VECTORS

Vector = (Magnitude)(Dirx^n)

Dirx^n given by unit vector in the dirx^n of vector (Magnitude = 1)

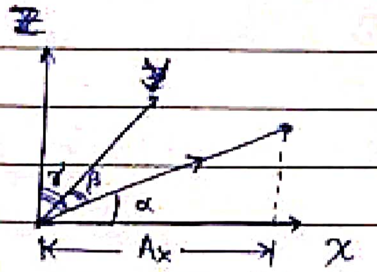
Representation —

Vector : $\vec{A}$ Magnitude : $|\vec{A}|$ or A

Dirx^n : $\hat{A} = \left(\frac{\vec{A}}{A}\right)$ Geometric :

Rectangular Components of Vector —

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$



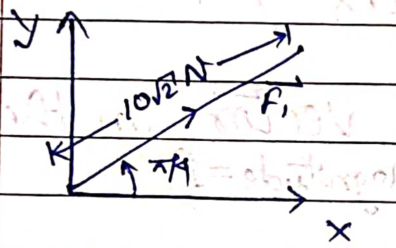
$A_x = A\cos(\alpha)$
$A_y = A\cos(\beta)$
$A_z = A\cos(\gamma)$

$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$ $\cos^2(\alpha) + \cos^2(\beta) + \cos^2(\gamma) = 1$

$\cos(\alpha)$, $\cos(\beta)$, $\cos(\gamma)$ are called dirx^n cosines.

Q) If $A = 3\hat{i} - 4\hat{j} + 5\hat{k}$. find angle b/w $\vec{A}$ and X axis.

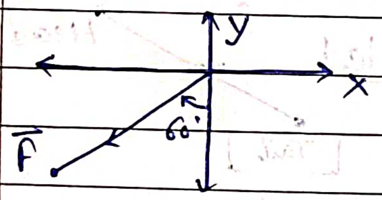
A) $\cos(\alpha) = \dfrac{A_x}{A} = \dfrac{3}{\sqrt{9+16+25}} \Rightarrow \cos(\alpha) = \dfrac{3}{5\sqrt{2}}$

Q) 

If $F_1 = 10\sqrt{2}\,N$ and ∠ b/w $\vec{F_1}$ and X axis is $\pi/4$. find $\vec{F_1}$.

A) $\vec{F_1} = F_x\hat{i} + F_y\hat{j} = (F)\left(\cos(\pi/4)\hat{i} + \sin(\pi/4)\hat{j}\right)$

$\Rightarrow \boxed{\vec{F_1} = 10\hat{i} + 10\hat{j}}$

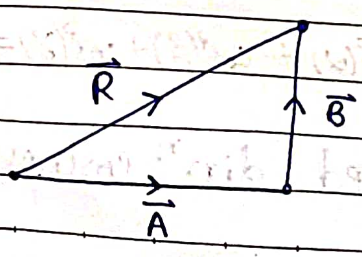
Q) 

If $F = 20N$ and ∠ b/w $\vec{F}$ and (-ve) y axis is $60°$. find $\vec{F}$.

A) $\vec{F} = (20)\left(\cos(60°)\hat{j} - \sin(60°)\hat{i}\right) \Rightarrow \boxed{\vec{F} = (10\hat{j} + 10\sqrt{3}\hat{i})(-1)}$

Addition of Vectors –

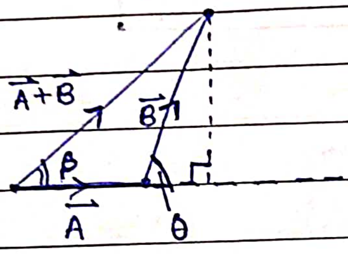
1. Triangle Law –



$\vec{R} = (\vec{A} + \vec{B})$

Join head of one vector to tail of another.

