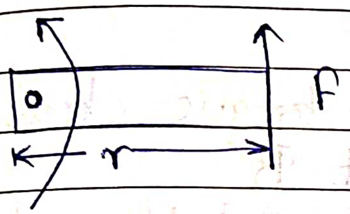
Rotation

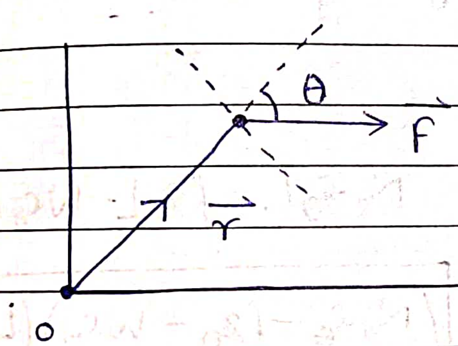
Torque
Moment of Inertia
Angular Momentum
Rolling

Torque –

Turning effect of force is called torque.



↺ $\Rightarrow \tau > 0$
↻ $\Rightarrow \tau < 0$

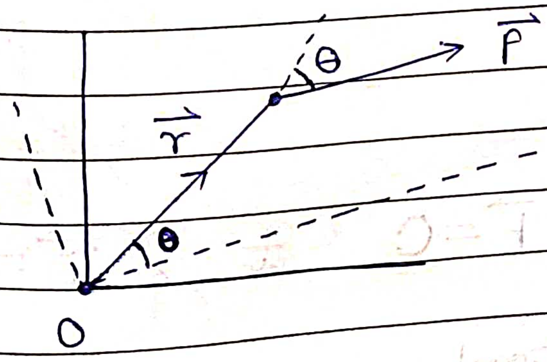


$$\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$$

Dirx^n of $\vec{r}$ from axis of rotation to pt. of application of force.

$$\boxed{\tau = Fr\,s_\theta}$$

We can also divide $\vec{r}$ along & $\perp$ $\vec{F}$.



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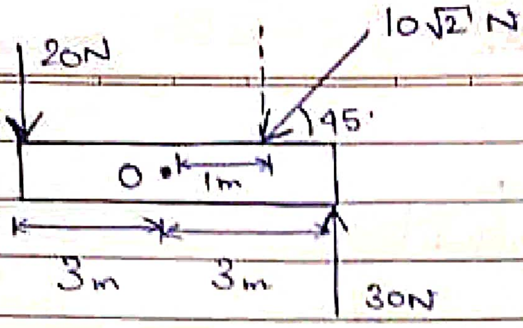
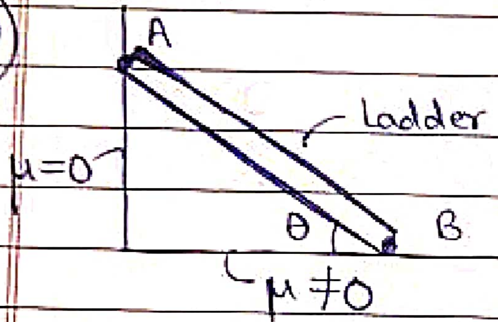
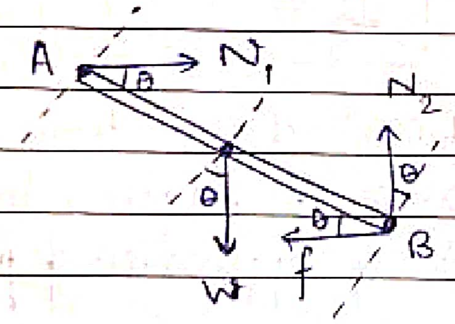
Q) 

Find torque about O.

A) $\tau = +(20)(3) + (30)(3) - (10)(1) = 140 \text{ Nm}$

Q) 

Find torque about A & B.
Length of ladder = L.

A) 

A: $N_2 \cos\theta L - f\sin\theta L - W\cos\theta \frac{L}{2}$

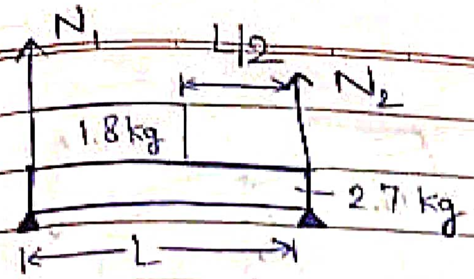
$= \boxed{\left(N_2\cos\theta - f\sin\theta - \frac{W\cos\theta}{2}\right)(L)}$

B: $W\cos\theta \frac{L}{2} - N_1\sin\theta L = \boxed{\left(\frac{W\cos\theta}{2} - N_1\sin\theta\right)(L)}$

Equilibrium –

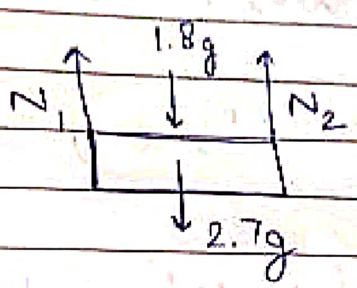
For equilibrium, $\underbrace{\sum \vec{F} = 0}_{\text{translational}}$ & $\underbrace{\sum \vec{\tau} = 0}_{\text{rotational}}$ about every pt.

Q)



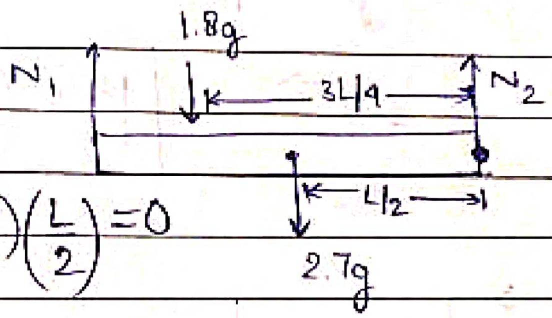
Find N_1 & N_2.

A) Translational :



$N_1 + N_2 = (4.5)g$

Rotational :

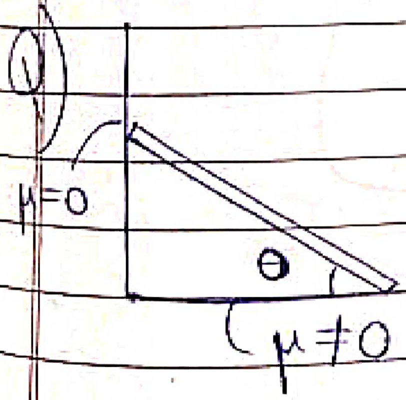


$-(N_1)(L) + (1.8g)\left(\frac{3L}{4}\right) + (2.7g)\left(\frac{L}{2}\right) = 0$

$\Rightarrow N_1 = 27$

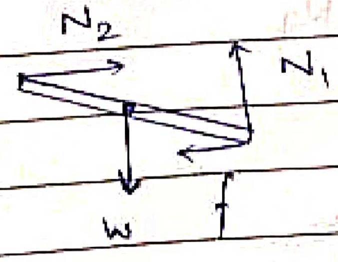
$\Rightarrow N_2 = 18$

Q)



Ladder in eq.
Find μ

A) Translational :



$N_1 = W$

$N_2 = f$

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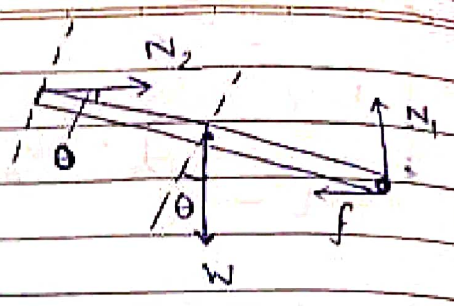


Rotational:

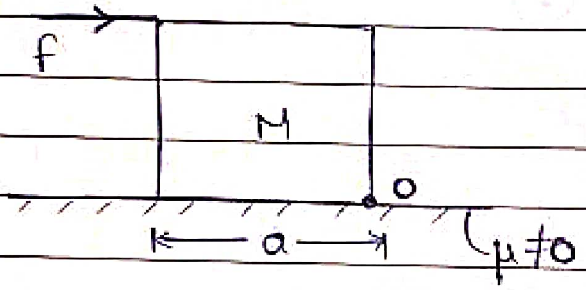
$$-N_2 S_\theta L + W C_\theta \frac{L}{2} = 0$$

$$\Rightarrow \quad W = 2N_2 t_\theta$$

Now, $f \leq \mu N_1 = \mu W \Rightarrow N_2 \leq \mu (2N_2 t_\theta)$

$$\Rightarrow \quad \boxed{\mu \geq \left(\frac{T_\theta}{2}\right)}$$

☆Q)



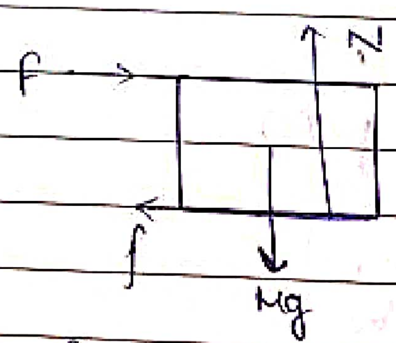
Find f_{min} s.t. obj. is just tilted about O.

A) ☆ N shifts towards O to balance torque due to f.

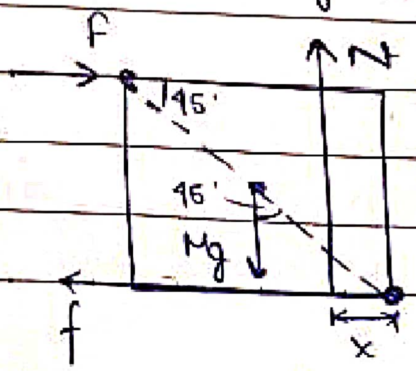
Translational:

$f = f$

$N = Mg$



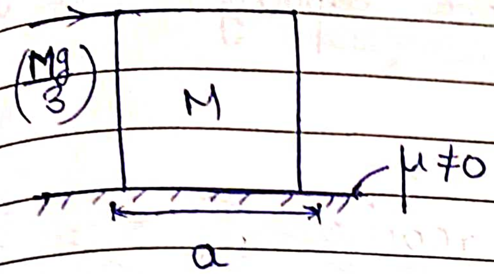
Rotational:



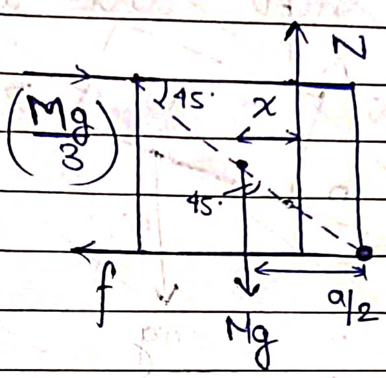
$$-f \cdot \frac{a\sqrt{2}}{\sqrt{2}} + \frac{Mg}{\sqrt{2}} \cdot \frac{a}{\sqrt{2}} - Nx = 0$$

$$\Rightarrow \quad f = \left(\frac{Mg}{2}\right) - \left(\frac{Mgx}{a}\right)$$

Now $x \geq 0 \Rightarrow$ If $\boxed{f > \frac{Mg}{2}}$ then obj. tilts/topples

Q) 

Find shift in normal

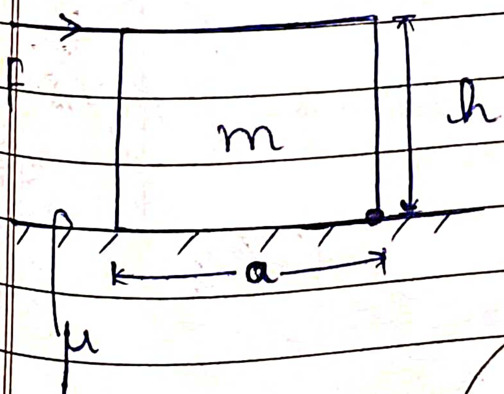


A) $N = Mg$

$$-\frac{Mg}{3\sqrt{2}} \cdot a\sqrt{2} + \frac{Mg}{\sqrt{2}} \cdot \frac{a}{\sqrt{2}} - N\left(\frac{a}{2} - x\right) = 0 \Rightarrow \boxed{x = a/3}$$

21/9/22

Conditions



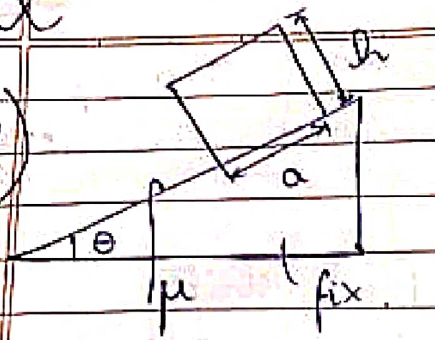
($\tau_{net} \neq 0$)

Toppling : $F \geq \left(\frac{mga}{2h}\right)$

Slipping : $F \geq \mu mg$

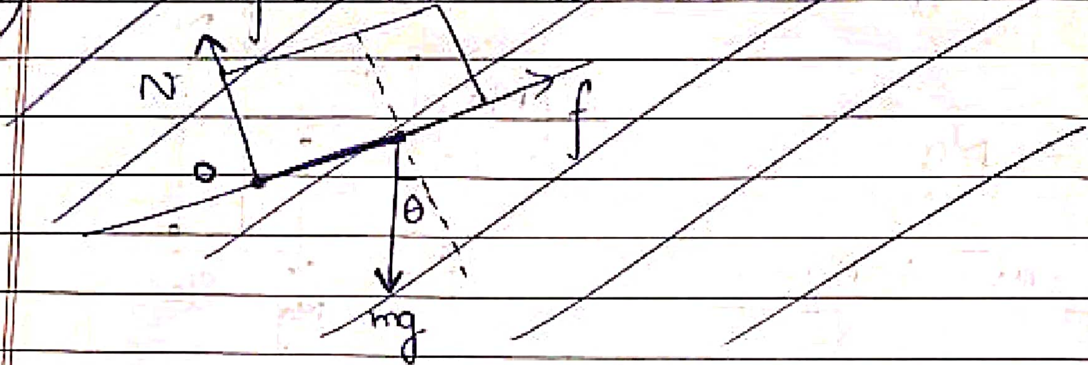
Toppling before slipping : $\mu \geq \left(\frac{a}{2h}\right)$

($\tau_{net} \neq 0$ & $F \leq \mu mg$)

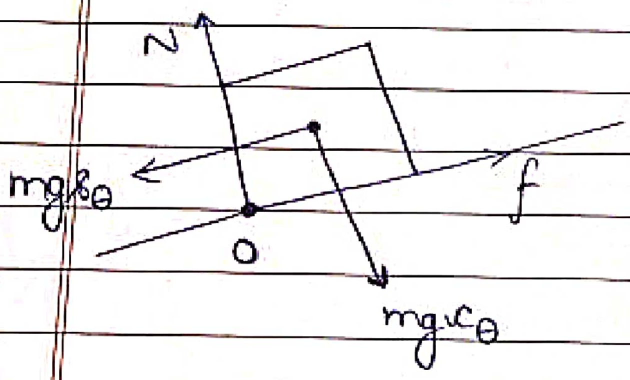
Q) 

find condt. for toppling before slipping.

A) ~~N shifts till it reaches O.~~



N shifts till it reaches O



Toppling : $(mg s_\theta)\left(\frac{h}{2}\right) \geq (mg c_\theta)\left(\frac{a}{2}\right)$

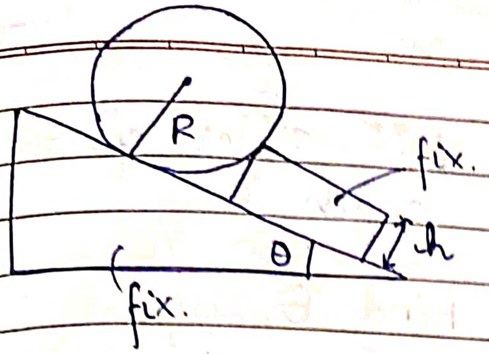
$\Rightarrow t_\theta \geq \left(\frac{a}{h}\right)$

Slipping : $\tan(\theta) \geq \mu$

Toppling before Slipping : $\left(\frac{a}{h}\right) \leq t_\theta \leq \mu$

$$\Rightarrow \boxed{\mu \geq \left(\frac{a}{h}\right)}$$

Q)



Find h s.t. Sphere doesn't topple.

A) ~~Since toppling not there,~~ $\cancel{\tau_{\sigma} \geq \tau_{\Omega}}$

We find condt. for toppling, then inverse the inequality.

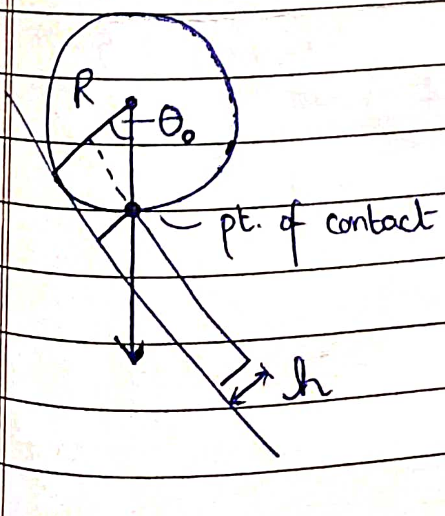
Since obj. lift, friction $= 0$.
Normal $= 0$

~~Normal shifts to pt. of contact.~~

$\Rightarrow$ Only gravity produces τ.

for limiting condt. $\tau = 0$.

☆ Pass 'mg' thru pt. of contact



For toppling, $\theta \geq \theta_0$

where $\left(\frac{R-h}{R}\right) = \cos\theta_0$

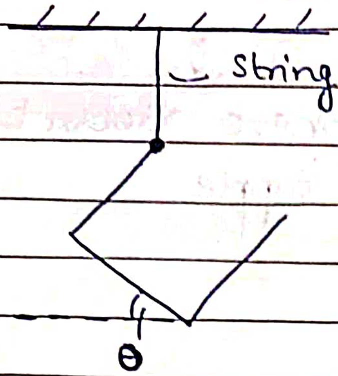
$\Rightarrow$ $\boxed{h \leq R(1-\cos\theta_0)}$

$\Rightarrow$ for not toppling, $\boxed{h \geq R(1-\cos\theta)}$

Q14

Date: Page:

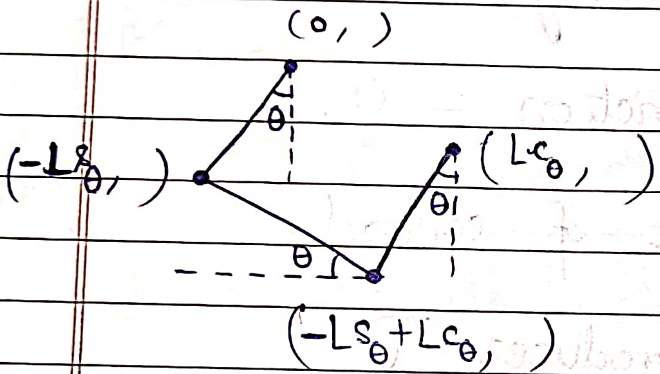
☆ Q)



All sides of length L.

