# Waves

**Wave** — It is propogation of energy & momentum from one one place to another w/o transport of mass.

Waves
- Mechanical
  - Medium req. for propogation
  - Eg – Sound
- Non-Mechanical
  - Medium NOT req for propogation
  - Eg – EM waves

**Transverse Wave** — (Disp. of particle) ⊥ (Disp. of wave)

Observed in Solid & on Surface of Liq.

**Longitudnal Wave** — (Disp. of particle) || (Disp. of Wave).

Observed in Solid, Liq. & Gas

Travelling / Progressive Wave — Disturbance once created moves upto $\infty$.

Eqn. of Travelling Wave

$$\boxed{y = f(ax \pm bt)} \qquad \{a, b \neq 0\}$$

$y$ — (Disp. of particle)

$x$ — (Posn. of particle along wave's axis of propagation) i.e (Disp. of wave)

Also, $y$ should be finite ; $\forall\ x, t$

Eg: $y = \dfrac{1}{1+(x-vt)^2}$ ✓, $y = e^{-(x-vt)^2}$ ✓, $y = \tan(x-vt)$ ✗,

$y = \ln(x-vt)$ ✗, $y = A\sin(\omega t - kx)$ ✓, ...

If 'f' is Periodic fxn. $\Rightarrow$ Plane Progressive Wave

DATE
PAGE

## Plane Progressive Wave

$$\boxed{y = A\sin(\omega t - kx)}$$

$\omega = \left(\frac{2\pi}{T}\right)$ , $k = \left(\frac{2\pi}{\lambda}\right)$ , $\left(\frac{\omega}{k}\right) = \left(\frac{\lambda}{T}\right)$

(Time period of particles in SHM)

(Propogation Const.)

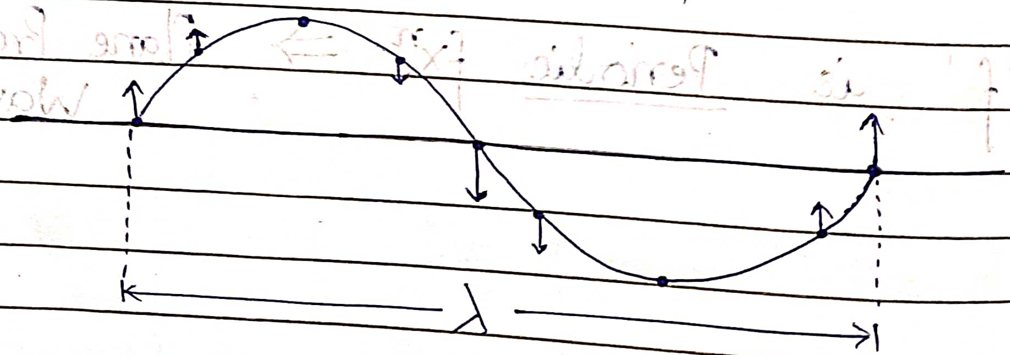
(Wavelength of Wave)

Now, $v = \lambda \nu = \frac{\lambda}{T} = \frac{\omega}{k}$

$$\Rightarrow \boxed{v = \left(\frac{\omega}{k}\right)} ; \boxed{v = \left(\frac{\partial x}{\partial t}\right)}$$

In a given medium, vel. is const.

Wavelength ($\lambda$) — Dist. b/w 2 particles vibrating in same phase



Particles travel with same amplitude, same time period but diff. phase.

$$\boxed{(\text{Phase Diff.}) = \left(\frac{2\pi}{\lambda}\right)(\Delta x)}$$

(b/w any 2 particles of wave) (Dist. b/w particles)

Now,
$$y = A \sin(\omega t - kx)$$

$$\Rightarrow \left(\frac{\partial y}{\partial t}\right) = A \cos(\omega t - kx) \cdot \omega$$

