}\right)^2 + \frac{1}{2} \cdot 5 \cdot \left(\frac{5}{2}\right)^2 = 5gh - 3gh$

$\Rightarrow$ $\boxed{h = 5/4}$

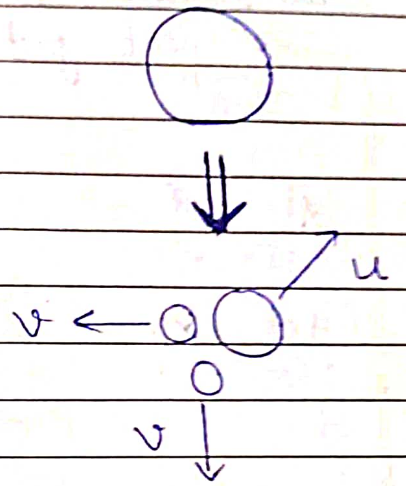


Conservation of Momentum

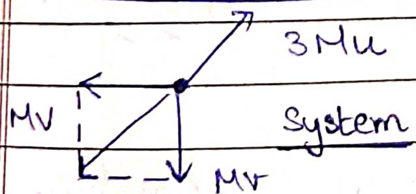
If $\vec{F}_{ext} = 0 \Rightarrow \frac{d\vec{p}}{dt} = 0 \Rightarrow \boxed{\vec{p} = \text{Const}}$

(Can't use for Impulsive forces, as they do work quickly.)

Q) A mass explodes into 3 parts with mass ratio 1:1:3. Parts with equal mass are moving ⊥ to each other with v. Find speed of 3rd part.



A) Just before & after collision, Fext not able to do any work, hence $\vec{p}$ consrv.



$p_f = p_i = 0 \Rightarrow 3Mu = Mv\sqrt{2}$

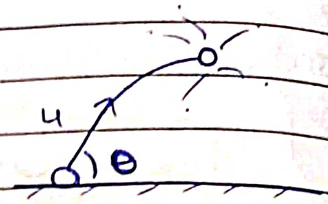
$\Rightarrow \boxed{u = \frac{v\sqrt{2}}{3}}$

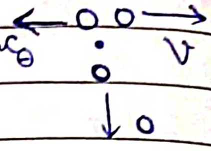
Q) If total mass = 5M in above Q, find energy released by explosion

A) $\Delta E = E_f - E_i = \left(\frac{1}{2} \cdot 3M \cdot \frac{2v^2}{9} + \frac{1}{2} \cdot M \cdot v^2 + \frac{1}{2} M v^2\right) - 0$

$\Rightarrow \Delta E = \frac{4v^2 M}{3}$

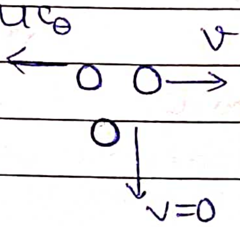
Q) At highest pt., obj breaks into 3 equal parts. One falls in vertically down with 0 init. vel., another retraces its path. Find vel. of 3rd part just after explosion.





A) At highest pt, before exp., $p = 3Muc_\theta$

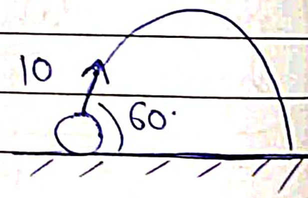
After exp,



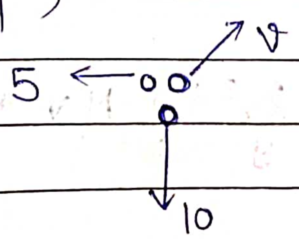
$Mv - Muc_\theta = 3Muc_\theta$

$\Rightarrow \boxed{v = 4uc_\theta}$

Q) At highet pt. obj breaks into 3 equal parts. First fall vertally down with 10 init. vel. 2nd part retraces its path. Find vel. of third just after collision



A) After exp.,



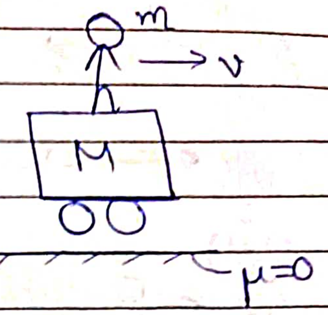
$P_{before} = 3M10c_{60°} = 15M(\rightarrow)$

$P_{after} = 5M(\leftarrow) + 10M(\downarrow) + Mv(\nearrow)$

$\Rightarrow Mv_y = 10M, \quad Mv_x = 20M$

$v_x = 20$
$\Rightarrow v_y = 10 \Rightarrow \boxed{v = 10\sqrt{5}}$

Q) Person jumps with vel. 'v' wrt state of car before jumping. Find v_{car} just after jump.