Find θ.

A) τ due to gravity only ⇒ τ = 0 when 'mg' thru string



Let string be $x=0$ line.

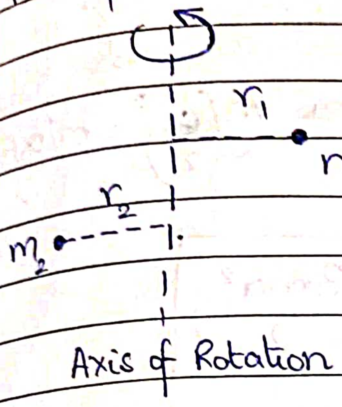
$\Rightarrow$ CoM $(x_{coor}) = 0$

$$\Rightarrow \left(\frac{-Ls_\theta}{2}\right) + \left(-Ls_\theta + \frac{Lc_\theta}{2}\right) + \left(Lc_\theta - \frac{Ls_\theta}{2}\right) = 0$$

$$\boxed{t_\theta = \left(\frac{3}{4}\right)} \Leftarrow$$

Moment of Inertia –

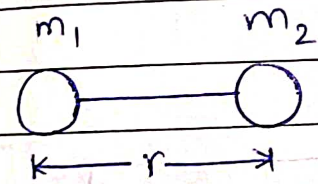
For pt. masses,



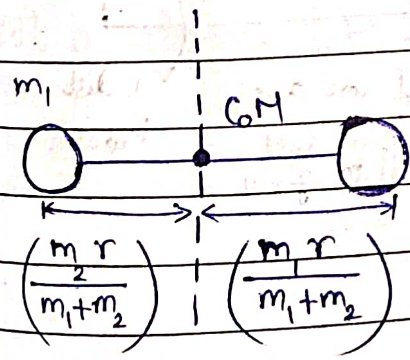
$$\boxed{I = m_1 r_1^2 + m_2 r_2^2}$$

$r = \perp$ dist. of mass from AoR.

I depends on <u>Mass</u> & <u>Distr. of Mass about AoR</u>

Q) 

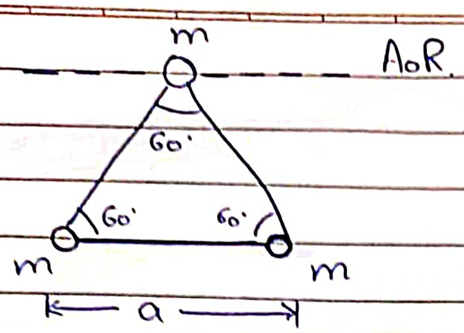
Find MoI about axis thru CoM $\perp$ to line joining the masses

A) 

$$I = m_1\left(\frac{m_2 r}{m_1+m_2}\right)^2 + m_2\left(\frac{m_1 r}{m_1+m_2}\right)^2$$

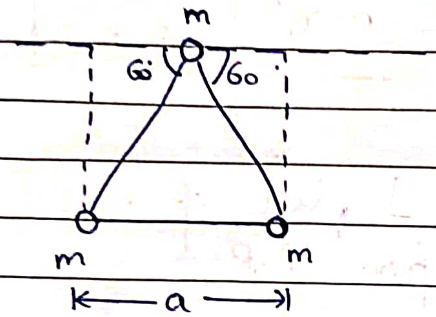
$$\Rightarrow \boxed{I = \left(\frac{m_1 m_2}{m_1+m_2}\right) r^2}$$

Q)



find MoI.

A)

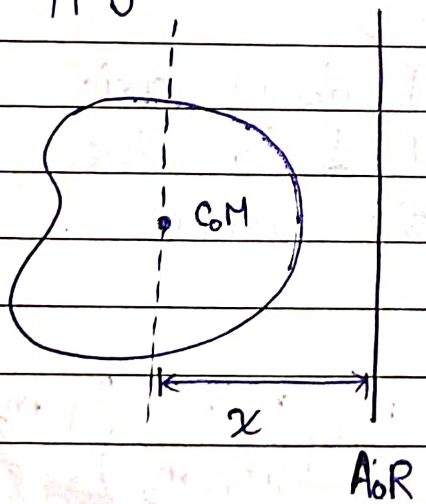


$$I = m(0)^2 + m\left(\frac{a\sqrt{3}}{2}\right)^2 + m\left(\frac{a\sqrt{3}}{2}\right)^2$$

$$\Rightarrow \boxed{I = \left(\frac{3ma^2}{2}\right)}$$

Theorem of || axis —

Apply when AoR NOT pass thru CoM.



$$\boxed{I = I_{CoM} + Mx^2}$$

I about given axis.

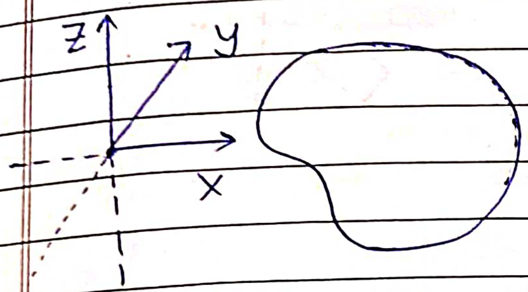
I wrt axis thru CoM || to given axis.

⊥ dist. b/w axes.

Theorem of $\perp$ axis —

Apply only for plane bodies.

Iff body in XY plane,

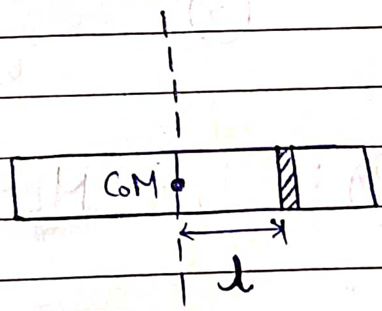


$$\boxed{I_z = I_x + I_y}$$

* All these axes should pass thru same pt. (NOT necessarily CoM)

MoI of Diff. Objs. — $\left(I = \int I_{element}\right)$

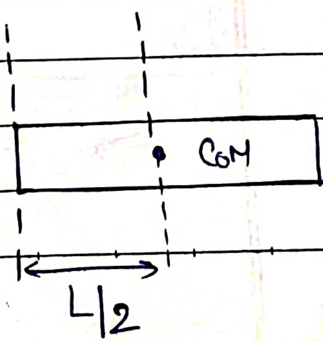
1) Thin Rod :



Abt axis thru CoM $\perp$ to Rod,

$$I = \int l^2 \, dm = \int_{-L/2}^{L/2} l^2 \cdot \frac{m}{L} \, dl \Rightarrow \boxed{I = \left(\frac{mL^2}{12}\right)}$$

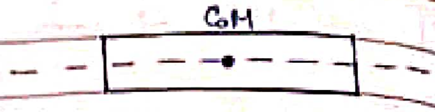
Abt axis thru edge $\perp$ to Rod,



$$I = \left(\frac{mL^2}{12}\right) + m\left(\frac{L}{2}\right)^2 \Rightarrow \boxed{I = \left(\frac{mL^2}{3}\right)}$$

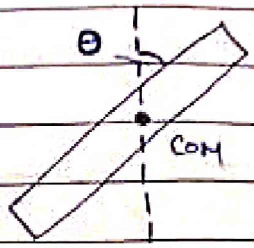
Date: ______ Page ______

Abt axis thru Rod,



Rod thin $\Rightarrow$ $\boxed{I = 0}$

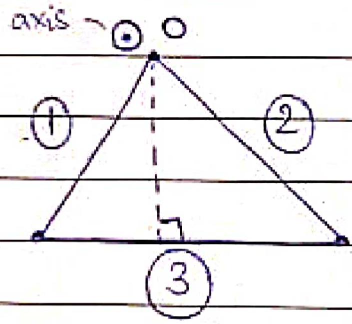
Abt axis at $\angle = \theta$,



$$\boxed{I = \frac{mL^2}{12} \sin^2(\theta)}$$

27/9/22

Q)



All rods identical.
Mass = M, Length = L.

Find MoI wrt. axis thru O normal to plane.

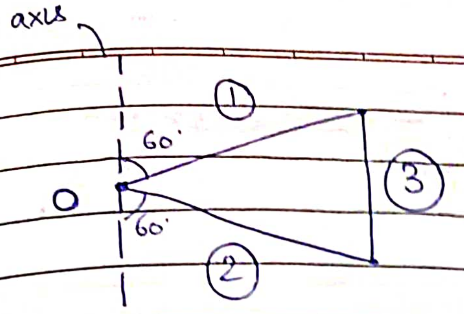
A) ① : $I_1 = \frac{ML^2}{3}$ ② : $I_2 = \frac{ML^2}{3}$

③ : By || axis theorem, $I_3 = \frac{ML^2}{12} + M\left(\frac{\sqrt{3}L}{2}\right)^2$

$$\Rightarrow I_3 = \frac{5ML^2}{6}$$

$$\Rightarrow I_{net} = \frac{ML^2}{3} + \frac{ML^2}{3} + \frac{5ML^2}{6} \Rightarrow \boxed{I_{net} = \left(\frac{3ML^2}{2}\right)}$$

Q)



All rods identical
Mass $= M$, Length $= L$.

Find MoI. wrt. axis thru O. || to opp side.

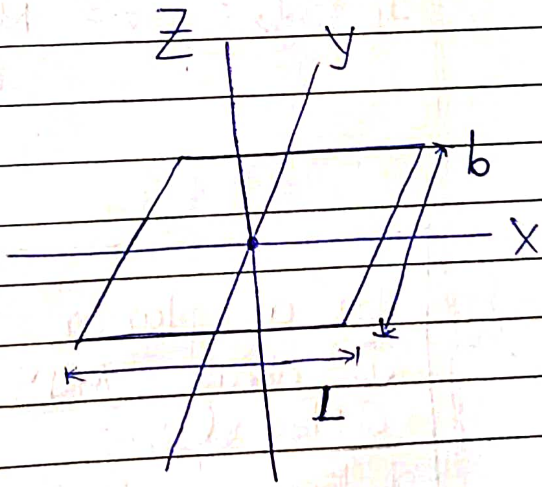
By || axis theorem,

A) 3 : $I_3 = 0 + M\left(\frac{\sqrt{3}L}{2}\right)^2 \Rightarrow I_3 = \frac{3ML^2}{4}$

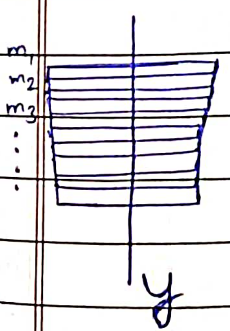
1 & 2 : $I_1 = I_2 = \frac{ML^2}{3}\sin^2 60° \Rightarrow I_1 = I_2 = \frac{ML^2}{4}$

$\Rightarrow I_{net} = \frac{3ML^2}{4} + \frac{ML^2}{4} + \frac{ML^2}{4} \Rightarrow \boxed{I_{net} = \frac{5ML^2}{4}}$

2) Rectangular Lamina :



Abt y axis,



★ Adding bunch of rods.

$I_y = \frac{(m_1 + m_2 + \ldots)L^2}{12}$

$\Rightarrow \boxed{I_y = \left(\frac{ML^2}{12}\right)}$

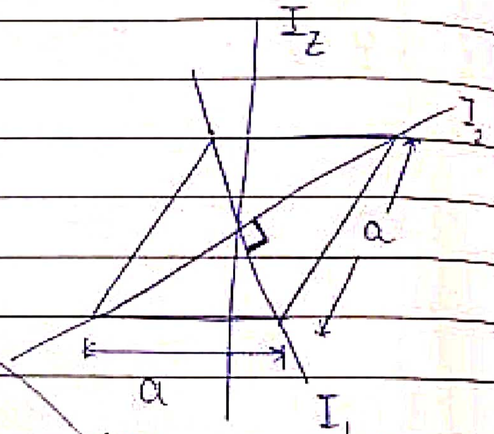
Similarly,

$\Rightarrow \boxed{I_x = \left(\frac{Mb^2}{12}\right)}$

By $\perp$ axis theorem, $\boxed{I_z = \left(\frac{M(L^2+b^2)}{12}\right)}$

☆ If mass element integrated **ALONG** axis, or several masses added along axis then expression for I does **NOT** change.

Abt. axis thru diag. (in a square)

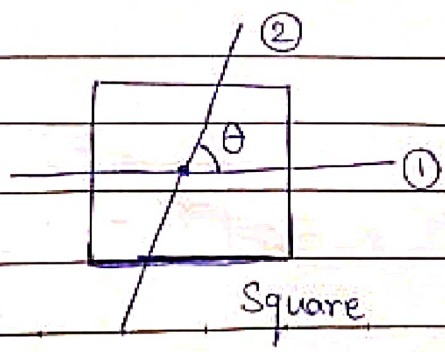


Since $I_1 \perp I_2$, by $\perp$ axis theorem.

$$I_1 + I_2 = I_z = \frac{Ma^2}{6} \Rightarrow$$

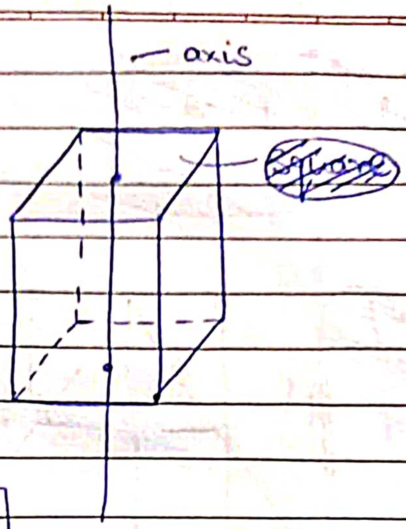
☆ $\boxed{I_1 = I_2 = I_x = I_y = \left(\frac{ma^2}{12}\right)}$

☆ In a planar symmetric body, I about **ANY** axis thru **CoM** lying **IN the PLANE** of body is **SAME**.



$I_1 = I_2$

3) Cuboid : ~~if face sq.~~ Length - L Breadth - b

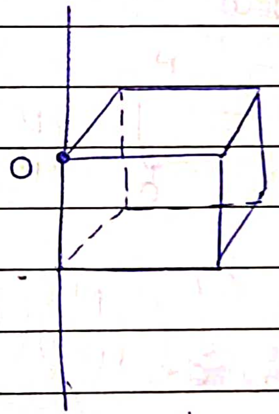


Adding bunch of lamina.

$$I = \frac{(m_1+m_2+\dots)}{12}(\cancel{a}b^2+L^2)$$

$$\Rightarrow \boxed{I = \left(\frac{M}{12}\right)(L^2+b^2)}$$ axis || to height

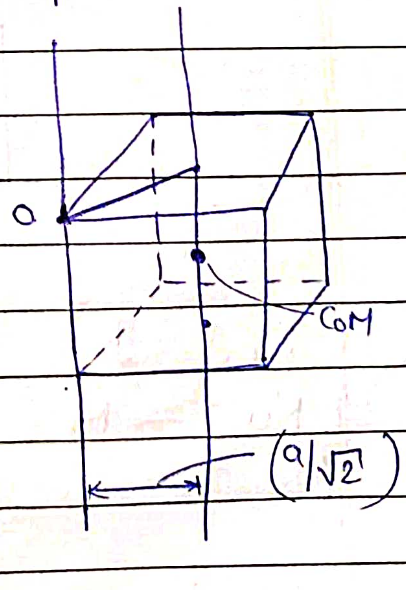
Q) Cube with side 'a'. find I about O.



A) By || axis theorem,

$$I = \left(\frac{M}{12}\right)(a^2+a^2) + M\left(\frac{a}{\sqrt{2}}\right)^2$$

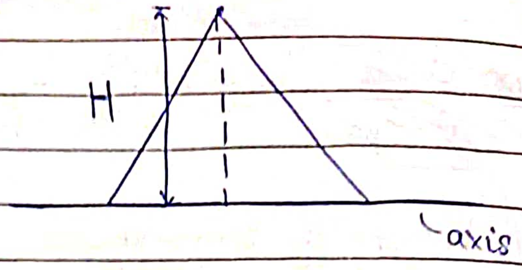
$$\Rightarrow \boxed{I = \left(\frac{2Ma^2}{3}\right)}$$



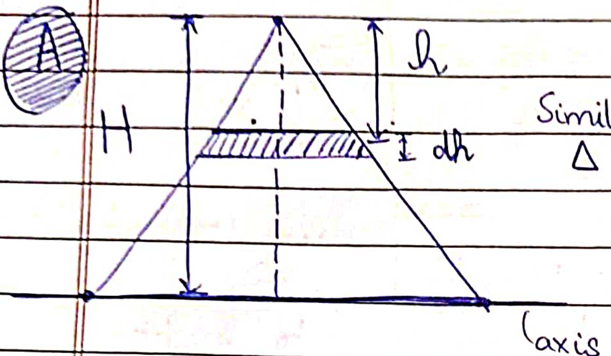
4) Triangle :

Date: ________ Page:

Q) Find I.
Mass = M



A)



Similar Δ: $\dfrac{m}{h^2} = \dfrac{m+dm}{(h+dh)^2} = \dfrac{M}{H^2}$

$\Rightarrow \dfrac{dm}{2h\,dh} = \dfrac{M}{H^2}$

$\Rightarrow dm = \left(\dfrac{2M}{H^2}\right) h\,dh$

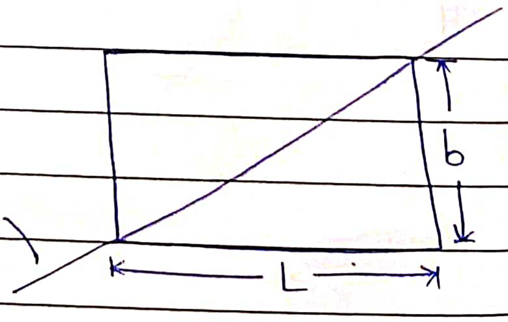
Now,

$$I = \int_0^H \left(\frac{2M}{H^2}\right) h (H-h)^2\,dh$$

$$= \int_0^H \left(\frac{2M}{H^2}\right)(Hh^2 - h^3)\,dh = \left(\frac{2M}{H^2}\right)\left[\frac{Hh^3}{3} - \frac{h^4}{4}\right]_0^H$$

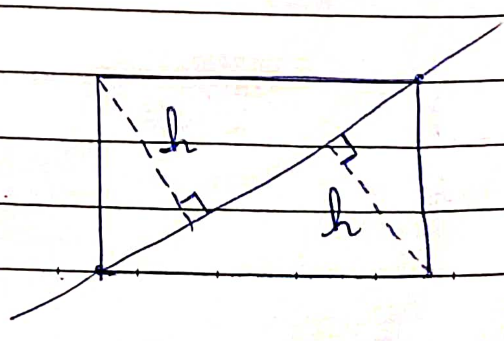
$$\Rightarrow \boxed{I = \left(\frac{MH^2}{6}\right)}$$

Q) Find I.
Mass = M.