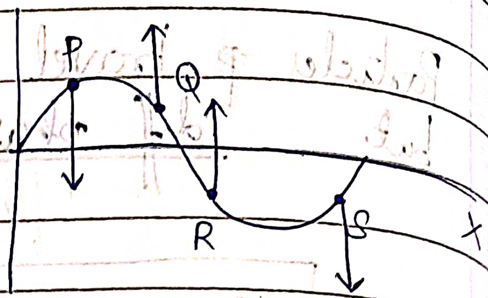
$$\Rightarrow \boxed{v_p = \omega A \cos(\omega t - kx)}$$

Now,
$$\left(\frac{\partial y}{\partial x}\right) = (-k) A \cos(\omega t - kx)$$

$$\Rightarrow v_p \Big/ \left(\frac{\partial y}{\partial x}\right) = \left(\frac{-\omega}{k}\right) = (-v)$$

$$\Rightarrow \boxed{v_p = (-v)\left(\frac{\partial y}{\partial x}\right)}$$

Q) Wave moving in +X dirx$^n$.
Find dirx$^n$ of motion of P, Q, R, S.




A) Given $v > 0$. Now, $\left(\frac{\partial y}{\partial x}\right)_{P \& S} > 0$ & $\left(\frac{\partial y}{\partial x}\right)_{Q \& R} < 0$ <

$\Rightarrow$ $V_{particle}$ : $(P \& S) < 0$ & $(Q \& R) > 0$

$\Rightarrow$ P & S move ~~Up~~ Down
Q & R move ~~Down~~ Up

Now, $a_p = \left(\frac{\partial v_p}{\partial t}\right) = \left(\frac{\partial}{\partial t}\right)\left(-v \cdot \partial y/\partial x\right)$

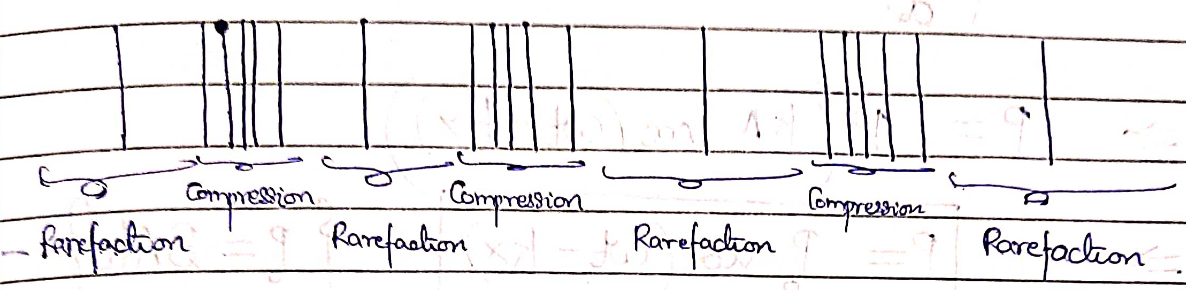
$$= (-v)\left(\frac{\partial}{\partial t}\left(\partial y/\partial x\right)\right)$$

$$= (-v)\left(\frac{\partial^2 y}{\partial x^2}\right)\left(\frac{\partial x}{\partial t}\right)$$

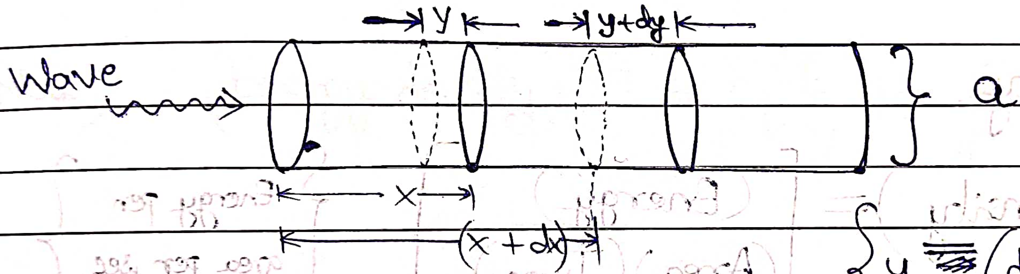
$$\Rightarrow \boxed{a_p = \left(v^2\right)\left(\frac{\partial^2 y}{\partial x^2}\right)}$$

# Sound

It is longitudinal wave in air.