☆ $\angle$ b/w vectors to be calc. only if they have same initial pt. (Co-initial vectors)



$$|\vec{A}+\vec{B}| = \sqrt{A^2+B^2+2AB\cos(\theta)}$$

$$\tan(\beta) = \left(\frac{B\sin(\theta)}{A+B\cos(\theta)}\right)$$

☆ $|\vec{A}+\vec{B}| \in [|A-B|, A+B]$

If $\cos(\theta)=1 \Rightarrow$ vectors $\parallel$, then $|\vec{A}+\vec{B}|_{max} = A+B$	If $\cos(\theta)=\cancel{0}(-1) \Rightarrow$ vectors anti$\parallel$, then $|\vec{A}+\vec{B}|_{min} = |A-B|$

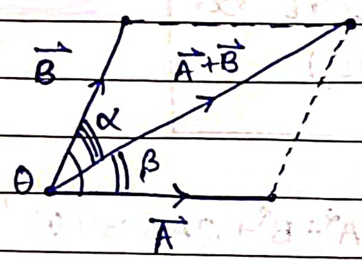
Q) Let $F_1=3N$, $F_2=5N$, $F_3=6N$. Can $\vec{F_1}+\vec{F_2}+\vec{F_3}$ be equal to $\vec{0}$?

A) $2 \leq |\vec{F_1}+\vec{F_2}| \leq 8 \Rightarrow |\vec{F_1}+\vec{F_2}|$ can be equal to $|\vec{F_3}|=6$.

In such a case, if $\angle$ b/w $(\vec{F}+\vec{F_3})$ and $(\vec{F_3})$ is π, we get :-

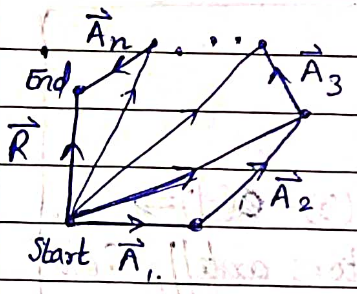
$$\boxed{\vec{F_1}+\vec{F_2}+\vec{F_3}=\vec{0}}$$

2. **||gm Law** — This is for co-initial vectors



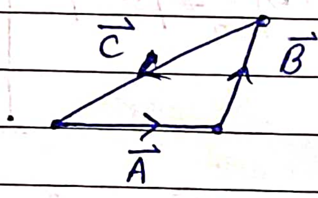
If	$A = B \Rightarrow$	$\alpha = \beta$
If	$A > B \Rightarrow$	$\alpha > \beta$
If	$A < B \Rightarrow$	$\alpha < \beta$

3. **Poly^n Law** —



$\vec{R} = (\vec{A}_1 + \ldots + \vec{A}_n)$

Join head to tail of succesive vectors.



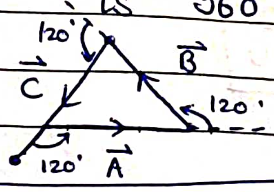
$\vec{A} + \vec{B} + \vec{C} = \vec{0}$ null vector

(Magnitude = 0
Dirx^n = Unspecified)

☆ If closed poly^n formed $\Rightarrow$ Sum of $\angle$ b/w consecutive vectors is 360°.

Note: Sum of $\angle$ b/w consecutive vectors is 360° $\not\Rightarrow$ closed poly^n formed

Eg:



$\vec{A} + \vec{B} + \vec{C} \neq \vec{0}$

$\boxed{\text{\&} \ (A_1 \wedge A_2) = (A_2 \wedge A_3) = \ldots = (A_n \wedge A_1)}$

Date: ______ Page: ______

★ If $|\vec{A_1}| = |\vec{A_2}| = \ldots = |\vec{A_n}|$ & $\sum_{i=1}^{n-1} (\vec{A_i} \wedge \vec{A_{i+1}}) + (\vec{A_n} \wedge \vec{A_1}) = 360°$,

then a closed polyⁿ is formed.

Q) 'N' forces of mag. 'F' are acting on a body. Angle b/w any 2 consecutive vectors is '$2\pi/N$'. Find resultant force.