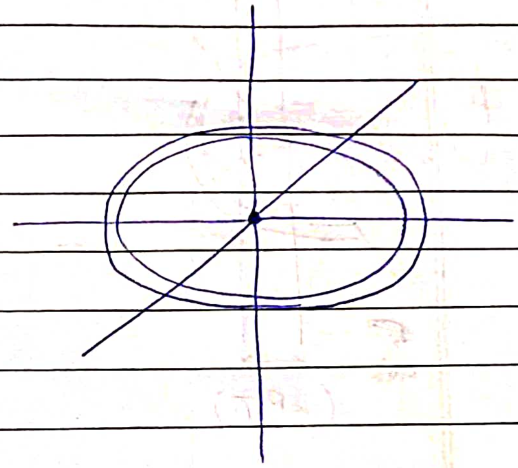
A) Consider as 2Δs.

$\sqrt{L^2+b^2} \cdot h = bL$



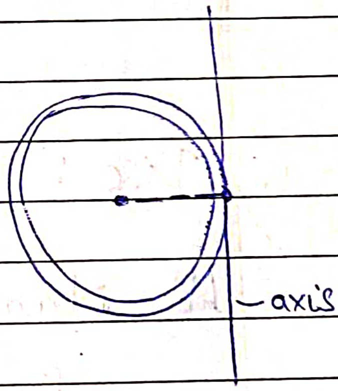
$$I_{net} = (2)\left(\frac{M}{2}\right)\left(\frac{h^2}{6}\right) \Rightarrow \boxed{I = \left(\frac{ML^2b^2}{6(L^2+b^2)}\right)}$$

5) Ring :



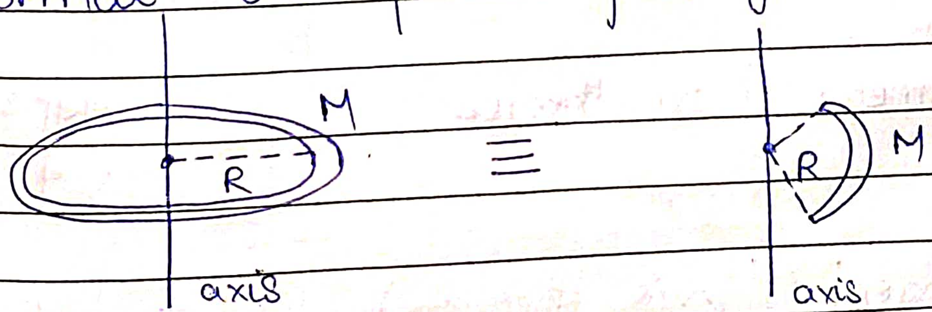
Abt Normal, $\boxed{I = MR^2}$

Abt Diameter, ($\perp$ axis theorem) $\boxed{I = \left(\frac{MR^2}{2}\right)}$

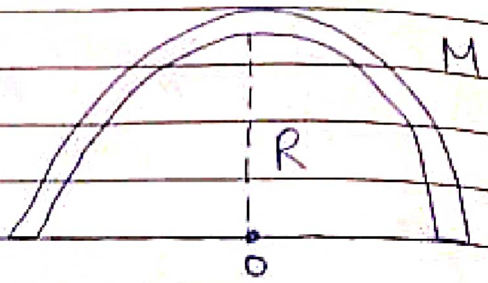


Abt Tangent, (|| axis theorem) $\boxed{I = \left(\frac{3MR^2}{2}\right)}$

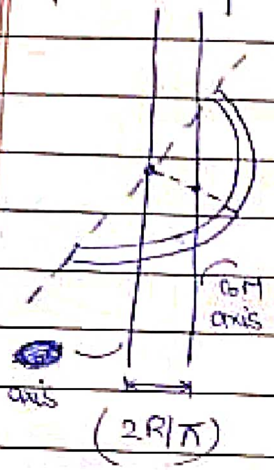
☆ MoI of Ring/Disc or part of ring/disc of Same mass & radius is Same about axis thru centre normal to plane of obj.



Q) find MoI about axis thru CoM and normal to plane of ring



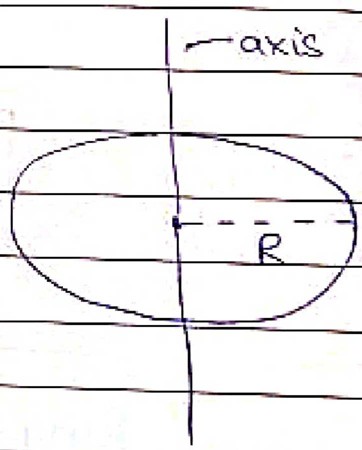
A)



$$I_{CoM} + M\left(\frac{2R}{\pi}\right)^2 = MR^2$$

$$\boxed{I_{CoM} = (MR^2)\left(1 - \frac{4}{\pi^2}\right)}$$

6) Disc :



Abt Normal,

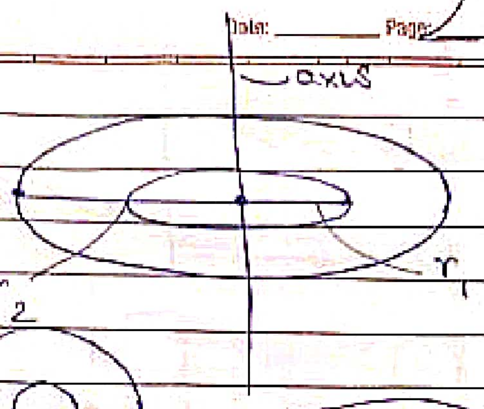
$$I = \int_0^R \left(\frac{M}{\pi R^2}\right)(2\pi r \cdot r^2)\, dr$$

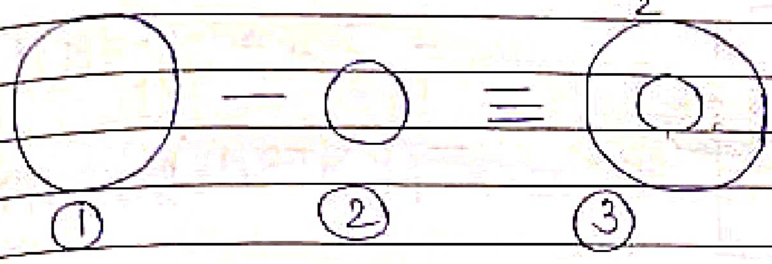
$$\Rightarrow \quad \boxed{I = \left(\frac{MR^2}{2}\right)}$$

Abt Diameter ($\perp$ axis theorem), $\boxed{I = \left(\frac{MR^2}{4}\right)}$

Abt Tangent ($\parallel$ axis theorem), $\boxed{I = \left(\frac{5MR^2}{4}\right)}$

Q) Axis normal to plane. MoI = ?



A) 

$$\rho = \frac{M}{\pi(r_2^2 - r_1^2)}$$

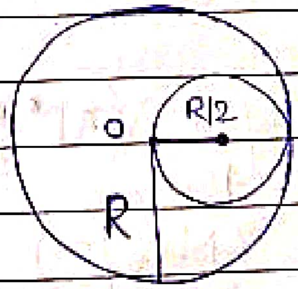
① : $I_1 = \frac{M_1 r_2^2}{2} = \frac{\rho(\pi r_2^2) r_2^2}{2} = \left(\frac{\rho\pi}{2}\right)(r_2^4)$

② : $I_2 = \left(\frac{\rho\pi}{2}\right)(r_1^4)$

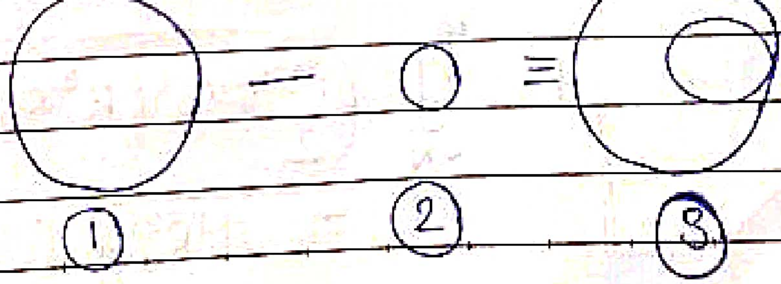
③ : $I = I_1 - I_2 = \left(\frac{\rho\pi}{2}\right)(r_2^4 - r_1^4)$

$$\Rightarrow \boxed{I = \left(\frac{M}{2}\right)(r_1^2 + r_2^2)}$$

Q) Mass of complete disc was M. Find MoI, if axis normal to plane



A) $\rho = \left(\frac{M}{R^2}\right)$



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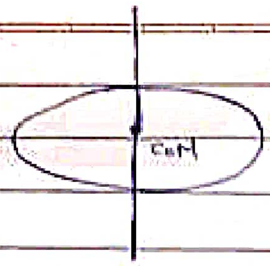
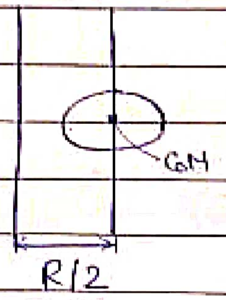
(1) :



$$I_1 = \left(\frac{\rho}{2}\right)(\pi R^2)(R^2)$$

(2) :



$$I_2 = I_{CoM} + \rho(\pi R^2/4)(R^2/4)$$

$$= \left(\frac{\rho}{2}\right)(\pi R^2/4)\left(\frac{R^2}{4}\right) + \rho\left(\frac{\pi R^2}{4}\right)\left(\frac{R^2}{4}\right)$$

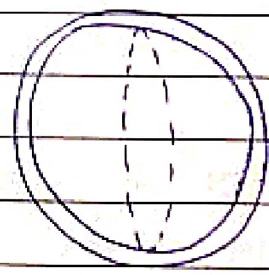
$$\Rightarrow I_2 = \left(\frac{3\rho}{32}\right)(\pi R^2)(R^2)$$

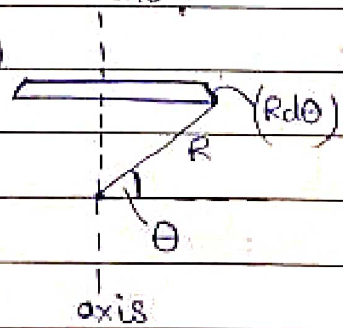
(3) : $I = I_1 - I_2 = \frac{MR^2}{2} - \frac{3MR^2}{32}$

$$\Rightarrow \boxed{I = \left(\frac{13MR^2}{32}\right)}$$

7) Sphere :

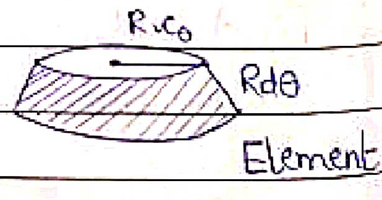
Consider ring as element.





$$dm = \left(\frac{M}{4\pi R^2}\right)(2\pi R c_\theta)(R\,d\theta)$$

$$= \left(M c_\theta/2\right)$$



$$I = \int_{-\pi/2}^{\pi/2} \left(\frac{M}{2}\right)(c_\theta)(R^2 c_\theta^2)\,d\theta = \left(\frac{MR^2}{8}\right)\int_{-\pi/2}^{\pi/2}\left(c_{3\theta} + 3c_\theta\right)d\theta$$

$$= \left(\frac{MR^2}{8}\right)\left[\frac{s_{3\theta}}{3} + 3s_\theta\right]_{-\pi/2}^{\pi/2}$$

$$\Rightarrow \boxed{I = \left(\frac{2MR^2}{3}\right)}$$ abt any axis thru CoM.

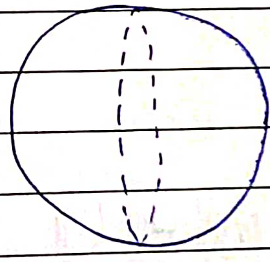
Abt tangent, (|| axis theorem) $\boxed{I = \left(\frac{5MR^2}{3}\right)}$

---

8) Solid Sphere :

Consider hollow sphere as element.

$$dm = \left(\frac{3M}{4\pi R^3}\right)(4\pi r^2)\,dr$$

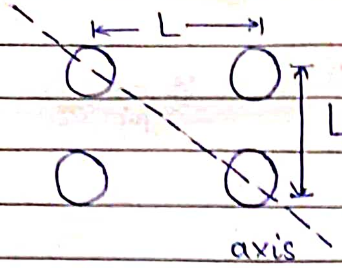


$$I = \int_0^R \left(\frac{2}{3}r^2\right)\left(\frac{3M}{4\pi R^3}\right)(4\pi r^2)\,dr$$

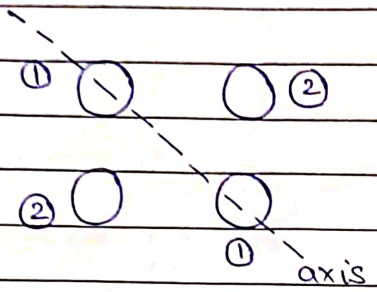
$$= \left(\frac{2M}{R^3}\right)\int_0^R r^4\,dr \Rightarrow \boxed{I = \left(\frac{2MR^2}{5}\right)}$$

abt any axis thru CoM.

Abt tangent, (|| axis theorem) $\boxed{I = \left(\frac{7MR^2}{5}\right)}$

Q) 

4 solid spheres each of mass M & radius R. find MoI.

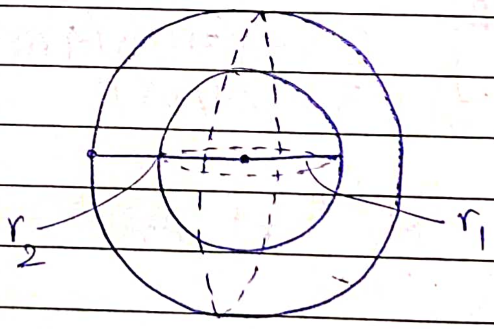
A) 

① : $I_1 = \left(\frac{2MR^2}{5}\right)$

② : $I_2 = \left(\frac{2MR^2}{5}\right) + M\left(\frac{L}{\sqrt{2}}\right)^2$

$I = 2I_1 + 2I_2 \Rightarrow \boxed{I = \left(\frac{8MR^2}{5}\right) + ML^2}$

Q) find MoI.



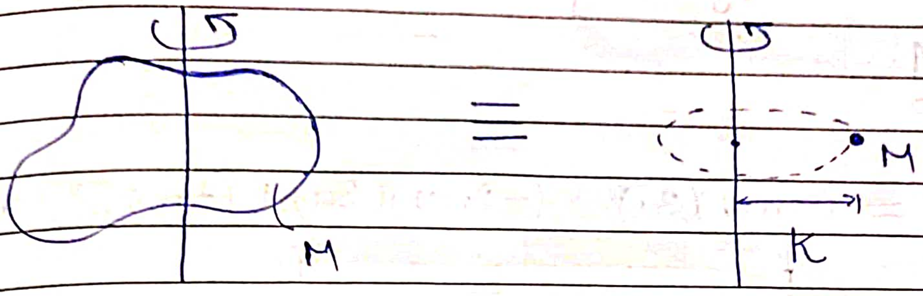
A) $\rho = \frac{3M}{4(r_2^3 - r_1^3)\pi}$

$$I = \left(\frac{2}{5}\right)\left(\frac{4\pi r_2^3}{3}\rho\right)\left(r_2^2\right) - \left(\frac{2}{5}\right)\left(\frac{4\pi r_1^3}{3}\rho\right)\left(r_1^2\right)$$

$$= \left(\frac{2}{5}\right)\left(\frac{M r_2^5}{(r_2^3 - r_1^3)}\right) - \left(\frac{2}{5}\right)\left(\frac{M r_1^5}{(r_2^3 - r_1^3)}\right)$$

$$\Rightarrow \boxed{I = \left(\frac{2M}{5}\right)\left(\frac{r_2^5 - r_1^5}{r_2^3 - r_1^3}\right)}$$

Radius of Gyration



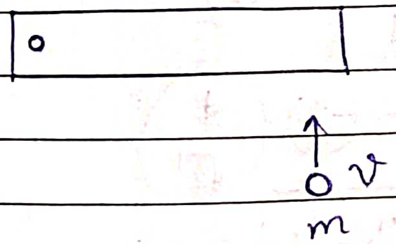
$$\boxed{I = Mk^2}$$

Radius of gyration

Angular Momentum

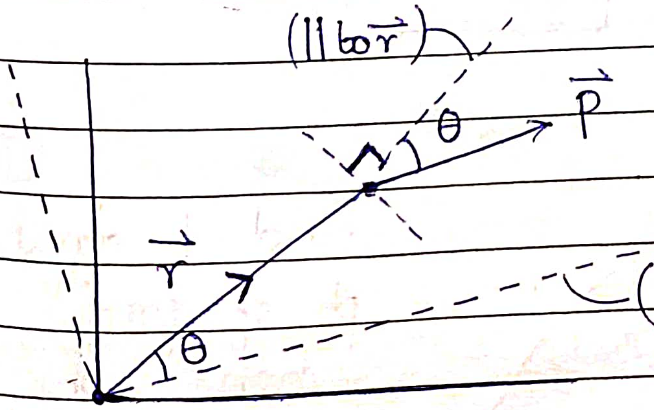
Turning effect of momentum is known as angular momentum

~~due to translation~~

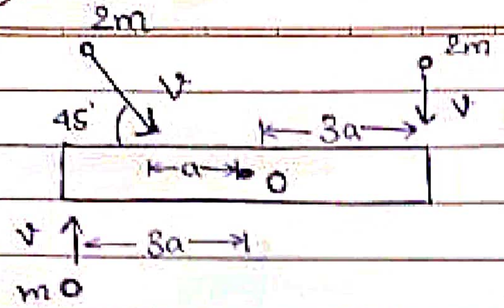


$$\boxed{\vec{L} = \vec{r} \times \vec{p}}$$

$\Rightarrow$ $\boxed{L = rp\,s_\theta}$



We can either take component of $\vec{p}$, or of $\vec{r}$.

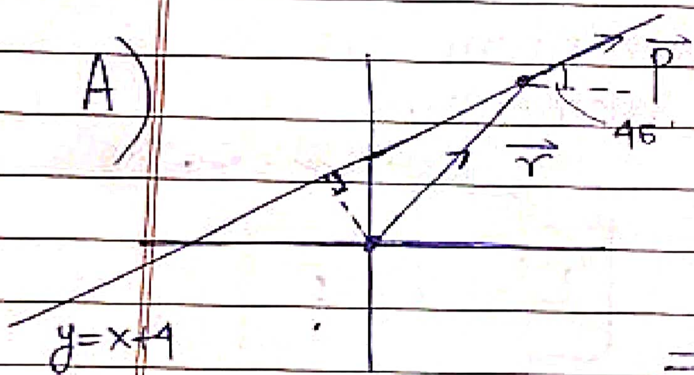
Q) 

Find L about O.