Since $P$ is changing $\Rightarrow$ Pressure Wave




$\{ y = $ (disp. of air layer at dist. 'x') $\}$

$$\left.\begin{aligned}(\text{Init. Vol.}) &= a \cdot dx \\ (\text{Final Vol.}) &= a\,(dx + dy)\end{aligned}\right\} \quad (\Delta V) = a \cdot dy$$

$$(\text{Strain}) = \left(\frac{-\Delta V}{V}\right) = \left(\frac{-a\,dy}{a\,dx}\right) \Rightarrow \epsilon = \left(\frac{-dy}{dx}\right)$$

Now, $B = \dfrac{P}{\left(\dfrac{-\Delta V}{V}\right)} \Rightarrow \boxed{P = B\left(\dfrac{-dy}{dx}\right)}$

Now, $y = A\sin(\omega t - kx)$

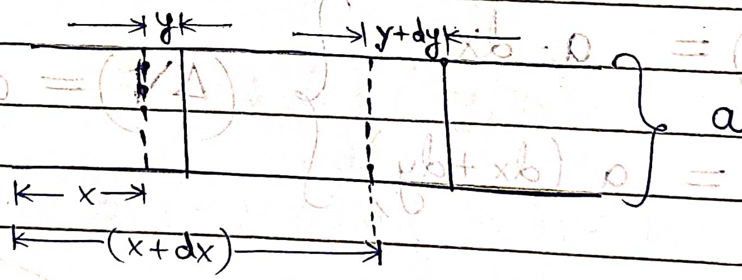
$$\Rightarrow \left(\frac{dy}{dx}\right) = (-kA)\cos(\omega t - kx)$$

$$\Rightarrow P = B\left(kA\cos(\omega t - kx)\right)$$

$$\Rightarrow \boxed{P = \underbrace{P_0}_{\text{(Pressure amplitude)}}\cos(\omega t - kx)} \quad \{P_0 = BAk\}.$$

Intensity

$$\left(\text{Intensity}\right) = \left[\frac{(\text{Energy})}{(\text{Area})(\text{Time})}\right] \quad \left\{\begin{array}{l}\text{Energy per}\\ \text{area per sec.}\end{array}\right\}$$




$(\text{Init. Energy}) = u \cdot a \cdot dx \qquad (\text{Final Energy}) = u \cdot a \cdot (dy + dx)$

$$\Rightarrow \left(\begin{array}{c}\text{Rate of Flow}\\ \text{of Energy}\end{array}\right) = u \cdot a \cdot \left(\frac{dy}{dt}\right) = u \cdot a \cdot v = \frac{1}{2}\rho\omega^2A^2\, a \cdot v$$

$$\Rightarrow \boxed{(\text{Intensity}) = \frac{1}{2}\rho\omega^2A^2v = \left(\frac{P_0^2}{2\rho v}\right)}$$

where $\boxed{v = \sqrt{B/\rho}}$

## Energy of Wave

$$V_p = \omega A \cos(\omega t - kx)$$

$$\Rightarrow KE = \frac{1}{2} m V_p^2 = \frac{1}{2} m \omega^2 A^2 \cos^2(\omega t - kx)$$

$$\Rightarrow KE_{max} = \frac{1}{2} m \omega^2 A^2$$

$$\Rightarrow \boxed{E_{wave} = \frac{1}{2} m \omega^2 A^2}$$

(Total Energy)

Now, $(\text{Energy density}) = u = \left(\frac{E}{V}\right) = \frac{1}{2}\left(\frac{m}{V}\right)\omega^2 A^2$

$$\Rightarrow \boxed{u = \frac{1}{2} \rho \omega^2 A^2}$$

Q) $y_1 = A\sin(2\omega t - kx)$, $y_2 = 2A \sin(\omega t - 2kx)$

find $I_1 / I_2$.