A) Mag. of all forces equal ? ⇒ ✓

Sum of Angles $= N\left(\frac{2\pi}{N}\right) = 2\pi = 360°$ ⇒ ✓

All angles equal ? ⇒ ✓

Therefore, resultant is $\vec{0}$

Q) 'N-1' forces of mag. 'F' are acting on a body. Angle b/w any 2 consecutive vectors is '$2\pi/N$'. Find magnitude of resultant.

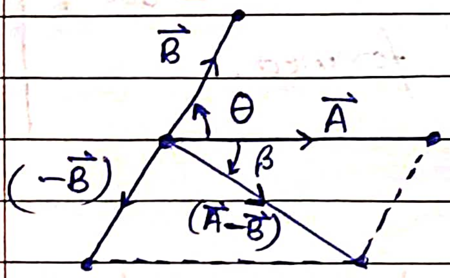
A) Let $\vec{F_i}$ be the ith force. If there were 'N' forces as in above Q, we would have.

$$\vec{F_1} + \ldots + \vec{F_N} = \vec{0}$$

$$\Rightarrow \vec{F_1} + \ldots + \vec{F_{N-1}} = -\vec{F_N}$$

$$\Rightarrow |\vec{F_1} + \ldots + \vec{F_{N-1}}| = |-\vec{F_N}| = F$$

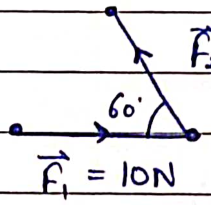
Subtraction of Vectors –



$$|\vec{A}-\vec{B}| = \sqrt{A^2+B^2-2AB\cos(\theta)}$$

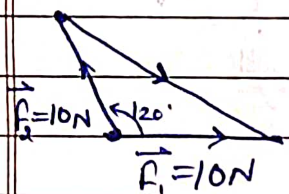
$$\tan(\beta) = \left(\frac{B\sin(\theta)}{A-B\cos(\theta)}\right)$$

Q)



$|\vec{F_1}-\vec{F_2}| = ?$

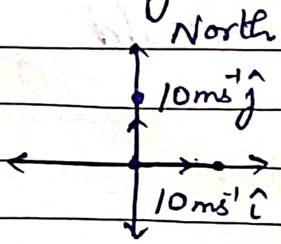
A)



$$|\vec{F_1}-\vec{F_2}| = \sqrt{10^2+10^2-2\cdot10\cdot10\cos(120°)}$$
$$= 10\sqrt{3}$$

Q) Car moving towards East turns to the North w/o changing speed. If its speed is $10ms^{-1}$, find change in velocity.

A)

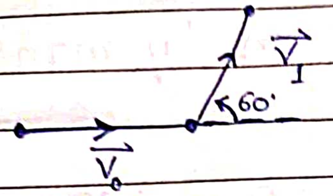


$$\vec{v_1} = 10\,ms^{-1}\,\hat{j}$$
$$\vec{v_0} = 10\,ms^{-1}\,\hat{i}$$

$$\boxed{\Delta\vec{v}} = \vec{v_1} - \vec{v_0} = \boxed{(10ms^{-1})(-\hat{i}+\hat{j})}$$

Q) A body is moving East $10 ms^{-1}$. It turns by 60° to North. find $\Delta\vec{v}$ (w/o changing speed)

A)



$\vec{v}_0 = 10\hat{i}$

$\vec{v}_1 = 5\hat{i} + 5\sqrt{3}\hat{j}$

$(\vec{v}_1 - \vec{v}_0) = \boxed{-5\hat{i} + 5\sqrt{3}\hat{j} = \Delta\vec{v}}$

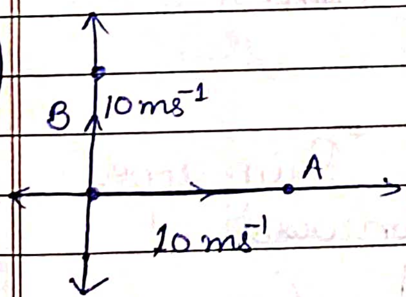
$|\vec{v}_1 - \vec{v}_0| = 10\ ms^{-1}$; $\Delta = 60^\circ$ North of East.