A) $L = (-mv)(3a) + (-2mv)(3a) + (+mv\sqrt{2})(a)$

$\Rightarrow \boxed{L = -(9-\sqrt{2})mva}$

Q) A body is moving in a st. line $y=(x+4)$ with momentum p. Find L about origin

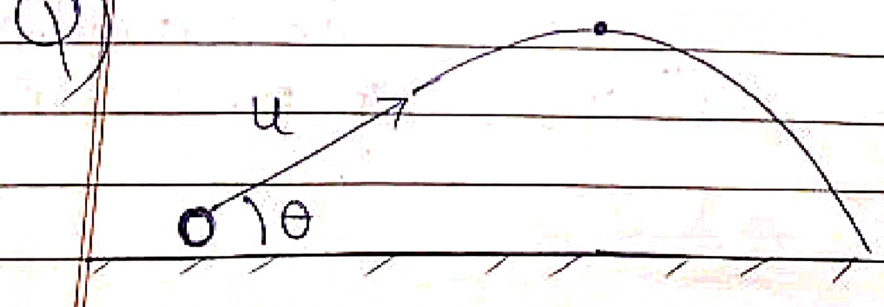
A) 

Divide $\vec{r}$.

$L = p(r \sin\theta)$

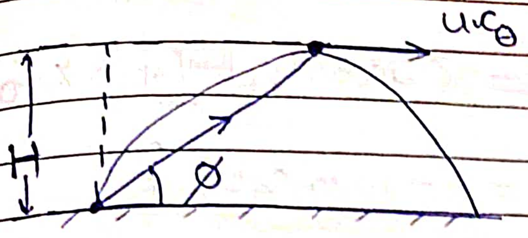
$\Rightarrow L = (-p)\left(\frac{4}{\sqrt{2}}\right)$

$\Rightarrow \boxed{L = (-2\sqrt{2})p}$

Q) 

Find L about pt. of proj. when obj at highest pt.

A)

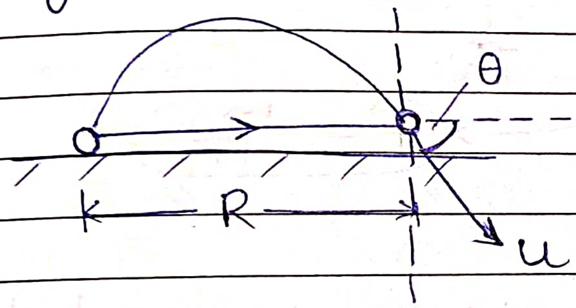


$$L = (-mu c_\theta)\left(\frac{u^2 s_\theta^2}{2g}\right)$$

$$\Rightarrow \boxed{L = \left(-\frac{mu^3}{2g}\right)\left(s_\theta^2 c_\theta\right)}$$

Q) In above Q, find L about pt. of proj. when obj. about to hit the ground.

A)



$$L = (-mu s_\theta)\left(\frac{2u^2 s_\theta c_\theta}{g}\right)$$

$$\Rightarrow \boxed{L = \left(-\frac{2mu^3}{g}\right)\left(s_\theta^2 c_\theta\right)}$$

Q) In above Q, find L about pt. of proj. at time 't'.

A) $\vec{v} = \langle uc_\theta,\ us_\theta - gt \rangle$; $\vec{r} = \langle uc_\theta t,\ us_\theta t - \frac{1}{2} g t^2 \rangle$

$$\vec{L} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v}) = m \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ uc_\theta t & uc_\theta & 0 \\ \left(us_\theta t - \frac{1}{2}gt^2\right) & (us_\theta - gt) & 0 \end{vmatrix}$$

$$\Rightarrow \boxed{\vec{L} = \left(-\frac{1}{2}\right)\left(mugc_\theta t^2\right)\hat{k}}$$

Date: Page:

Alternate Sol^n: $\vec{\tau} = \langle u c_\theta t,\ u s_\theta t - \frac{1}{2} g t^2 \rangle \times \langle 0, -mg \rangle$

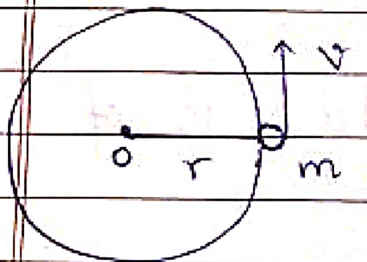
$\Rightarrow \quad \vec{\tau} = (-mug\, c_\theta t)\,\hat{k}$

$\Rightarrow \quad \vec{L} = \left(\int (-mug\, c_\theta)\, t\, dt \right) \hat{k}$

$\Rightarrow \quad \boxed{\vec{L} = \left(\frac{-1}{2} \right) (mug\, c_\theta)\, t^2 \hat{k}}$

- $\boxed{\vec{\tau} = \left(\frac{d\vec{L}}{dt} \right)}$ This is similar to $\vec{F} = d\vec{p}/dt$.

- Assume an obj. in circular motion about fix. pt.



About O,

$L = mvr = m\omega r^2$
$= (mr^2)\,\omega$

(Pure Rotation) $\Rightarrow \boxed{\vec{L} = I\vec{\omega}}$ about pt. of Rotation

due to rotation.

- If obj. both translating & ~~revolving~~ rotating,

$\boxed{\vec{L} = (\vec{r} \times \vec{p}) + I\vec{\omega}}$

MoI of obj. about CoM, axis $\parallel$ to AoR

[L of CoM about pt.] [L about CoM]

Q) 

Find L about O

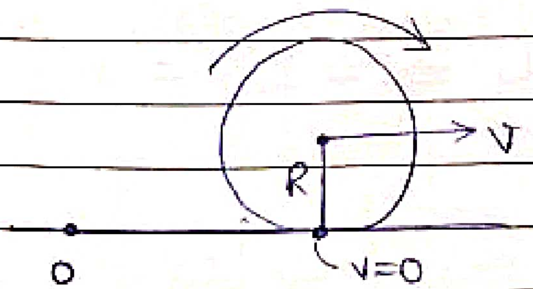
A) $$\vec{L} = (\vec{r} \times \vec{p}) + I\vec{\omega}$$

$$= \left(\frac{L}{2}\right)\left(M \cdot \frac{\omega L}{2}\right) + \left(\frac{ML^2}{12}\right)(\omega)$$

$$\Rightarrow \boxed{L = \left(\frac{M\omega L^2}{3}\right)}$$

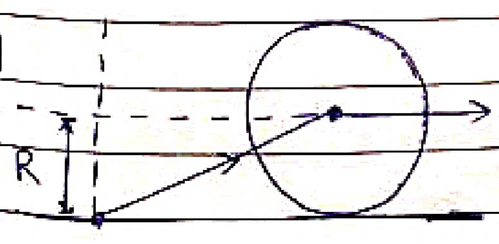
<u>Alternate Solⁿ</u>: Since pure rotation about O we can directly use $L = I\omega$

$$\Rightarrow L = \left(\frac{ML^2}{3}\right)(\omega) \Rightarrow \boxed{L = \left(\frac{M\omega L^2}{3}\right)}$$

Q) 

Mass = M.

Find L about O.

A) 

$$L = (\vec{r} \times \vec{p}) + I\vec{\omega}$$

$$= MvR + (MR^2/2)(v/R)$$

$$= \frac{3MvR}{2}$$

Date: Page:

Consrv. of Angular Momentum

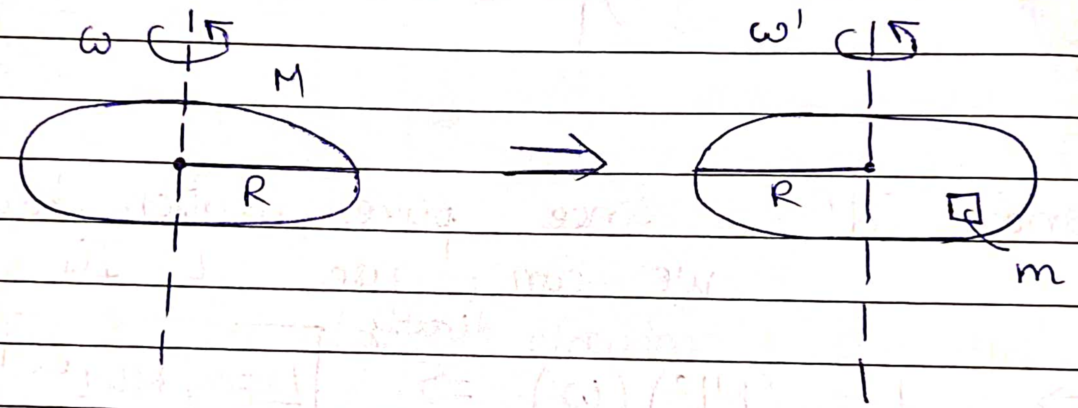
$$\tau_{ext} = 0 \Rightarrow \boxed{\vec{L} = \text{Const.}}$$

abt pl. where $\vec{\tau} = 0$.

For obj. in pure rotation, about CoM

$$\boxed{KE_{rot} = \frac{1}{2} I \omega^2 = \frac{L^2}{2I}}$$ abt CoM.

Q)

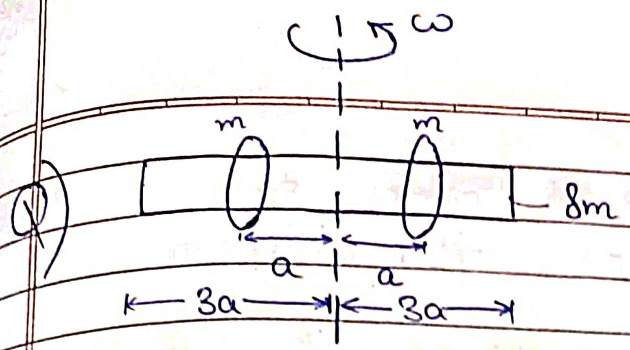


Small mass 'm' put on disc w/o disturbing it. Find ω'.

A) $\tau_{ext} = 0$ (abt CoM) $\Rightarrow L_0 = L_1$ (abt CoM) $\Rightarrow I_0 \omega = I_1 \omega'$

$$\Rightarrow \left(\frac{MR^2}{2}\right)\omega = \left(\frac{MR^2}{2} + mR^2\right)\omega' \Rightarrow \boxed{\omega' = \left(\frac{M}{M+2m}\right)\omega}$$

Q)



Find ω of rod when rings just leave the rod.

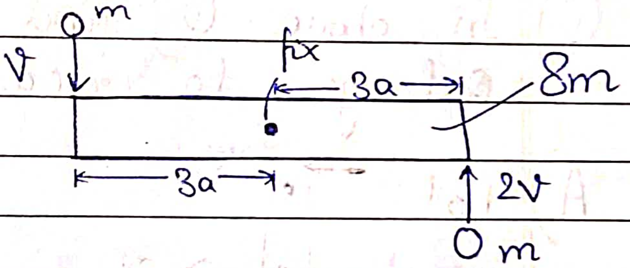
A) Consider radius of rings is negligible.

$\tau_{ext} = 0$ (abt. CoM) $\Rightarrow$ $L_0 = L_1$ (abt. CoM) $\Rightarrow$ $\omega\left(\frac{(8m)(6a)^2}{12} + 2ma^2\right)$

$= \omega'\left(\frac{(8m)(6a)^2}{12} + 2m(3a)^2\right)$

$$\Rightarrow \boxed{\omega' = \left(\frac{13\omega}{21}\right)}$$

Q) Balls strikes the rod & it gets embedded



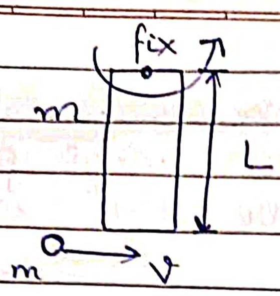
Find ω of rod just after collision.

A) Just after collision, L consrv. of system

$$(mv)(3a) + (2mv)(3a) = \left[\frac{(8m)(6a)^2}{12} + m(3a)^2 + m(3a)^2\right]\omega$$

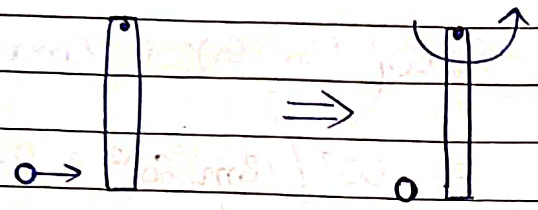
$$\Rightarrow \boxed{\omega = \left(\frac{3}{14}\frac{v}{a}\right)}$$

☆ Since in this case CoM remains same, we don't write $(\vec{r} \times \vec{p})$ term.
But if CoM changes, we have to include it.

Q) 

Ball comes to rest after collision.

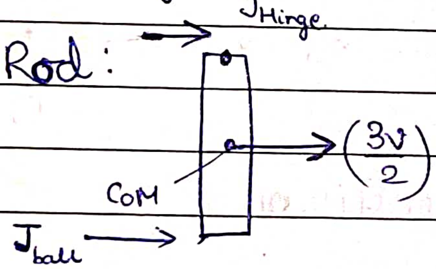
Find ω of rod just after collision.

A) 

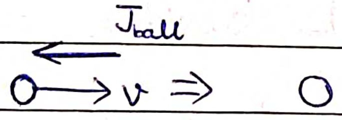
$$mvL = \left(\frac{mL^2}{3}\right)\omega$$

$$\Rightarrow \boxed{\omega = \left(\frac{3v}{L}\right)}$$

Q) In above Q, find impulse given by hinge to rod.

A) Rod: 

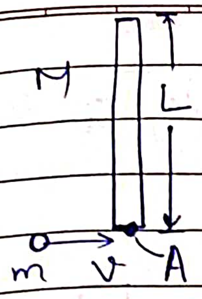
$$\left(\frac{3mv}{2}\right) = J_{Hinge} + J_{ball}$$

Ball: 

$$-J_{ball} = -mv$$

$$\Rightarrow \boxed{J_{Hinge} = \left(\frac{mv}{2}\right)}$$

Q)



Rod kept on horizontal frictionless surface.
Mass m stops after collision.

Find ω, just after collision.

A) Since no $\tau_{ext.}$, L consrv. abt. every pt.

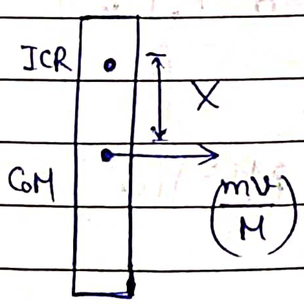
About CoM, $L_0 = L_1$

$$\Rightarrow mv\left(\frac{L}{2}\right) = \left(\frac{mv}{M}\right)(M)(0) + \left(\frac{ML^2}{12}\right)\omega$$

$$\Rightarrow \boxed{\omega = \left(\frac{6mv}{ML}\right)}$$

Q) In above Q, find inst. centre of rotation.

A)



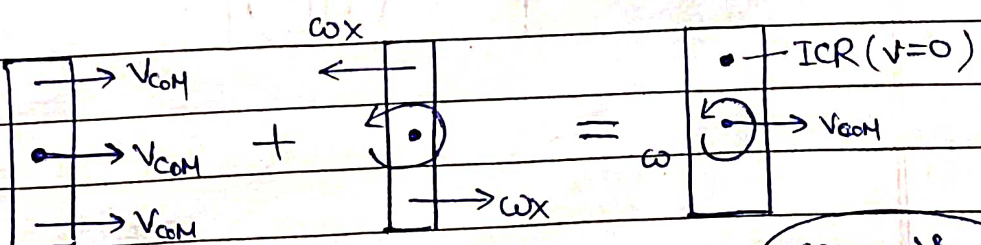
CoM moves with ω wrt. ICR.

$$\Rightarrow \omega x = \left(\frac{mv}{M}\right) \Rightarrow \boxed{x = \left(\frac{mv}{M\omega}\right)}$$

$$\Rightarrow \boxed{x = L/6}$$

Alternate:

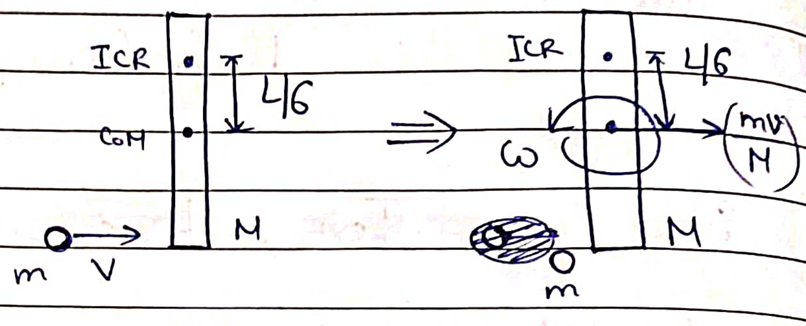
Motion =



$\omega x = v_{CoM}$

Q) In above Q, find ω by applying consrv of L at A & ICR.

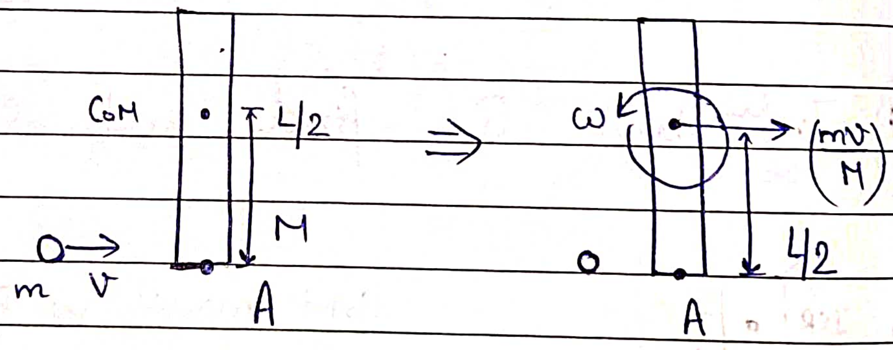
A) About ICR,



$$L_0 = L_1 \Rightarrow mv\left(L/6 + L/2\right) = \left(\frac{mv}{M}\right)(M)\left(\frac{L}{6}\right) + \left(\frac{ML^2}{12}\right)\omega$$

$$\Rightarrow \boxed{\omega = \left(\frac{6mv}{ML}\right)}$$

About A,



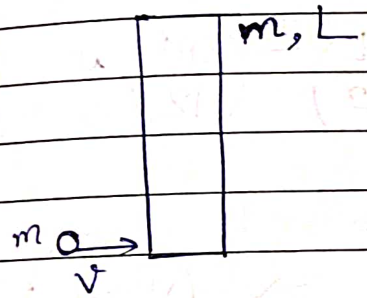
$$L_0 = L_1 \Rightarrow 0 = M\left(\frac{mv}{M}\right)\left(\frac{L}{2}\right) - \left(\frac{ML^2}{12}\right)(\omega)$$

$$\Rightarrow \boxed{\omega = \left(\frac{6mv}{ML}\right)}$$

Q) In above Q, find dist. moved by rod when it has turned 90°.