A) $\left(I_1 / I_2\right) = \left[\frac{1/2\, \rho\, (2\omega)^2 \cdot A^2 \cdot (2\omega/k)}{1/2\, \rho\, \omega^2 \cdot (2A)^2 \cdot (\omega/2k)}\right] = 4$

## Sound Level

$$\boxed{(S.L.) = 10 \log_{10}\left(\frac{I}{I_0}\right)}$$

Units : dB (decibals)

$I_0 = $ (Threshold freq. for human ear) $= 10^{-12}\ W/m^2$

Since $I \propto A^2 \Rightarrow \boxed{(S.L.) = 20 \log_{10}\left(\frac{A}{A_0}\right)}$

Q) $10\ dB + 10\ dB = \ ?\ dB.$

A) $10 = 10 \log_{10}\left(\frac{I}{I_0}\right) \Rightarrow I = 10 I_0$

Now, $I_{new} = 2I = 20 I_0$

$\Rightarrow 10 \log_{10}\left(\frac{I_{new}}{I_0}\right) = (10)(1 + \log_{10}(2))$

$\Rightarrow \boxed{(S.L)_{new} \approx 13\ dB}$

# Speed of Wave in Gas

$$\boxed{v = \sqrt{\frac{(\text{Elasticity of Medium})}{(\text{Density of Medium})}}}$$

~~In air/gas,~~

**Newton's formula:** $v = \sqrt{\frac{P}{\rho}} \approx 280\ ms^{-1}$ (for air) ✗

(assuming propogation of wave is IT process)

$PV = \text{Const.} \Rightarrow P\Delta V + \Delta P\, V = 0 \Rightarrow \frac{\Delta P}{(-\Delta V/V)} = P = B$

**Laplace Correction:** $v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}} \approx 343\ ms^{-1}$ (for air) ✓

(assuming propogation of wave is AdB process)

$PV^{\gamma} = \text{Const.} \Rightarrow \Delta P \cdot V^{\gamma} + \gamma \cdot P \cdot V^{(\gamma-1)}\Delta V = 0 \Rightarrow \frac{\Delta P}{(-\Delta V/V)} = \gamma P = B$

If $P$ change $\Rightarrow$ No effect on $v$

(at a given temp.) (as $P\uparrow \Rightarrow \rho\uparrow$ & $RT/M = \text{Const.}$)

If $T$ change $\Rightarrow \left(\frac{v_{t^\circ C}}{v_{0^\circ C}}\right) = \sqrt{\frac{273+t}{273}} \approx \left(1 + t/546\right)$

$\Rightarrow \Delta v = \left(\frac{v_{0^\circ C}}{546}\right) t$

$\Rightarrow \boxed{\Delta v \approx (0.6)\, t}$ $(ms^{-1})$

(as $(H_2O)_M = 18$)

If humidity,

| Dry Air | Moist Air |
| --- | --- |
| $M.W = 29$ | $M.W < 29$ |
| $\gamma = 1.41$ | $\gamma < 1.41$ |

(as $(H_2O)_\gamma = 1.33$)

M.W dec. MORE compared to $\gamma$. $\Rightarrow$ $\gamma/M$ inc.

$\Rightarrow$ Vel. of Wave INC. with humidity.