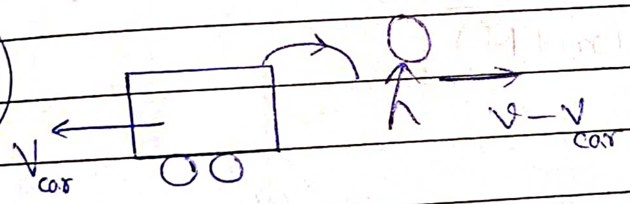
A) $P_{init} = 0 = P_{final} = mv - Mv_{car}$

$\Rightarrow v_{car} = \dfrac{mv}{M}$

(If only given 'wrt car', assume this)

Q) In above Q, $v_{car} = ?$ if person jumps with 'v' wrt car after jumping state of.

A)



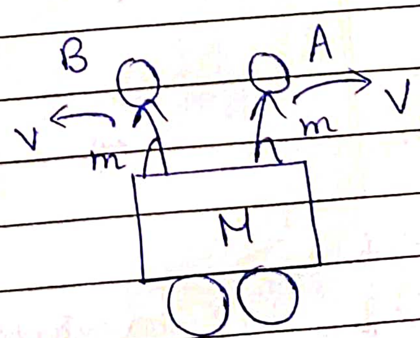
$P_{init} = 0 = P_{final}$

$= m(v - v_{car}) - Mv_{car}$

After jump

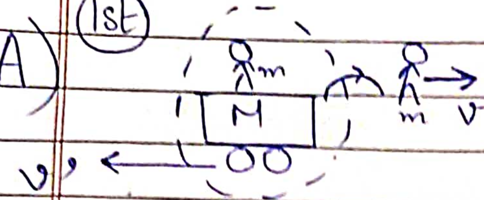
$\Rightarrow v_{car} = \left(\dfrac{mv}{M+m}\right)$

☆ Q) Persons jump with velocity 'v' wrt state of car before their resp. jump.



Find v_{car} after 2nd jump, if A jumps before B.

A) 1st



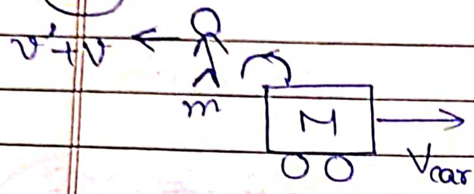
1st jump

$P_{init} = 0 = P_{final}$

$= mv - (m+M)v'$

$\Rightarrow \boxed{v' = \frac{mv}{m+M}}$

2nd



2nd jump

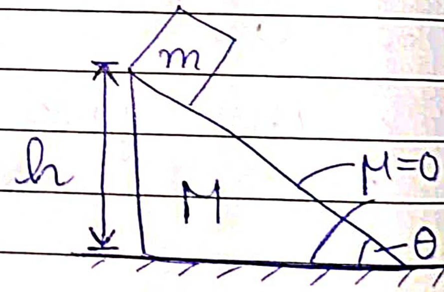
$P_{init} = P_{final}$

$\Rightarrow (M+m)v' = m(v'+v) - MV_{car}$

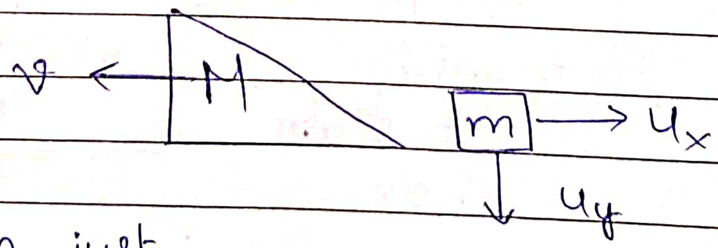
$\Rightarrow MV_{car} = m\left(\frac{mv}{M+m} + v\right) - (M+m)\left(\frac{mv}{M+m}\right)$

$\Rightarrow \boxed{V_{car} = \frac{m^2 v}{M(m+M)}}$

Q) Find vel. of M when m reaches bottom of incline.



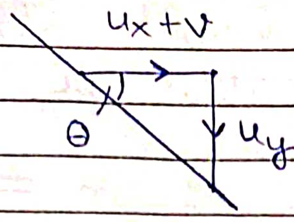
A)



When m just reach bottom.