A) $t = \frac{\theta}{\omega} = \frac{\pi/2}{(6mv/ML)} \Rightarrow t = \left(\frac{M}{mv}\right)\left(\frac{\pi L}{12}\right)$

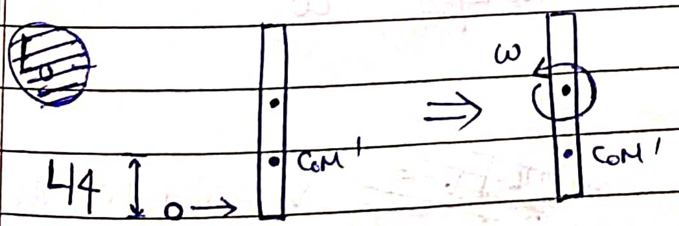
$d = vt = \left(\frac{mv}{M}\right)\left(\frac{M}{mv}\right)\left(\frac{\pi L}{12}\right) \Rightarrow \boxed{d = \left(\frac{\pi L}{12}\right)}$

Q) 

Mass gets embedded.

Find ω of system after collision.

A) ☆ We consrv. L about new CoM to eliminate $(\vec{r} \times \vec{p})$ term from $L = (\vec{r} \times \vec{p}) + I\omega$



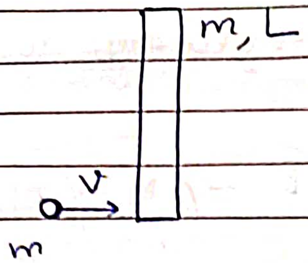
$\cancel{L_0 = L_1}$

$\cancel{\Rightarrow mv\left(\frac{L}{4}\right) = \omega}$

$L_0 = L_1 \Rightarrow mv\left(\frac{L}{4}\right) = \omega I = \omega\left(\frac{mL^2}{12} + \frac{mL^2}{16} + \frac{mL^2}{16}\right)$

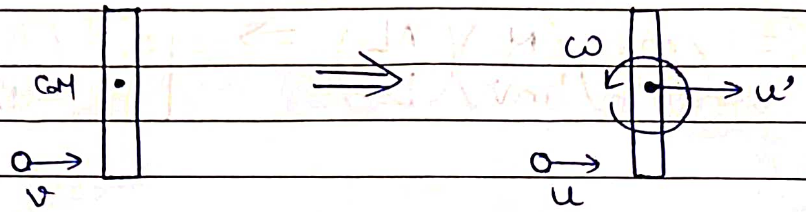
(rod abt its CoM) (due to ∥ axis theorem) (ball abt. CoM')

$\Rightarrow \boxed{\omega = \left(\frac{6v}{5L}\right)}$

Q) 

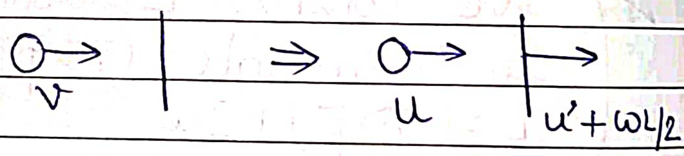
Collision is elastic

Find ω.

A) 

① $L_0 = L_1$ (about CoM) $\Rightarrow (mv)\left(\frac{L}{2}\right) = (mu)\left(\frac{L}{2}\right) + \left(\frac{mL^2}{12}\right)\omega$

② By Momentum Consrv., $mv = mu' + mu \Rightarrow v = u' + u$

③ ~~Bo~~ By Elastic Collision, 

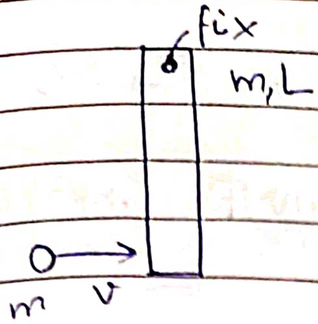
$e = 1 \Rightarrow v = u' + \omega L/2 - u.$

Using ② in ~~①~~ ③, $\cancel{\left(\frac{mL}{2}\right)v =}\ v = v + \frac{\omega L - 2u}{2}$

$\Rightarrow u = \omega L/4$

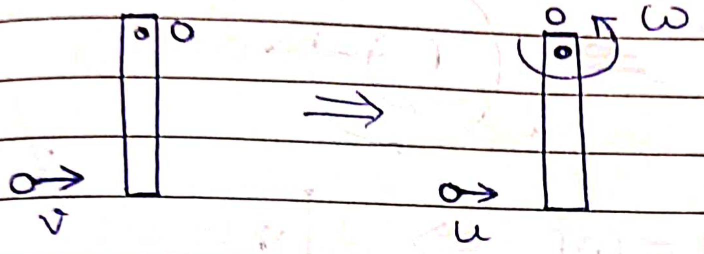
Into ①, $\left(\frac{mL}{2}\right)v = \left(\frac{mL}{2}\right)\left(\frac{\omega L}{4}\right) + \left(\frac{mL}{2}\right)\left(\frac{\omega L}{6}\right)$

$\Rightarrow \boxed{\omega = \cancel{\frac{3v}{2L}}}\ \left(\frac{12v}{5L}\right)$

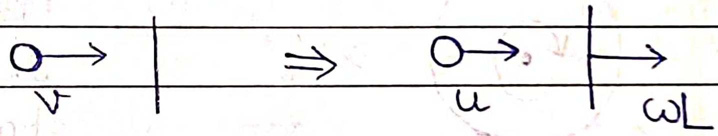
Q) 

Collision is elastic.

Find ω.

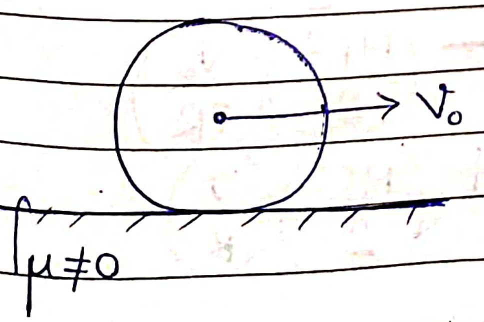
A) 

(1) $L_0 = L_1$ (about O) $\Rightarrow$ $mvL = muL + \left(\frac{mL^2}{3}\right)\omega$

(2) By Elastic Collision, 

$e = 1 \Rightarrow v = \omega L - u \Rightarrow u = (\omega L - v)$

Into (1), $v = (\omega L - v) + \left(\frac{\omega L}{3}\right) \Rightarrow \boxed{\omega = \left(\frac{3v}{2L}\right)}$

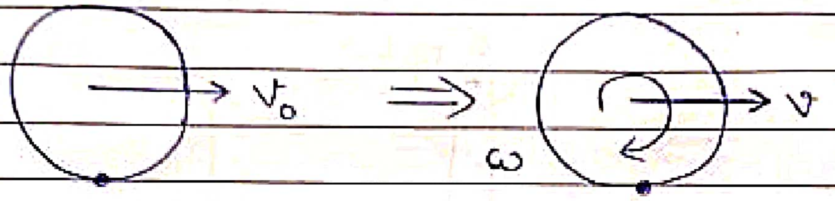
Q) 

Obj. initially sliding.

Find vel. of disc. when it start pure rolling. ($v = \omega R$)

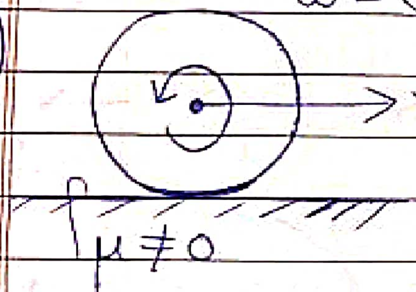
A) $\tau_{ext} = 0$ at lowest pt. as $\tau_f = 0$

$L_0 = L_1 \Rightarrow (mv_0)(R) = (mv)R + \left(\frac{mR^2}{2}\right)\omega$
(about lowest pt.)



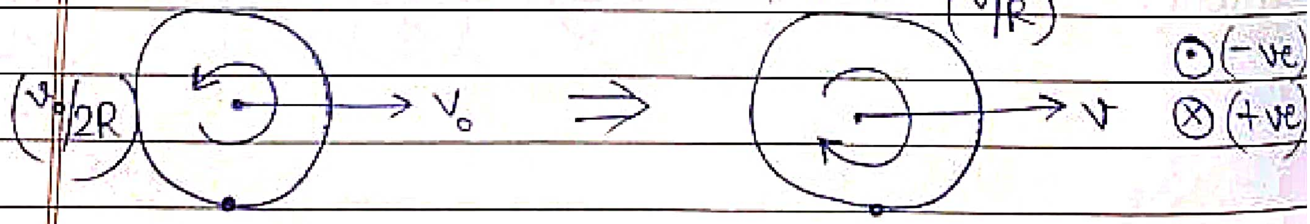
We have $v = \omega R \Rightarrow \boxed{v = \left(\frac{2v_0}{3}\right)}$

Q) $\omega = (v_0/2R)$



Find vel. of disc. when it starts pure rolling

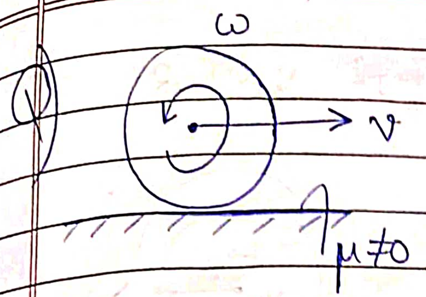
A) $\tau_{ext} = 0$ at lowest pt.



$L_0 = L_1 \Rightarrow +(Mv_0)(R) - \left(\frac{MR^2}{2}\right)\left(\frac{v_0}{2R}\right)$
(at lowest pt.)

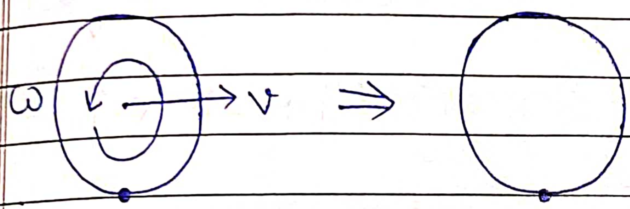
$= +(Mv)(R) + \left(\frac{MR^2}{2}\right)\left(\frac{v}{R}\right)$

$\Rightarrow \boxed{v = v_0/2}$



Find ω_{min} so that dirxⁿ of motion of disc reverses.

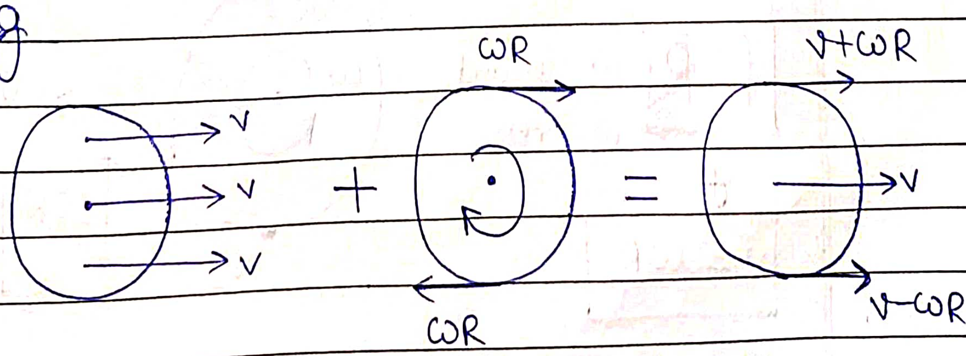
A) In limiting case $\omega = 0$ when $v = 0$.
for return back ω in ↻ when $v = 0$.



$L_0 = L_1 \Rightarrow -(Mv)(R) + \left(\frac{MR^2}{2}\right)(\omega) = 0$
(abt. lowest pt.)

$$\Rightarrow \boxed{\omega = 2v/R}$$

Rolling

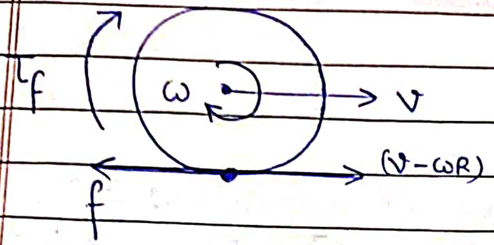


Pure Translation

Pure Rolling

Rolling

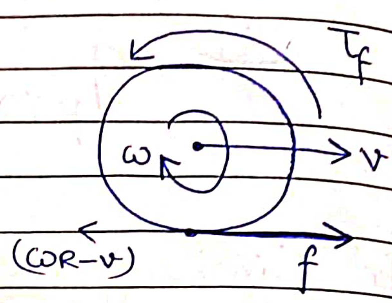
If $v > \omega R$



$\tau_f \parallel \omega \Rightarrow \omega \uparrow$

f anti$\parallel$ $v \Rightarrow v \downarrow$

If $v < \omega R$



τ_f anti$\parallel$ $\omega \Rightarrow \omega \downarrow$

$f \parallel v \Rightarrow v \uparrow$

So ultimately, if obj. roll on rough surface, we will get $\boxed{v = \omega R}$

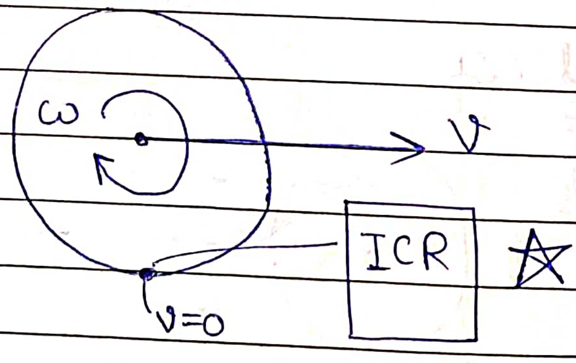
<u>Pure Rolling</u> : There is no rel. motion at pt. of contact.

<u>In this case</u>,

$\boxed{f = 0}$ as ground at rest

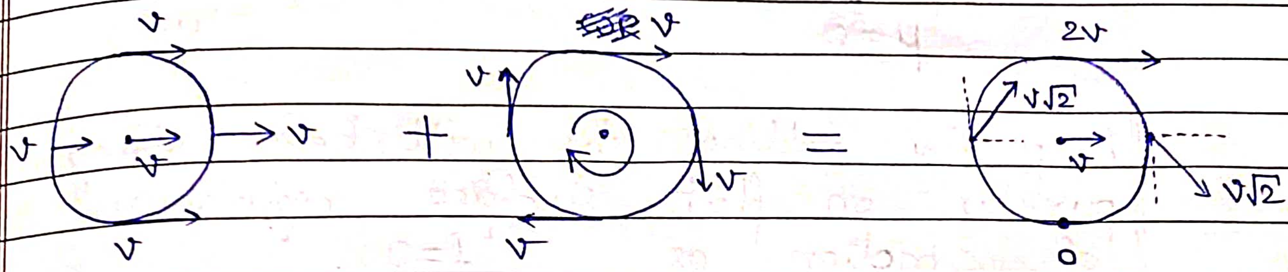
&

$\boxed{v = \omega R}$

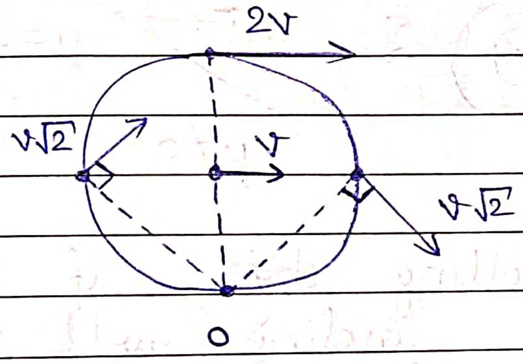


Now, we can find vel. of any pt. on obj.

By Translation + Rolling,



Also since lowest pt. ICR, we can use $v = \omega r$.



for any body which ~~is~~ both rotating & translating

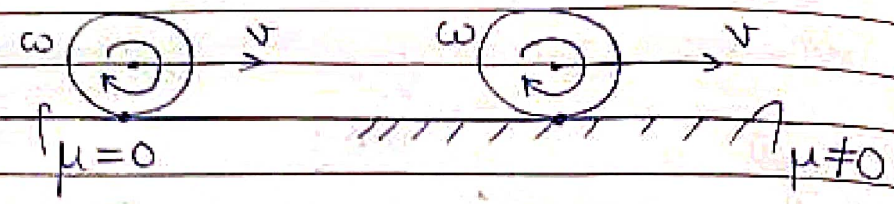
$$\boxed{KE_{body} = \frac{1}{2} M v_{CoM}^2 + \frac{1}{2} I \omega^2}$$

of body abt axis || IAR thru CoM

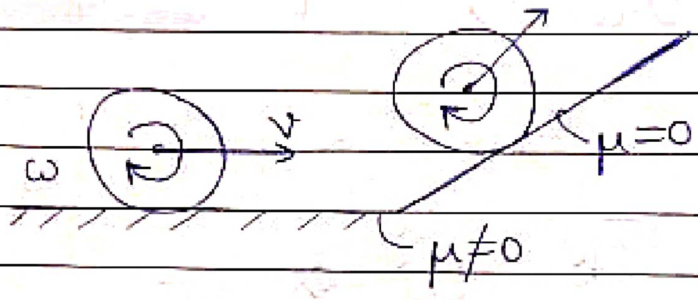
for purely rotating body,

$$\boxed{KE_{body} = \frac{1}{2} I \omega^2}$$

of body abt IAR

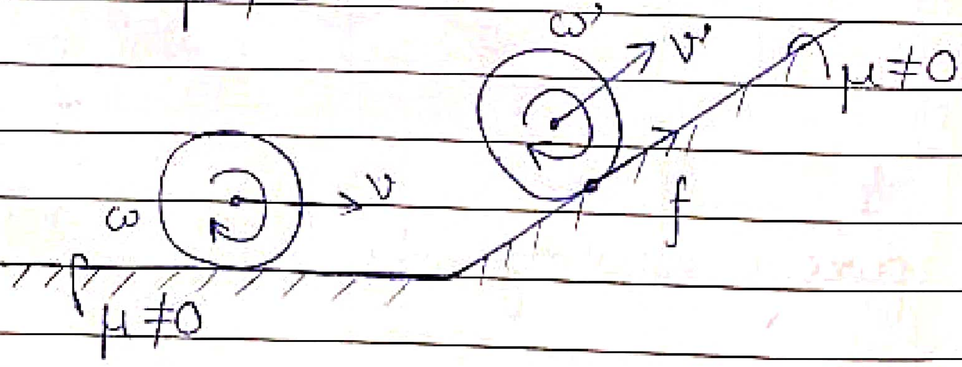


A purely rolling obj will keep rolling purely on flat surface regargless of friction as $\boxed{f=0}$.