1) Relative Velocity:

$\vec{V}_{A,B}$ = Velocity of $\vec{A}$ w.r.t $\vec{B}$ (observer is at $\vec{B}$)

$$\boxed{\vec{V}_{A,B} = \vec{V}_A - \vec{V}_B}$$

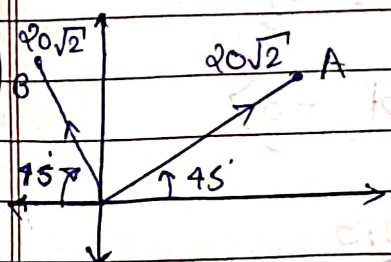
Q) Find $\vec{V}_{A,B}$



A) $\vec{V}_A = 10\hat{i}$ $\vec{V}_{A,B} = (10\hat{i} - 10\hat{j})$

$\vec{V}_B = 10\hat{j}$

Q) find $\vec{V}_{A,B}$



A) $\vec{V}_A = 20\langle 1, 1\rangle$

$\vec{V}_B = 20\langle -1, 1\rangle$

$\vec{V}_{A,B} = 20(\langle 1,1\rangle - \langle -1,1\rangle) = 20\langle 2,0\rangle$

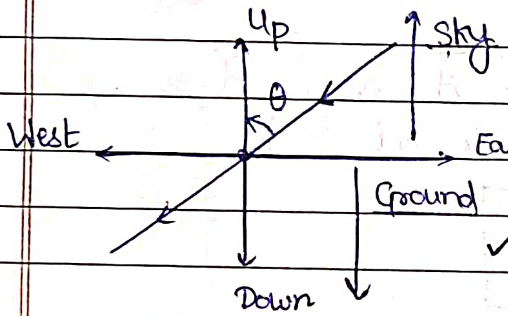
i) Rain-Man Problems:

We need ($\vec{V}_{rM}$), how rain appears to fall as seen by man.

Q) Person moving East $4\,ms^{-1}$. Rain falling vertically down, $3\,ms^{-1}$ speed. Find $\vec{V}_{rM}$

A) $\vec{V}_r = \langle 0, -3 \rangle$ $\qquad$ $\vec{V}_M = \langle 4, 0 \rangle$

$$\vec{V}_{rM} = \vec{V}_r - \vec{V}_M = \langle -4, -3 \rangle$$



$\tan(\theta) = 4/3$

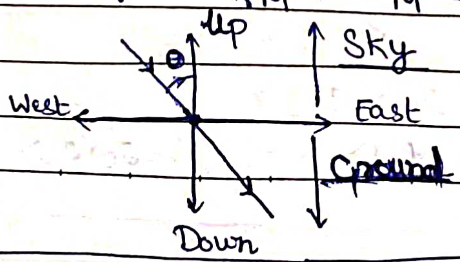
$\Rightarrow \theta \approx 53^\circ$

✓ Rain falling 53° east of Vertical. w.r.t. Man.

Q) Person moving East $4\,ms^{-1}$. Rain appears to fall in vertically downward dirx^n with $3\,ms^{-1}$. Find $\vec{V}_r$

A) $\vec{V}_M = \langle 4, 0 \rangle$ $\qquad$ $\vec{V}_{rM} = \langle 0, -3 \rangle$

$$\vec{V}_r = \vec{V}_{rM} + \vec{V}_M \Rightarrow \vec{V}_r = \langle 4, -3 \rangle$$



$\tan(\theta) = 4/3$

$\Rightarrow \theta \approx 53^\circ$ West of Vertical

Q) Person moving East $4ms^{-1}$. Rain appears to fall in vertically down dirx^n. When he doubles his speed, rain appears to come at 45° with vertical. Find velocity of rain.