Q) Speed of wave in $O_2 = 300\ ms^{-1}$ at $800\ °C$ find speed of wave in mix. $O_2 : He = 4:1$ by n at $0°C$

A) $M_{mix} = \left(\dfrac{4(32) + 1(4)}{4+1}\right) \Rightarrow M_{mix} = \cancel{26}\ (132/5)$

~~$\gamma_{mix} = \left(\dfrac{5(4)+3(1)}{4+1}\right) \Rightarrow \gamma_{mix} = \dfrac{23}{5}$~~

Now, $\left(\dfrac{v_{new}}{v_{old}}\right) = \sqrt{\dfrac{T_{new}}{T_{old}} \cdot \dfrac{\gamma_{new}}{\gamma_{old}} \cdot \dfrac{M_{old}}{M_{new}}}$

$= \sqrt{\dfrac{273}{573} \cdot \dfrac{33/23}{7/5} \cdot \dfrac{32}{132/5}}$

$\Rightarrow$ $\boxed{v_{new} = (300)\sqrt{\dfrac{273 \cdot 25.8}{573 \cdot 23 \cdot 7}}}$

$\left\{ f_{mix} = \left(\dfrac{5(4)+3(1)}{4+1}\right) = \dfrac{23}{5} \Rightarrow \gamma_{mix} = 1 + \dfrac{2 \cdot 5}{23} \Rightarrow \gamma_{mix} = \left(\dfrac{33}{23}\right) \right.$

## Speed of Wave in Solid

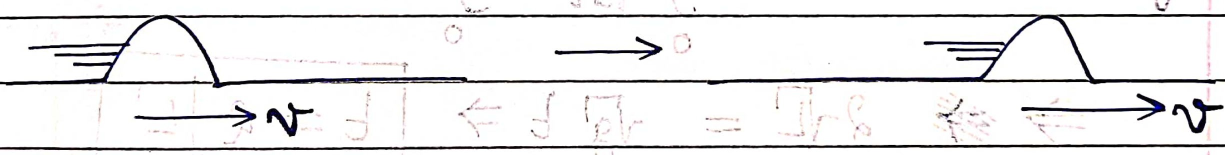
Transverse $\longrightarrow$ $\boxed{v = \sqrt{\dfrac{\eta}{\rho}}}$

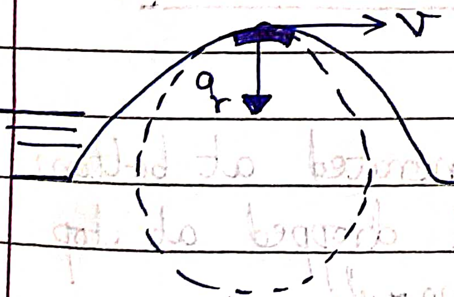
Longitudinal $\longrightarrow$ $\boxed{v = \sqrt{\dfrac{Y}{\rho}}}$

## Speed of Wave in Liq.

$$\boxed{v = \sqrt{\frac{B}{\rho}}}$$

## Speed of Wave in a Stretched String



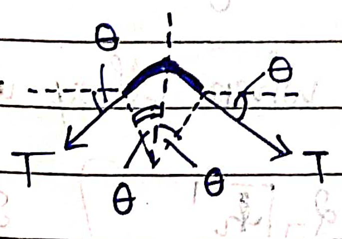




$$\boxed{v = \sqrt{\frac{T}{\mu}}}$$

(Tension in string)

(Mass per unit length)

FBD :

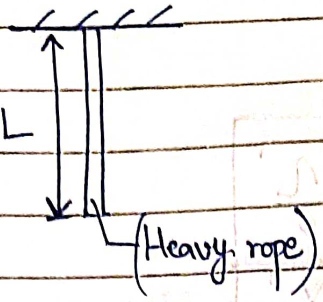



$$m = \mu R \cdot 2\theta$$

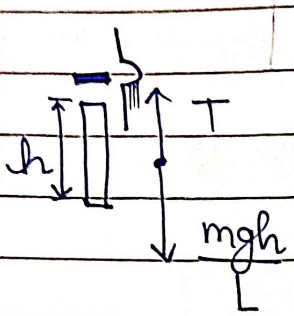
$$f_r = 2T \sin(\theta) \approx 2T\theta$$

Now, $a_r = \dfrac{v^2}{R} \Rightarrow \dfrac{(2T\theta)}{(\mu R \cdot 2\theta)} = \dfrac{v^2}{R} \Rightarrow v = \sqrt{T/\mu}$

DATE
PAGE

Q) 

Find vel. of wave at height 'h' from bottom.