A purely rolling obj., if it climbs on smooth incline will stop rolling purely as mg comp. changes 'v' but 'ω' same as $\tau_{mg} = 0$. Rolling with sliding happens.

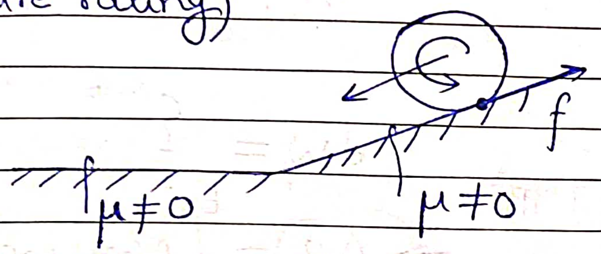
Hence to have obj. to roll purely we need $\mu \neq 0$.

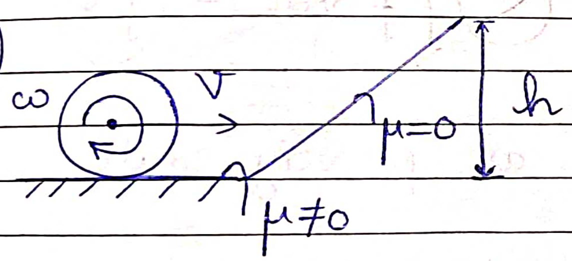


To reach pure rolling, f inc. v' & f dec. ω'. But in total vel. changes dirx^n and ultimately obj. returns.

vel. & angular vel. keep changing, but at every pt. there is pure rolling.

If obj. start from rest on rough incline, 'mg's comp. along incline inc. vel.
To inc. ω, friction in up dirxⁿ.
(for pure rolling)



Q)


Solid Sphere.
Pure rolling init.
Find min. vel.
so that obj. just reach top.

A) By Energy Consrv., $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh + \frac{1}{2}I\omega^2$

Since incline smooth, obj. keep rolling with ω.

$\Rightarrow \boxed{v = \sqrt{2gh}}$

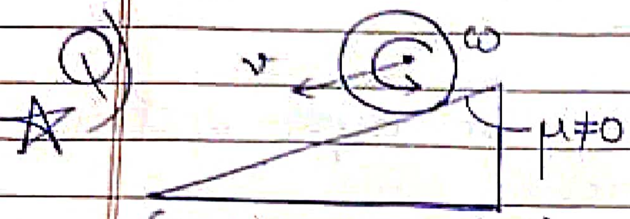
Q) If above Q, find v_{min} if incline is rough.

A) Since pure rolling, $v_{\text{lowest pt.}} = 0 \Rightarrow W_f = 0$
$\Rightarrow$ We can consrv. Energy.

There will always be pure rolling.

$\therefore E_0 = E_1 \Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2mR^2}{5}\right)\left(\frac{v}{R}\right)^2 = mgh \Rightarrow \boxed{v = \sqrt{\frac{10gh}{7}}}$

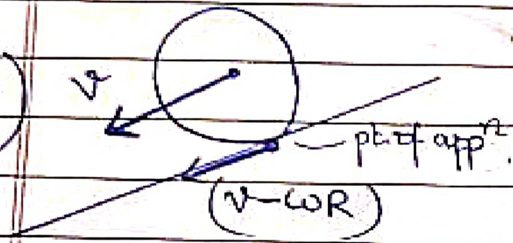
★ Q)



(μ is very less)

If centre moves dist. l, find work by friction. Given obj is rolling with slipping.

A)



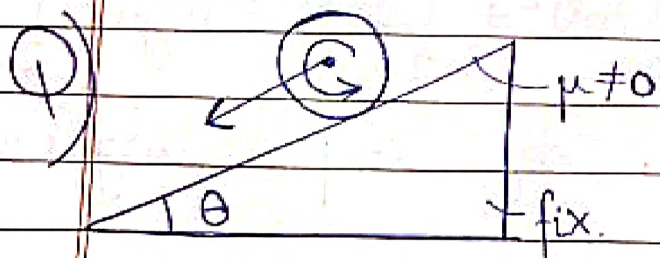
$dW_f = \overset{\text{Const}}{f}(v-\omega R)\,dt$

$\Rightarrow W_f = \int_0^t f(v-\omega R)\,dt$

$< \int_0^t f\,v\,dt = fl$

v & ω will keep changing but if $v > \omega R$ then it will remain so as μ very small.

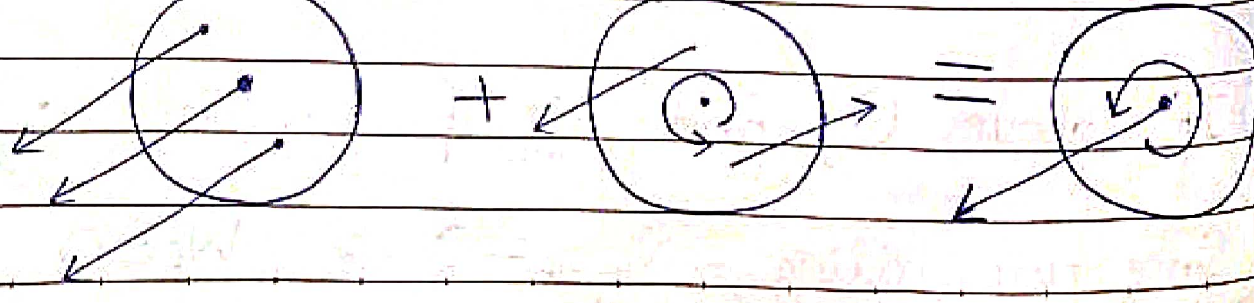
Q)



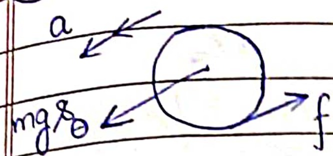
Pure rolling.

Find acc. of body.

A)

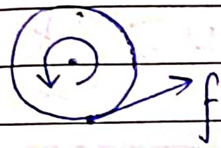


(1) Force condⁿ :



$$ma = mg\sin\theta - f \quad -(1)$$

(2) Torque condⁿ :

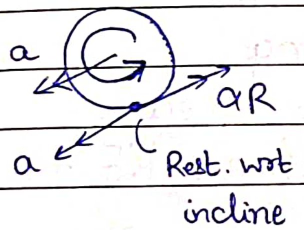


$$\tau = I\alpha$$
(about centre)

$$\Rightarrow \quad fR = (mk^2)\alpha \quad -(2)$$

(3) Pure rolling :

$$a = \alpha R \quad -(3)$$



Now, (3), (2) → 1 $\Rightarrow \quad ma = mg\sin\theta - \left(\frac{mk^2}{R^2}\right)a$

$$\Rightarrow \quad \boxed{a = \frac{g\sin\theta}{(1+(k/R)^2)}}$$

☆ In ANY Q, we can apply these eqⁿs :— Angular Momentum eqⁿ, Consrv. L (if no τ_{ext}) Force, Torque (abt CoM), Torque (abt ICR), Constraint, Energy Consrv (if no f_{ext}) & Momentum Consrv. (if no F_{ext})

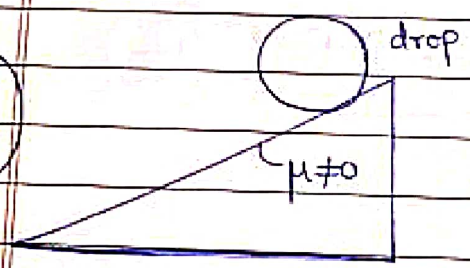
Q) Find friction in prev. Q & μ req. of incline

A) $mg s_\theta - ma = f = m\left[g s_\theta - \frac{g s_\theta}{(1+K^2/R^2)}\right]$

$\Rightarrow \boxed{f = \left(\frac{K^2}{K^2+R^2}\right) mg s_\theta}$

Now, $f \le \mu mg c_\theta \Rightarrow \boxed{\mu \ge \left(\frac{K^2}{K^2+R^2}\right) t_\theta}$

Q)



1) Sphere 2) Disc
Pure rolling.
When they reach bottom, whose KE more?

A) By Energy Consrv., $\Delta K + \Delta U = 0$

$\Rightarrow K_f = U_i \Rightarrow$ $\boxed{\text{Both have same K.E.}}$

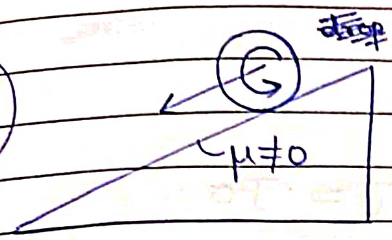
Q) In above Q, find v in general.

A) By Energy Consrv., $\frac{1}{2} mv^2 + \frac{1}{2} I\omega^2 = mgh$

$\Rightarrow \frac{1}{2} mv^2 + \frac{1}{2} \frac{mk^2 v^2}{R^2} = mgh \Rightarrow \boxed{v = \sqrt{\frac{2gh}{1+(K/R)^2}}}$

~~There~~ Here, 'k' is Radius of Gyration.

Q)



1) Sphere 2) Disc
Rolling with sliding.
Whose KE more?

A) Rolling with sliding $\Rightarrow$ Same time to reach bottom. Same friction force.

Now, $\tau = \left(\frac{dL}{dt}\right)$ (abt. CoM)

Since only friction produce torque $\Rightarrow \int \tau \, dt = \int I \, d\omega$
(Const.)

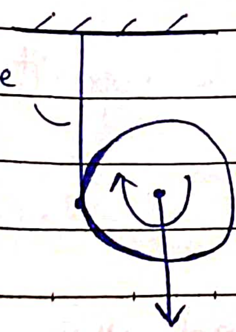
$$\Rightarrow \boxed{\tau t = I\omega}$$

Now, $KE = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2 + \frac{\tau^2 t^2}{2I}$

Since both obj. same time & same acc. $\Rightarrow v$ same

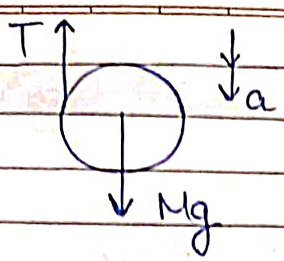
$\Rightarrow I\downarrow \Rightarrow \left(\frac{\tau^2 t^2}{2I}\right)\uparrow \Rightarrow KE\uparrow$

Q)



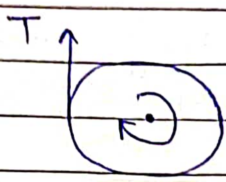
Cylinder rolling along with string unrolling. There is no slipping. Find acc. & tension in rope.

A) (1) Force :



$$Ma = Mg - T$$

(2) Torque :
(abt centre)

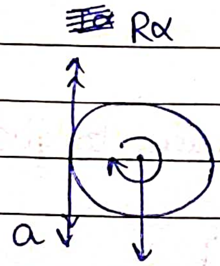


$$\tau = I\alpha$$

$$\Rightarrow TR = \left(\frac{MR^2}{2}\right)\alpha$$

$$\Rightarrow 2T = MR\alpha$$

(3) Pure Rolling :



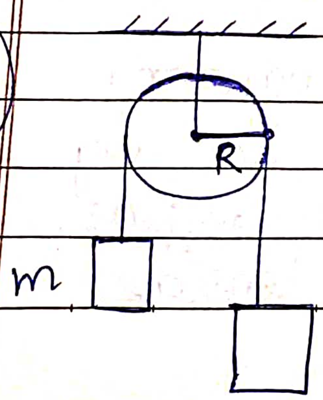
$$a = R\alpha$$

(3) → (2) ; $2T = Ma \Rightarrow T = \left(\frac{Ma}{2}\right)$ — (4)

(4) → (1) ; $Ma = Mg - \left(\frac{Ma}{2}\right) \Rightarrow \boxed{a = \left(\frac{2g}{3}\right)}$

Into (4), $\boxed{T = \left(\frac{Mg}{3}\right)}$

Q)



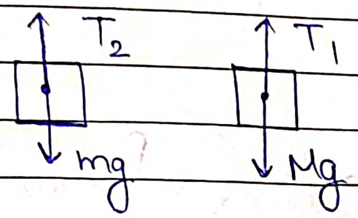
No slipping b/w rope & pulley.

MoI of pulley is I.

Given M > m, find acc.

A) Since rotating pulley, tension NOT same.

(1) **force** :

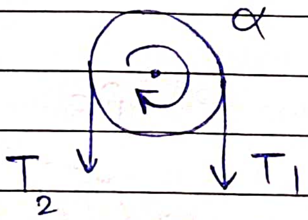


$$Mg - T_1 = Ma$$

$$T_2 - mg = ma$$

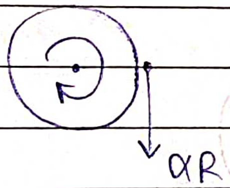
(2) **Torque** :

(abt centre)



$$T_1 R - T_2 R = I\alpha$$

(3) **Constraint** :



Pt. on pulley move with αR. Since in contact with string $\Rightarrow$ String Constraint

$$a = \alpha R$$

(3) $\longrightarrow$ (2) ;

$$(T_1 - T_2) = \left(\frac{Ia}{R^2}\right)$$

Adding with (1),

$$(M - m)g = (M+m)a + \left(\frac{Ia}{R^2}\right)$$

$$\Rightarrow \quad \boxed{a = \frac{(M-m)\,g}{(M+m) + (I/R^2)}}$$

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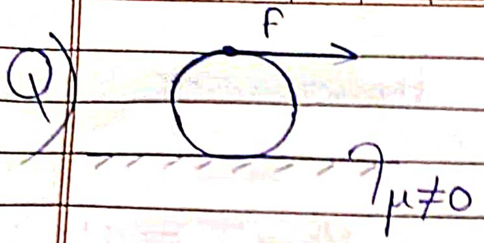
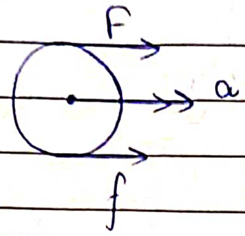
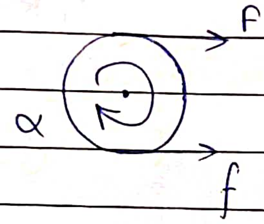
Q) 

Solid sphere. Pure rolling.
Find friction.

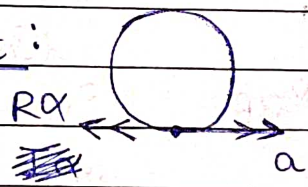
A) (1) Force: 

$$\boxed{(F+f) = Ma}$$

(2) Torque: 

$$FR - fR = I\alpha$$

$$\Rightarrow \boxed{(F-f) = \left(\frac{2MR\alpha}{5}\right)}$$

(3) Constraint: 

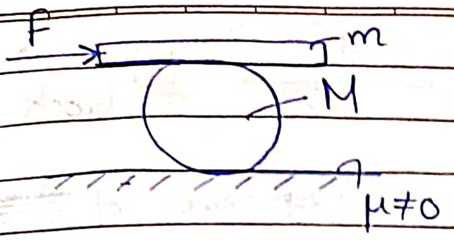
$$\boxed{a = R\alpha}$$

(3) → (2); $(F-f) = \left(\frac{2M}{5}\right)a$

Dividing with (1), $\left(\frac{F-f}{F+f}\right) = \left(\frac{2}{5}\right)$

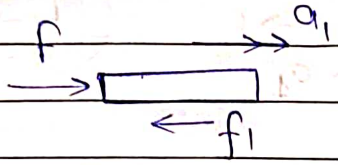
$$\Rightarrow \boxed{f = \left(\frac{3F}{7}\right)}$$

Q)

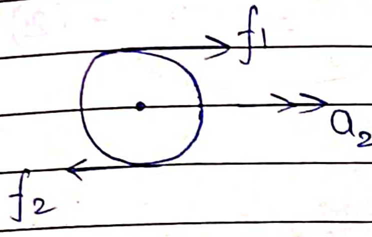


Solid sphere.
No slipping at any surface.
Find acc. of sphere

A) ① Force :

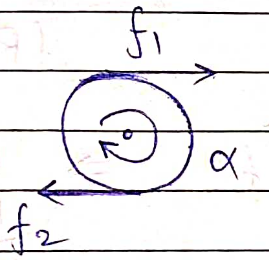


$$(F - f_1) = m a_1 \quad -①$$



$$(f_1 - f_2) = M a_2 \quad -②$$

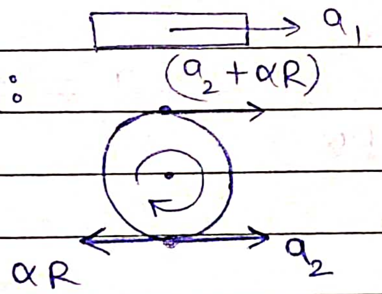
② Torque :



$$(f_1 R + f_2 R) = \left(\frac{2MR^2}{5}\right)\alpha$$

$$\Rightarrow (f_1 + f_2) = \left(\frac{2MR}{5}\right)\alpha \quad -③$$

③ Constraint :



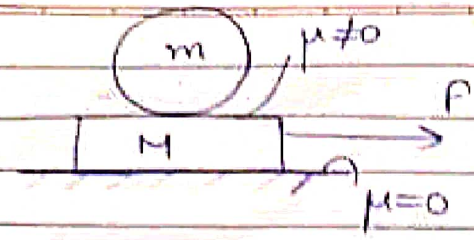
For pure rolling,

$$a_2 = \alpha R \quad -④$$
$$a_1 = a_2 + \alpha R \quad -⑤$$

④ → ⑤, ③ ⇒ $a_1 = 2a_2$ — ⑥ & $f_1 + f_2 = \left(\frac{2M}{5}\right)a_2$ — ⑦

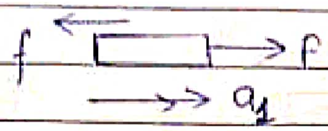
⑦ + ② ⇒ $f_1 = 7Ma_2/10$ — ⑧

⑧ & ⑥ → ① ⇒ $F - \frac{7Ma_2}{10} = 2ma_2$ ⇒ $\boxed{a_2 = \left(\frac{10F}{20m + 7M}\right)}$

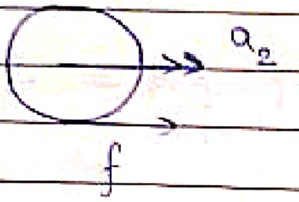
Q) 

No slipping b/w disc & block
Find acc. of ~~block~~ disc

A) (1) Force :

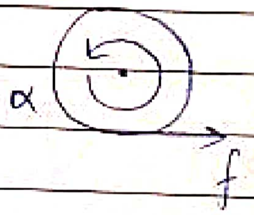


$$(F-f) = Ma_1 \quad -(1)$$



$$f = ma_2 \quad -(2)$$

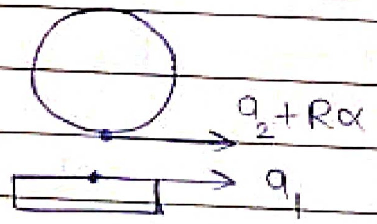
(2) Torque :
(abt CoM)



$$fR = \left(\frac{mR^2}{2}\right)\alpha$$

$$\Rightarrow f = \left(\frac{m}{2}\right)R\alpha \quad -(3)$$

(3) Constraint : for pure rolling,



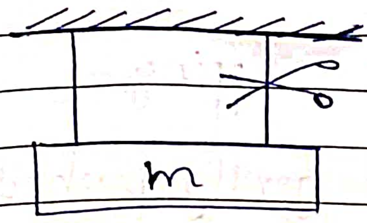
$$a_1 = a_2 + R\alpha \quad -(4)$$

Now, $m(4) - 2(3) + (1) + 3(2)$ gives

$$ma_1 - 2f + (F-f) + 3f = (ma_2 + mR\alpha) - mR\alpha + Ma_1 + 3ma_2$$

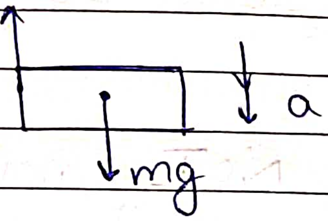
$$\Rightarrow F + (m-M)a_1 = 4ma_2$$

$$\Rightarrow F + \frac{(m-M)}{(M)}(F - ma_2) = 4ma_2 \Rightarrow \boxed{a_2 = \left(\frac{F}{3M+m}\right)}$$

Q) 

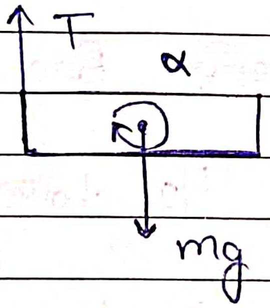
Find tension in string just after the cut.