Consrv. Momentum along X axis, $Mv = mu_x$

In M's frame, obj. move along incline



$t_\theta = \left(\frac{u_y}{u_x+v}\right)$

By Energy Consrv., $mgh = \frac{1}{2}Mv^2 + \frac{1}{2}mu_x^2 + \frac{1}{2}mu_y^2$

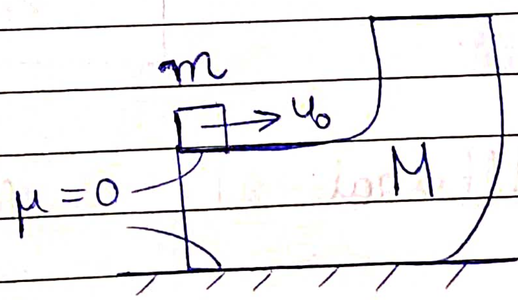
Now, $u_x = \left(\frac{M}{m}\right)v$ & $u_y = \left(\frac{M+m}{m}\right)vt_\theta$

Substituting, $mgh = \frac{M}{2}v^2 + \frac{m}{2}\cdot\frac{M^2v^2}{m^2} + \frac{m}{2}\cdot\left(\frac{M+m}{m}\right)^2 v^2 t_\theta^2$

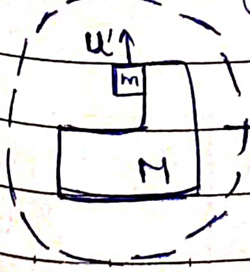
$\Rightarrow mgh = \left(\frac{1}{2m}\right)\left[Mmv^2 + M^2v^2 + (M+m)^2v^2t_\theta^2\right]$

$$\Rightarrow \boxed{v = \sqrt{\frac{2m^2gh}{Mm + M^2 + (M+m)^2 t_\theta^2}}}$$

Q) a) Find vel. of M when m is moving in vertical section



A) Since obj in contact, vel. along X axis same at last



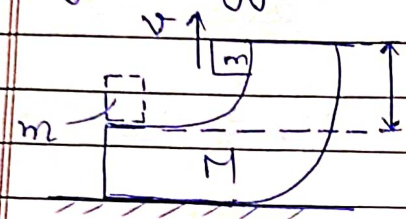
Consrv. Momentum along X axis.

$$\Rightarrow \boxed{v = \left(\frac{mu_0}{M+m}\right)}$$

b) Find max. height attained by mass from its initial pt.

A) Let up vel. of m = 'v'.

By energy Conserv., $\frac{1}{2}mu_0^2 = mgh + \frac{1}{2}(M+m)\left(\frac{mu_0}{M+m}\right)^2 + \frac{1}{2}mv^2$



$\Rightarrow mu_0^2 = 2mgh + \left(\frac{m^2u_0^2}{M+m}\right) + mv^2$

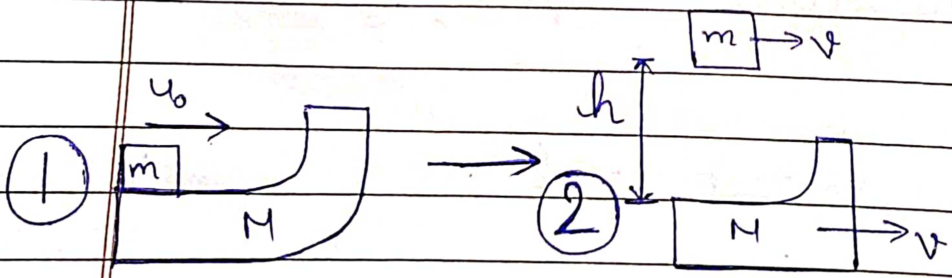
$$\Rightarrow \boxed{v^2 = \left(\frac{Mu_0^2}{M+m}\right) - 2gh}$$

Now, $h_{max.} = \frac{u_y^2}{2g} = \frac{Mu_0^2}{2g(M+m)} - h$

Wrt initial posⁿ., $H = h_{max} + h$

$$\Rightarrow \boxed{H = \frac{Mu_0^2}{2(M+m)g}}$$

☆ Alternat solⁿ – Apply Energy Consv. wrt CoM.



$E_1 = E_2 \Rightarrow \underbrace{\frac{1}{2}\left(\frac{Mm}{M+m}\right)u_0^2}_{(K_1)} = \underbrace{mgh}_{(U_2)} \Rightarrow \boxed{h = \frac{Mu_0^2}{2(M+m)g}}$