A) $\vec{V}_{M1} = \langle 4, 0 \rangle$ $\quad$ $\vec{V}_{rM_1} = \vec{V}_r - \vec{V}_{M1}$

$\Rightarrow \vec{V}_r = \vec{V}_{rM_1} + \langle 4, 0 \rangle$

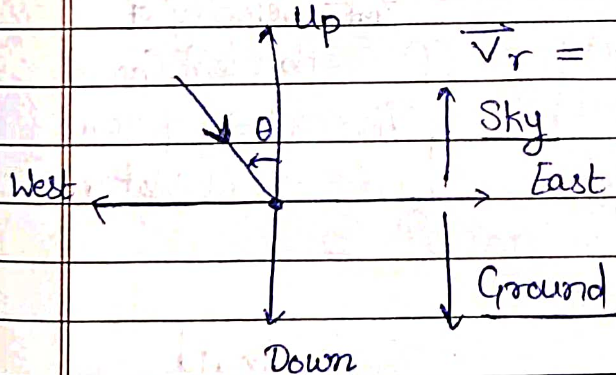
$\vec{V}_{M2} = \langle 8, 0 \rangle$ $\quad$ $\vec{V}_{rM_2} = \vec{V}_r - \vec{V}_{M2}$

$\Rightarrow \vec{V}_r = \vec{V}_{rM_2} + \langle 8, 0 \rangle$

$\vec{V}_{rM_1} = \vec{V}_{rM_2} + \langle 4, 0 \rangle \Rightarrow \langle 0, -a \rangle = \langle -b, -b \rangle + \langle 4, 0 \rangle$

$\Rightarrow \langle 0, -a \rangle = \langle 4-b, -b \rangle$

$\Rightarrow a = b$ and $b = 4$

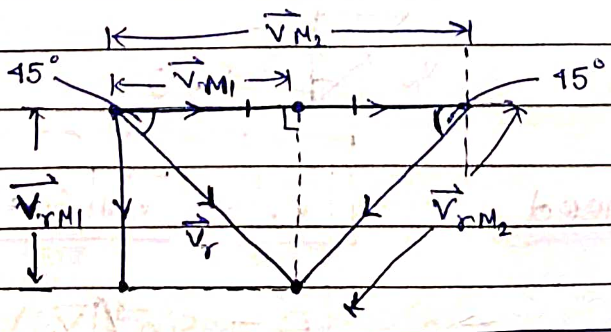


$\vec{V}_r = \langle 0, -4 \rangle + \langle 4, 0 \rangle = \langle 4, -4 \rangle$

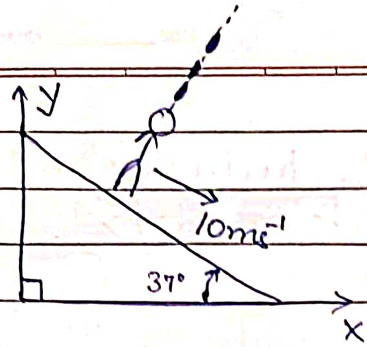
$\tan(\theta) = 1$

$\Rightarrow \theta = 45°$ West of Vertical

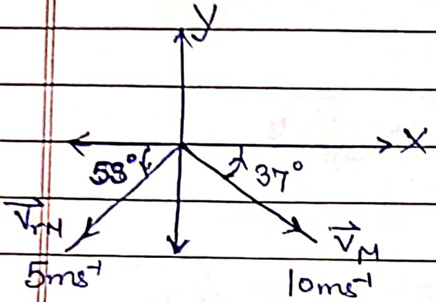
Sir's Sol^n:



$|\vec{V}_{M1}| = |\vec{V}_r| \cos(45°) \Rightarrow \boxed{|\vec{V}_r| = 4\sqrt{2} \quad \text{dirx}^n: \langle 1, -1 \rangle}$

Q) 

Rain appears to fall $5\,ms^{-1}$ in vertical dirx^n to person. Find $\vec{v}_r$.