A) Force:



$$mg - T = ma \quad \text{— (1)}$$

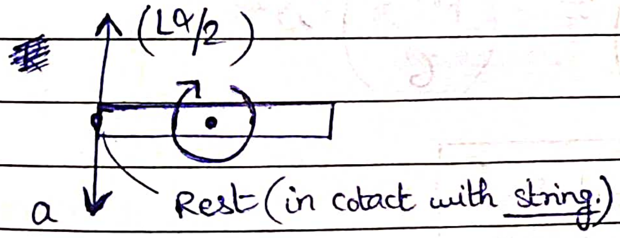
Torque: (abt CoM)



$$T\left(\frac{L}{2}\right) = \left(\frac{mL^2}{12}\right)\alpha$$

$$\Rightarrow \quad T = \left(\frac{mL}{6}\right)\alpha \quad \text{— (2)}$$

Constraint: Pure rotation abt. pt. in contact with string

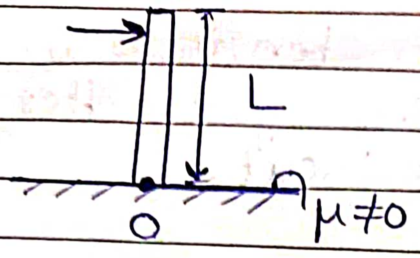


$$a = \left(\frac{L\alpha}{2}\right) \quad \text{— (3)}$$

(3) → (2) ; $T = (ma/3)$ — (4)

(4) → (1) ; $mg - T = 3T \Rightarrow \boxed{T = \left(\frac{mg}{4}\right)}$

Date: ______ Page:

Q) 

No slipping.

Rod gently pushed. Find ω of when it is abt to hit ground.

A) Since lowest pt. NOT move, work by friction on it is zero.

$\Rightarrow$ We can consrv. Energy!

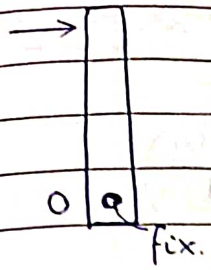
$E_0 = Mg\left(\frac{L}{2}\right)$ (CoM L/2 above ground)

$E_1 = \frac{1}{2}\left(\frac{ML^2}{3}\right)\omega^2$ (K.E. due to rot. as pure rot. abt. O)

$E_0 = E_1 \Rightarrow \left(\frac{MgL}{2}\right) = \left(\frac{ML^2}{6}\right)\omega^2$

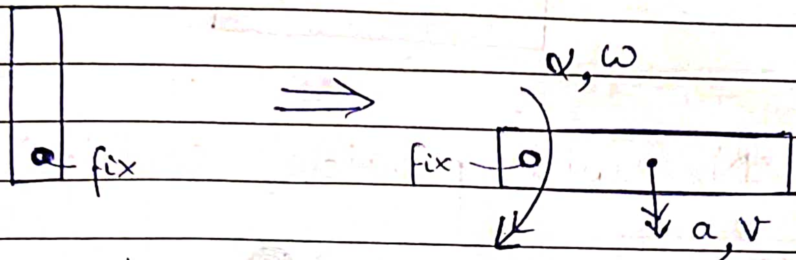
$\Rightarrow \boxed{\omega = \sqrt{\frac{3g}{L}}}$

Q)

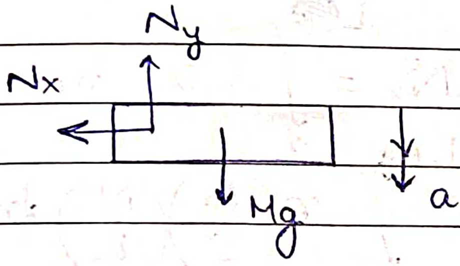


Find hinge rx^n when rod becomes horiz.

A)

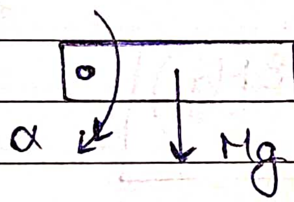


<u>force</u>:



$$Mg - N_y = Ma \quad \text{— (1)}$$

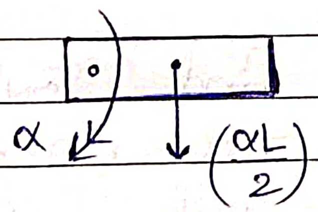
<u>Torque</u> : (abt O)



$$(Mg)\left(\frac{L}{2}\right) = \left(\frac{ML^2}{3}\right)(\alpha)$$

$$\Rightarrow \alpha = \left(\frac{3g}{2L}\right) \quad \text{— (2)}$$

<u>Constraint</u> :



$$a = \left(\frac{\alpha L}{2}\right) \quad \text{— (3)}$$

<u>Energy Consrv</u> :

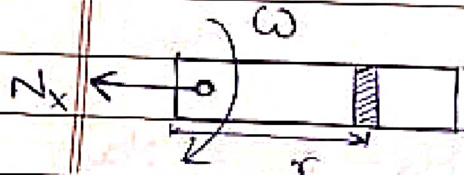
$$\left(\frac{MgL}{2}\right) = \frac{1}{2}\left(\frac{ML^2}{3}\right)\omega^2 \Rightarrow \omega = \sqrt{\frac{3g}{L}} \quad \text{— (4)}$$

(2) & (3) $\Rightarrow$ $a = 3g/4$

Into (1) $\Rightarrow$ $Mg - N_y = \left(\frac{3Mg}{4}\right)$

$\Rightarrow$ $\boxed{N_y = \left(\frac{Mg}{4}\right)}$

Now, N_x will provide centripetal force

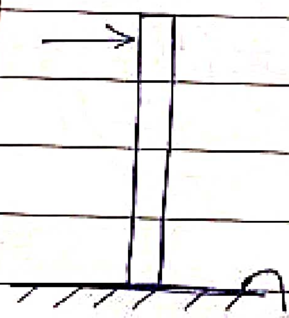


$$N_x = \int_0^L r\omega^2\, dm$$

$$\Rightarrow N_x = \int_0^L r\left(\frac{3g}{L}\right)\left(\frac{M}{L}\right) dr$$

$$\Rightarrow \boxed{N_x = \left(\frac{3Mg}{2}\right)}$$

☆ Q)



$\mu = 0$

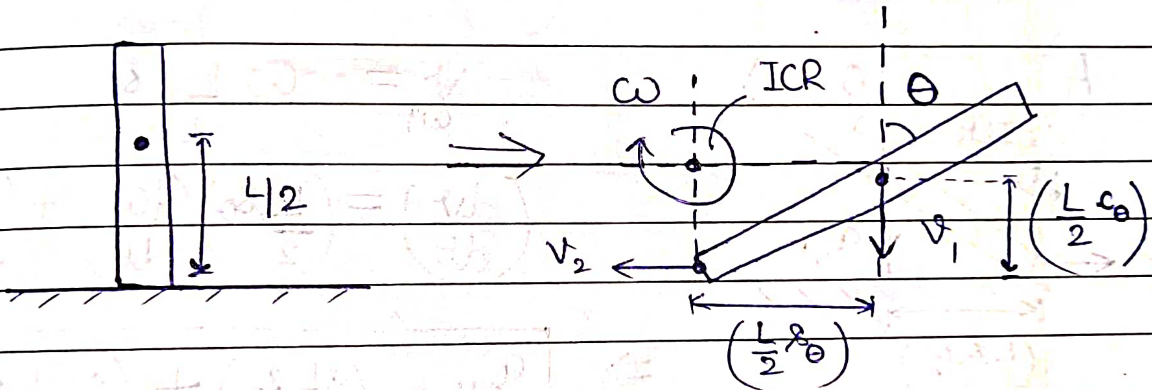
Find V_{COM} when rod makes θ with vertical.

A) In this Q, we need ICR !

Method to find ~~ICR~~ ICR / IAoR :

Find any 2 pts. with known velocities, draw $\perp$ to vel. at those pts.. The $\cap$ pt. is ICR / IAoR.

In this case we use CoM & lowest pt. as CoM move $\downarrow$ (only mg acts on it) and lowst pt. move $\leftarrow$ (it is on surface).



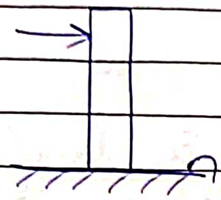
By Energy Consrv.,
$$\left(\frac{MgL}{2}\right) = \left(\frac{MgL\,c_\theta}{2}\right) + \frac{1}{2} I_{\text{abt. ICR}}\,\omega^2$$

By || axis theorem,
$$I_{ICR} = \left(\frac{ML^2}{12}\right) + M\left(\frac{L}{2}s_\theta\right)^2$$
$$\Rightarrow I_{ICR} = \left(\frac{ML^2}{12}\right)\left(3s_\theta^2+1\right)$$

Into above eq^n,
$$\left(\frac{MgL}{2}\right) = \left(\frac{MgL}{2}\right)c_\theta + \left(\frac{ML^2}{24}\right)\left(3s_\theta^2+1\right)\omega^2 \Rightarrow \boxed{\omega = \sqrt{\frac{12g(1-c_\theta)}{L(3s_\theta^2+1)}}}$$

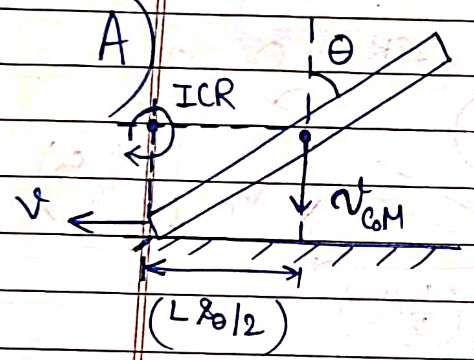
Now, $v_{CoM} = \omega\left(\frac{L s_\theta}{2}\right) \Rightarrow \boxed{v_{CoM} = \sqrt{\frac{3gL(1-c_\theta)}{(3s_\theta^2+1)}}}$

★ Q)



After rotating θ abt vertical, its angular acc. is α & angular vel. is ω. Find a_{CoM}.

A)



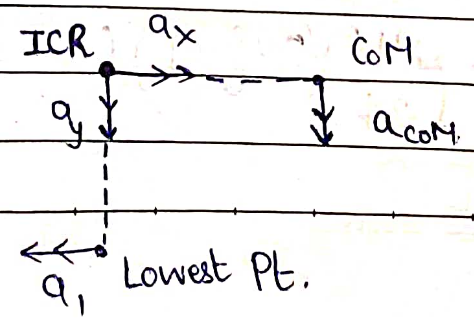
$$v_{CoM} = \omega \frac{L}{2} s_\theta$$

$$\left(\frac{dv}{dt}\right) = \left(\frac{L s_\theta}{2}\right)\left(\frac{d\omega}{dt}\right) + \left(\frac{L c_\theta}{2}\right)\left(\frac{d\theta}{dt}\right)(\omega)$$

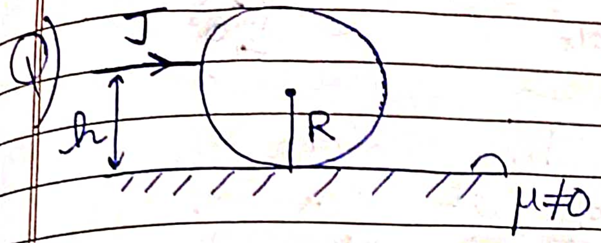
$$\Rightarrow \boxed{a_{CoM} = \left(\frac{\alpha L s_\theta}{2}\right) + \left(\frac{\omega^2 L c_\theta}{2}\right)}$$

We could have also applied formulae of Circular Motion in frame of ICR.

For this we need its acc., which can be found using condⁿ for rigid body.

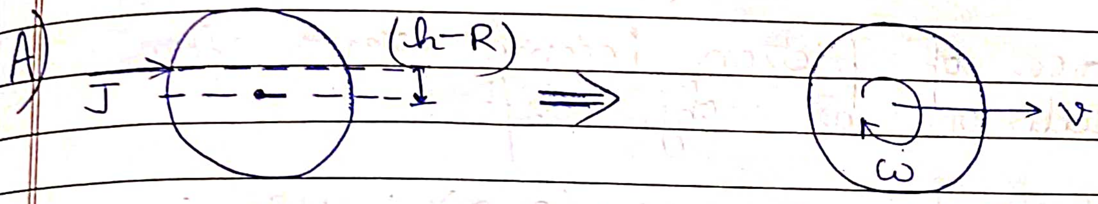


Q)



Impulse given to Solid Sphere.
Obj. start pure rolling
find 'h'.

A)



Translation : $Mv = J$ — ①

Rotation (abt CoM) : $J(h-R) = \left(\frac{3MR^2}{5}\right)\omega$ — ②

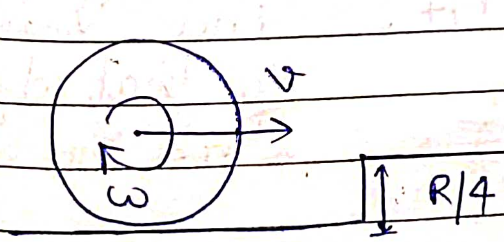
Angular Impulse

Constraint : $v = \omega R$. — ③

① × ② ÷ ③ : $\left(\frac{MvJ(h-R)}{v}\right) = J\left(\frac{3MR^2}{5}\right)\left(\frac{\omega}{\omega R}\right)$

$\Rightarrow \boxed{h = 7R/5}$

Q)

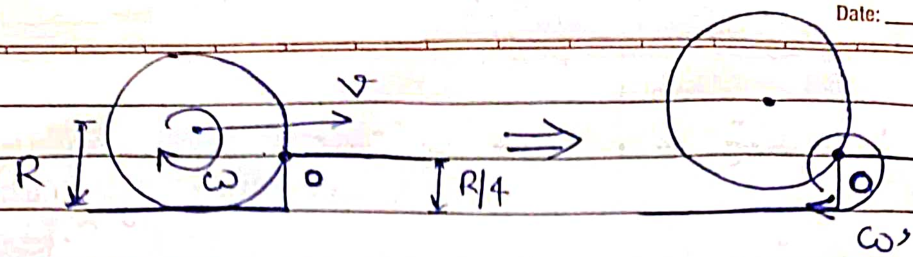


Disc. rolling purely. There is no jumping.

Find ω just after collision.

Date: Page:

A)



Since force applied by surface on obj., we take it as ref. for Consrv. of Angular Momentum.

Force of friction becomes zero after collision as obj. lifts.

$L_i = L_f$ (abt. O) $\Rightarrow Mv\left(R-\frac{R}{4}\right)+\left(\frac{MR^2}{2}\right)\omega$

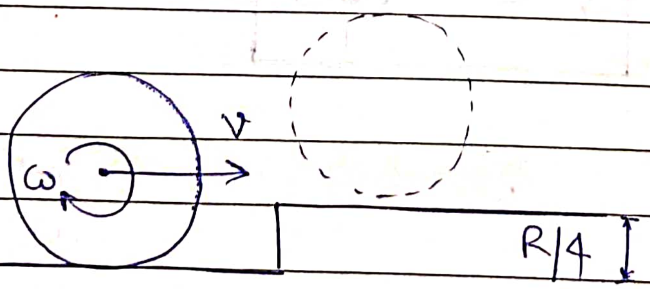
$= \left(\frac{3MR^2}{2}\right)\omega'$

Constraint: $v = \omega R$.

$\Rightarrow \left(\frac{3M}{4}\right)\omega R^2 + \left(\frac{MR^2}{2}\right)\omega = \left(\frac{3MR^2}{2}\right)\omega'$

$\Rightarrow \boxed{\omega' = \left(\frac{5\omega}{6}\right)}$

Q)



Disc. pure rolling. No jumping. Find v_{min} so that obj. climbs the stair.

A) By Energy Consrv., $E_i = E_f$.

$$E_i = \underbrace{+}_{MgR} \frac{1}{2} M v^2 + \frac{1}{2}\left(\frac{MR^2}{2}\right)\omega^2 = \left(\frac{3Mv^2}{4}\right) + MgR$$

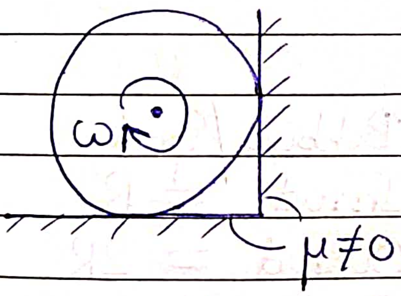
Finally obj. moving with $KE = K$.

$$E_f = Mg\left(R + \frac{R}{4}\right) + K = \left(\frac{5MgR}{4}\right) + K$$

$$E_i = E_f \Rightarrow \left(\frac{3Mv^2}{4}\right) + MgR = \left(\frac{5MgR}{4}\right) + K \geq \left(\frac{5MgR}{4}\right)$$

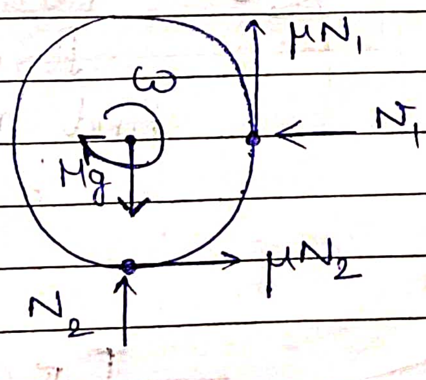
$$\Rightarrow \left(\frac{3Mv^2}{4}\right) \geq \left(\frac{MgR}{4}\right) \Rightarrow \boxed{v \geq \sqrt{\frac{gR}{3}}}$$

Q)



Find time after which it stops rotating.

A)



Translational Eq: $\mu N_1 + N_2 = Mg$ & $N_1 = \mu N_2$

$$\Rightarrow N_2 = Mg/(1+\mu^2)$$

Rotational : (abt. CoM) $\left(\frac{MR^2}{2}\right)\alpha = (\mu N_1)R + (\mu N_2)R$

$$\Rightarrow \alpha = \left(\frac{2\mu}{MR}\right)(N_1+N_2)$$

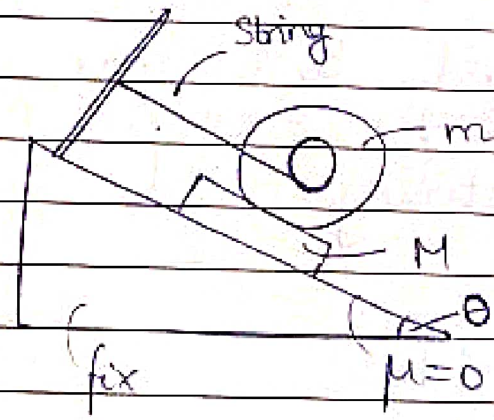
$$\Rightarrow \alpha = \frac{2\mu(1+\mu)\,N_2}{MR}$$

$$\Rightarrow \alpha = \frac{2g}{R}\frac{\mu(1+\mu)}{(1+\mu^2)}$$

Obj. stop after time 't'. Hence,

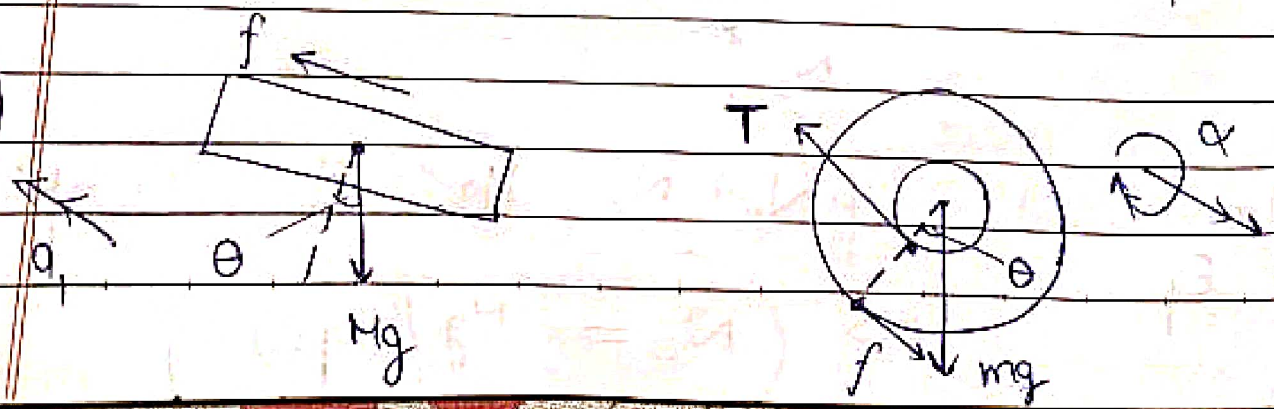
$\omega = \alpha t \Rightarrow$ $\boxed{t = \left(\frac{\omega R}{2g}\right)\frac{(1+\mu^2)}{\mu(1+\mu)}}$

Q1) ☆



Bobbin / Spool
Inrad. = R
Outrad. = 2R
MoI of bobbin = I.
Pure rolling.
Find m/M ratio
s.t. block move up.

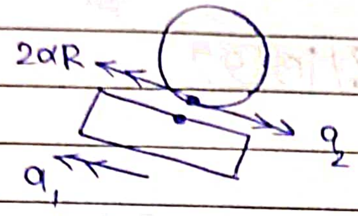
A)



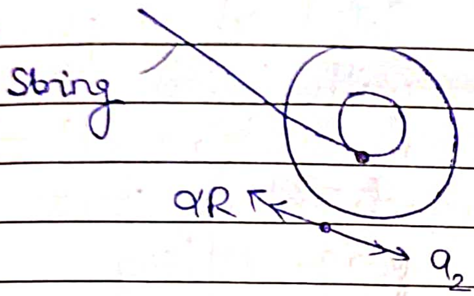
Translation : $Ma_1 = f - Mg\,\%_0$ (1) & $ma_2 = mg\,\%_0 + f - T$ (2)

Rotational : (abt. Centre) $I\alpha = TR - 2fR$ — (3)

Constraint :



$2\alpha R = a_1 + a_2$ — (4)
(Pure Rolling)



$a_2 = \alpha R$ — (5)
(String Constraint)

(4) & (5) $\Rightarrow$ $\boxed{a_1 = a_2 = \alpha R}$ — (6)

$m(1) + M(2) \Rightarrow (Mm)(a_1 + a_2) = (M+m)f - MT$

$\Rightarrow T = \left(1 + \frac{m}{M}\right)f - (m)(a_1 + a_2)$

Substituting (6) $\Rightarrow$ $T = \left(1 + \frac{m}{M}\right)f - 2m\alpha R$

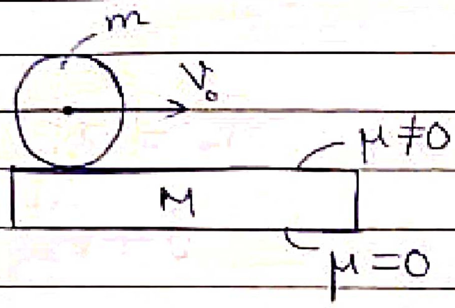
Using (3) $\Rightarrow$ $\left(\frac{I\alpha}{R}\right) + 2f = \left(1 + \frac{m}{M}\right)f - 2m\alpha R$

$\Rightarrow \left(\frac{m}{M} - 1\right)f = \left(\frac{I\alpha}{R} + 2m\alpha R\right) \propto \alpha$

We need $a_1 \geq 0 \Rightarrow \alpha \geq 0$

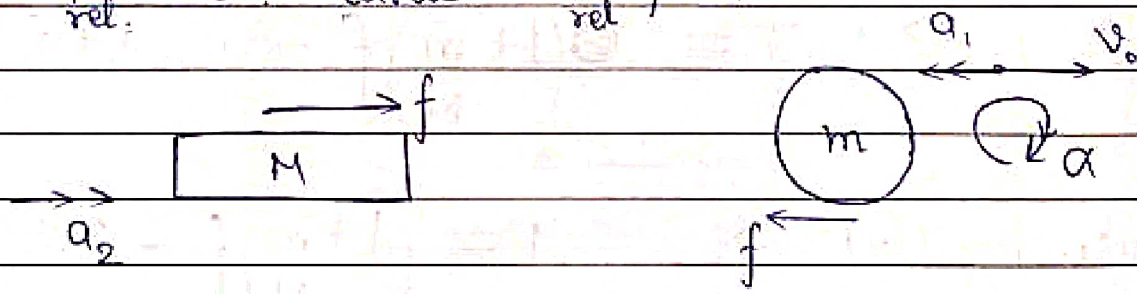
$$\Rightarrow \left(1-\frac{m}{M}\right)f \leq 0 \Rightarrow \boxed{m/M \geq 1}$$

28/10/22

Q) 

Solid sphere pushed with vel. v find time after which it start pure rolling.

A) At time 't', pure rolling start if $v_{rel} = 0$. Now suddenly friction will change so as to ensure $a_{rel} = 0$. Earlier $a_{rel} \neq 0$.

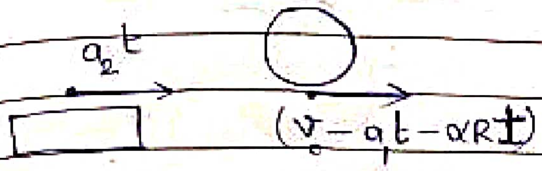


Translation : $f = Ma_2$, $f = ma_1$

Rotation : $\left(\frac{2mR^2}{5}\right)\alpha = fR$

$$\Rightarrow \alpha = \left(\frac{5f}{2mR}\right)$$

Constraint : $v_{rel} = 0$ (NOT $a_{rel} = 0$)



$$a_2 t = v_0 - a_1 t - \alpha R t$$

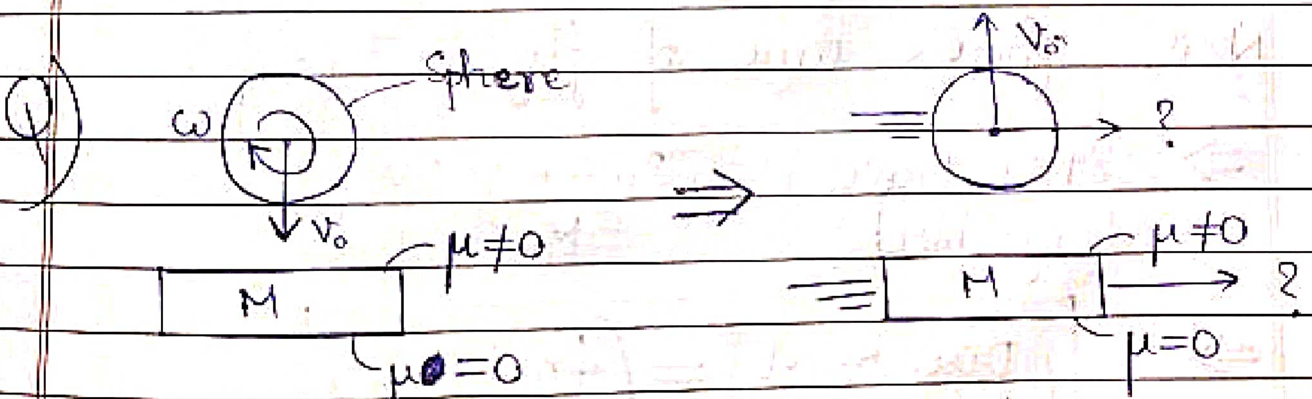
$$\Rightarrow \left(\frac{f}{M}\right)t = v_0 - \left(\frac{f}{m}\right)t - \left(\frac{5f}{2m}\right)t$$

$$\Rightarrow \quad t = \frac{2v_0 Mm}{f(7M+2m)}$$

As sliding happening before t, $f = \mu mg$

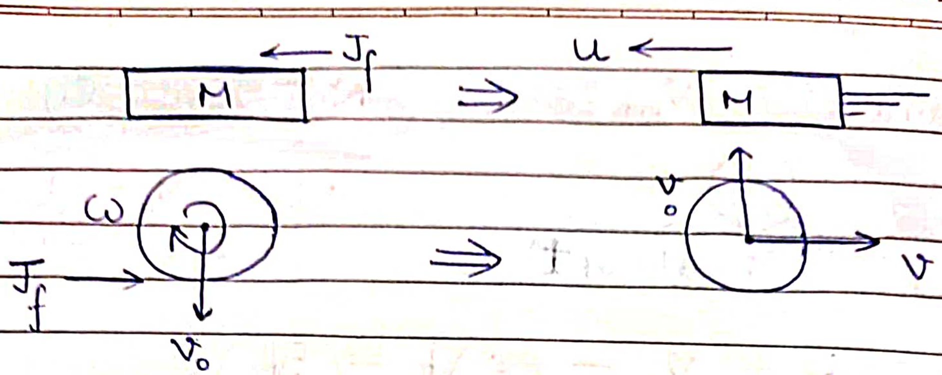
$$\Rightarrow \boxed{t = \frac{2v_0 M}{\mu g(7M+2m)}}$$

Q)



After 1st rebound, vel. normal to block is reversed. Find dist. travel by block b/w 1st & 2nd rebound.

A)



Impulse : $Mu = J_f$, $mv = J_f$

Angular Impulse : $0 - \left(\frac{2mR^2}{5}\right)\omega = +J_f R$

$$\Rightarrow J_f = \left(\frac{2m\omega R}{5}\right)$$

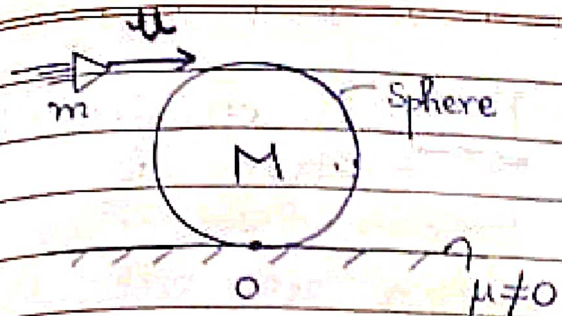
$$\Rightarrow u = \left(\frac{2m\omega R}{5M}\right), \quad v = \left(\frac{2\omega R}{5}\right)$$

Now, sphere's time of flight $= \left(\frac{2v_0}{g}\right)$

$$\Rightarrow \left(\begin{array}{c}\text{Dist. travel} \\ \text{by block}\end{array}\right) = \left(\frac{2m\omega R}{5M}\right)\left(\frac{2v_0}{g}\right)$$

$$\Rightarrow \boxed{\left(\begin{array}{c}\text{Dist. travel} \\ \text{by block}\end{array}\right) = \left(\frac{4m\omega R v_0}{5Mg}\right)}$$

Q)



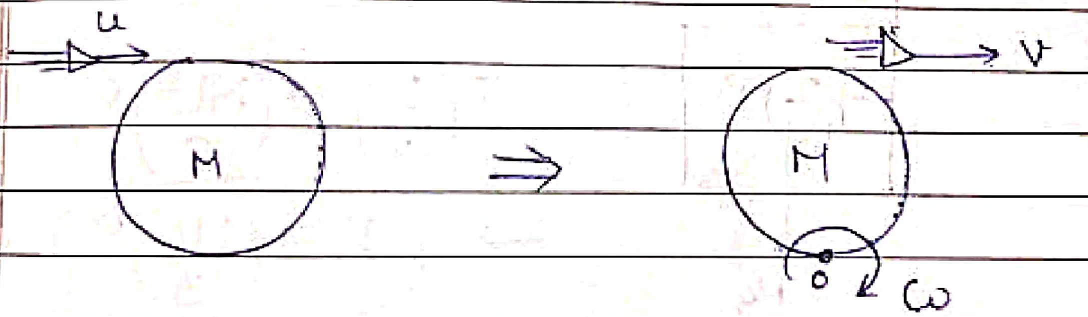
Bullet just grazes. Sphere starts pure rolling. Assume bullet's rel. vel. with sphere's surface to be zero at time of grazing.

A)

Angular Momentum Consrv. (abt O) :

$$mu(2R) = \left(\frac{3MR^2}{2}\right)\omega + mv(2R)$$

$$\Rightarrow 4mu = (3MR)\omega + 4mv$$



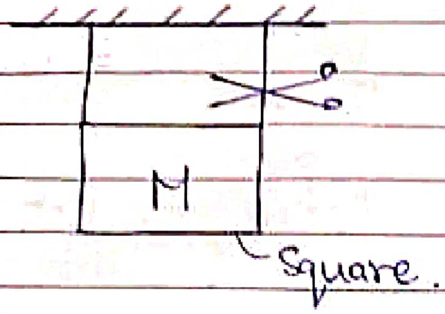
Constraint : $v = \omega(2R) \Rightarrow R\omega = v/2$

($v_{rel.}$ b/w bullet & mass $= 0$)

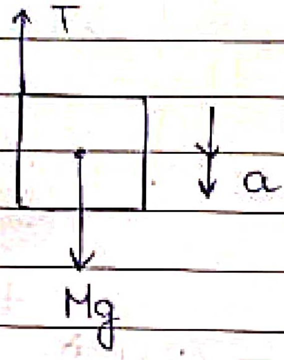
Combining,

$$\boxed{v = \left(\frac{8mu}{3M + 8m}\right)}$$

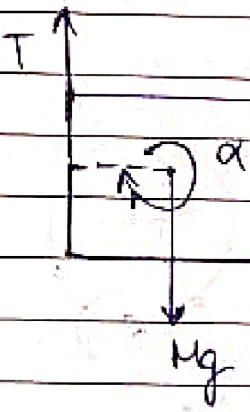
$$\boxed{\omega = \frac{4mu}{R(3M+8m)}}$$

Q) 

Find tension in ~~other~~ string at time when other string is cut.

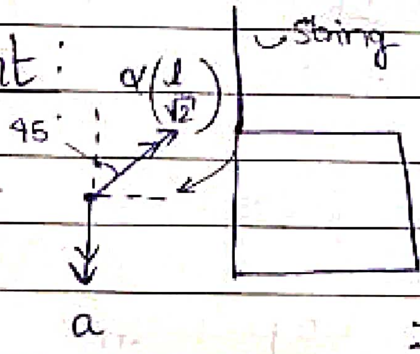
A) Force : 

$$Ma = Mg - T \quad -(i)$$

Torque : 

$$T\left(\frac{L}{2}\right) = \left(\frac{ML^2}{6}\right)\alpha$$

$$\Rightarrow \quad T = \left(\frac{ML}{3}\right)\alpha \quad -(ii)$$

Constraint : 

'a' along string is zero (String Constraint)

$$\Rightarrow \quad a = \left(\frac{\alpha l}{\sqrt{2}}\right) \&\ 45^\circ$$

$$\Rightarrow \quad 2a = \alpha l \quad -(iii)$$

(ii) & (iii) $\Rightarrow$ $T = 2Ma/3$

Into (i), $Ma = Mg - \frac{2Ma}{3} \Rightarrow Mg = \frac{5Ma}{3}$

$$\Rightarrow \left(\frac{2Ma}{3}\right) = \left(\frac{2}{5}\right)Mg$$

$$\Rightarrow \boxed{T = \left(\frac{2Mg}{5}\right)}$$

Imp. Pts —

1) (Angular Impulse) = $\vec{r} \times$ (Impulse)