A 

$\vec{V}_M = 10\langle\cos(37°), -\sin(37°)\rangle$

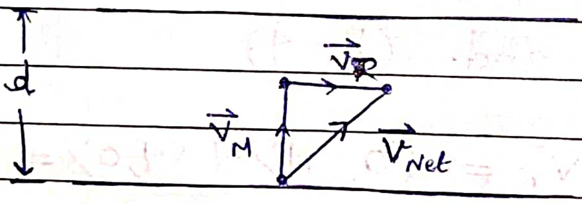
$\vec{V}_{rM} = 5\langle-\cos(53°), -\sin(53°)\rangle$

$\vec{V}_r = \vec{V}_M + \vec{V}_{rM}$

$= \langle 8, -6\rangle + \langle -3, -4\rangle$

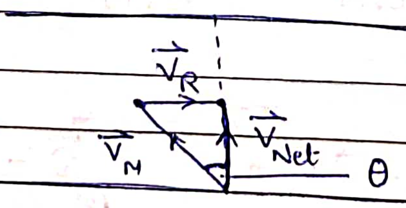
$\Rightarrow \boxed{\vec{V}_r = \langle 5, -10\rangle}$

ii) River-Boat Problems :



$\vec{V}_R$ = Velocity of River
$\vec{V}_{NH}$ = Velocity of Man w.r.t. Ground
$\vec{V}_M$ = Velocity of Man w.r.t Water.

C-1: Shortest Path.



'$\vec{V}_{Net}$' should be straight.

We need $\boxed{|\vec{V}_r| = |\vec{V}_M|\sin(\theta)}$

$\Rightarrow \boxed{\theta = \sin^{-1}\left(\frac{|\vec{V}_R|}{|\vec{V}_M|}\right)}$ with normal along upstream

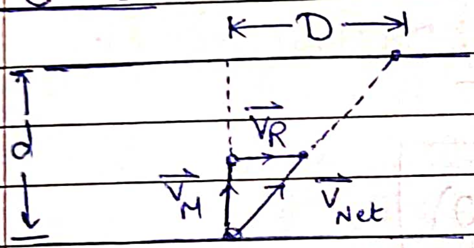
Time to Cross River $= \left(\dfrac{d}{|\vec{v}_M|\cos(\theta)}\right)$

$$= \boxed{\left(\dfrac{d}{\sqrt{v_M^2 - v_R^2}}\right)}$$

Velocity of Person w.r.t Ground $= (v_M \cos(\theta))$

$$= \boxed{\sqrt{v_M^2 - v_R^2}}$$

C-2: Shortest Time.



'$\vec{v}_M$' should be straight.

Time to Cross River $= \boxed{\left(\dfrac{d}{v_M}\right)}$

Drift $= D = \boxed{\left(\dfrac{v_R}{v_M}\right)(d)}$

Velocity of Person w.r.t. Ground $= \boxed{\sqrt{v_M^2 + v_R^2}}$

If $(v_R > v_M)$, then min. drift when

$\boxed{\theta = \sin^{-1}\left(\dfrac{v_M}{v_R}\right)}$ (Using diff.)

Date: ______ Page: ______

Multiplication of Vectors –

1. With Scalar : $\boxed{\vec{B} = \lambda \vec{A}}$

$\lambda > 0$	$\lambda < 0$
$\vec{B} \parallel \vec{A}$	$\vec{B}$ anti $\parallel \vec{A}$

2. With Vector :

i) Scalar / Dot Product –

$$\boxed{\vec{A} \cdot \vec{B} = AB \cos(\theta)}$$

$\theta \rightarrow \angle$ b/w $\vec{A}$ & $\vec{B}$

$\theta = \pi/2$, $\vec{A} \perp \vec{B}$	$\theta = 0$, $\vec{A} \parallel \vec{B}$
$\vec{A} \cdot \vec{B} = 0$	$\vec{A} \cdot \vec{B} = AB$ $\Rightarrow \vec{A} \cdot \vec{A} = A^2$

Q) $\vec{A} = \langle 3, -4, 5 \rangle$
$\vec{B} = \langle 1, 2, -1 \rangle$
$\vec{A} \cdot \vec{B} = ?$

A) $\vec{A} \cdot \vec{B} = 3 \cdot 1 + (-4) \cdot 2 + 5 \cdot (-1)$
$= 3 - 8 - 5 = -10$

Usage —

i) To find ∠ b/w vectors : $\boxed{\cos(\theta) = \left(\frac{\vec{A}\cdot\vec{B}}{AB}\right)}$

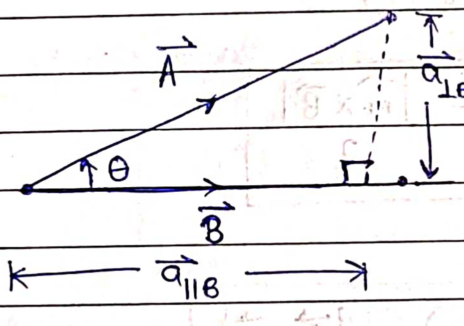
Q) $\vec{A} = \langle 3,4,0\rangle$; $\theta = ?$
$\vec{B} = \langle 1,-1,0\rangle$

A) $\cos(\theta) = \frac{3\cdot1 - 4\cdot1}{5\cdot\sqrt{2}}$

$= \left(-1/5\sqrt{2}\right)$

---

ii) Projection/Component of a vector on another :

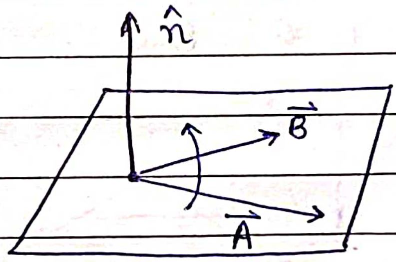


$\boxed{\vec{A} = \vec{a}_{\parallel B} + \vec{a}_{\perp B}}$

$\boxed{\vec{a}_{\parallel B} = \left(\frac{\vec{A}\cdot\vec{B}}{B}\right)\left(\frac{\vec{B}}{B}\right)}$

23/03/2022

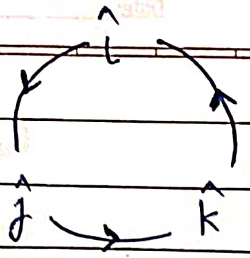
ii) Cross Product —



$\boxed{\vec{A}\times\vec{B} = AB\sin(\theta)\ \hat{n}}$ → dirx^n

'$\hat{n}$' is normal to plane containing $\vec{A}$ & $\vec{B}$.

⇒ $(\vec{A}\times\vec{B}) \perp \vec{A}$ & $(\vec{A}\times\vec{B}) \perp \vec{B}$



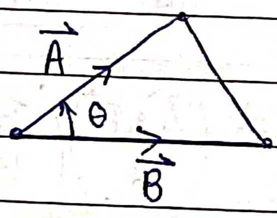
$$\boxed{\begin{aligned} \hat{i} \times \hat{j} &= \hat{k} \\ \hat{j} \times \hat{k} &= \hat{i} \\ \hat{k} \times \hat{i} &= \hat{j} \end{aligned}}$$

Q) $\vec{A} = \langle 3, 4, 1 \rangle$
$\vec{B} = \langle 1, -3, 4 \rangle$
$\vec{A} \times \vec{B} = ?$

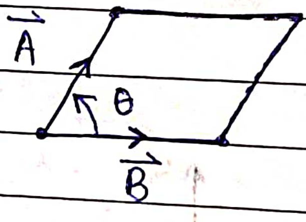
A) $$\begin{array}{c} \begin{matrix} (+) & (-) & (+) \end{matrix} \\ \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 1 \\ 1 & -3 & 4 \end{vmatrix} \end{array} = \langle 19, -11, -13 \rangle$$

Usage –

i) <u>To find area of Δ & $||gm$:</u>



$$\boxed{\begin{pmatrix} \text{Area} \\ \text{of } \Delta \end{pmatrix} = \frac{|\vec{A} \times \vec{B}|}{2}}$$



$$\boxed{\begin{pmatrix} \text{Area} \\ \text{of } ||gm \end{pmatrix} = |\vec{A} \times \vec{B}|}$$