A) 

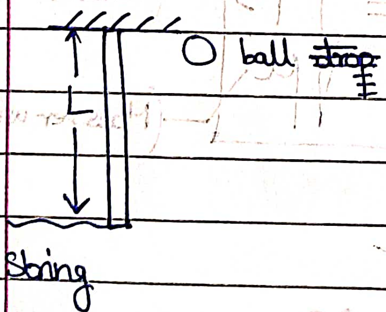
$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{mgh/L}{m/L}}$$

$$\Rightarrow \boxed{v = \sqrt{gh}}$$

Q) In above Q, find time taken by wave to reach top of the rope.

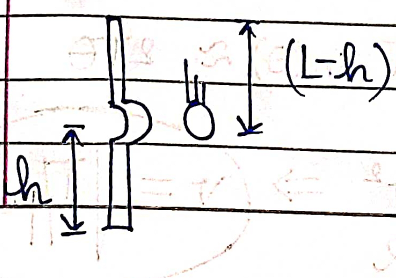
A) $v = \sqrt{gh} \Rightarrow \int_0^L \frac{dh}{\sqrt{h}} = \int_0^t \sqrt{g}\, dt$

$$\Rightarrow 2\sqrt{L} = \sqrt{g}\, t \Rightarrow \boxed{t = 2\sqrt{\frac{L}{g}}}$$

Q) 

Wave generated at bottom & ball dropped at top simultaneously.

Find time when they cross.

A) 

$$t_{wave} = 2\sqrt{\frac{h}{g}}$$

$$t_{ball} = \sqrt{\frac{2(L-h)}{g}}$$

Same

$\Rightarrow 4h = 2(L-h) \Rightarrow h = L/3$

$\Rightarrow \boxed{t = 2\sqrt{L/3g}}$

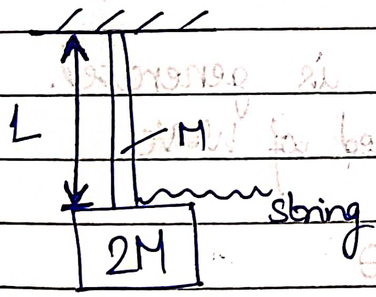
Alternate — $v = \sqrt{gh} \Rightarrow v^2 = gh$

$\Rightarrow 2v\left(\frac{dv}{dh}\right) = g$

$\Rightarrow \boxed{a_{wave} = \left(\frac{g}{2}\right)}$ (upward)

~~Use~~ Now, use rel. acc. Both objs. $V_{init} = 0$.

Q)




Wave generated.

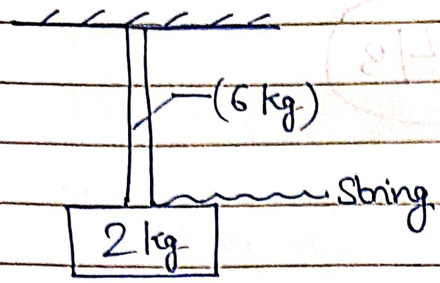
Find time taken to reach top.

A) $v = \sqrt{\frac{(2Mg + M/L \cdot gh)}{(M/L)}} = \sqrt{(2L+h)g}$

$\Rightarrow \int_0^L \frac{dh}{\sqrt{2L+h}} = \int_0^t \sqrt{g}\, dt \Rightarrow (2)(\sqrt{3L} - \sqrt{2L}) = \sqrt{g}\, t$

$\Rightarrow \boxed{t = (2)(\sqrt{3} - \sqrt{2})(\sqrt{L/g})}$

Q)




Wave of wavelength 'λ' is generated.

Find wavelength when it reaches top.

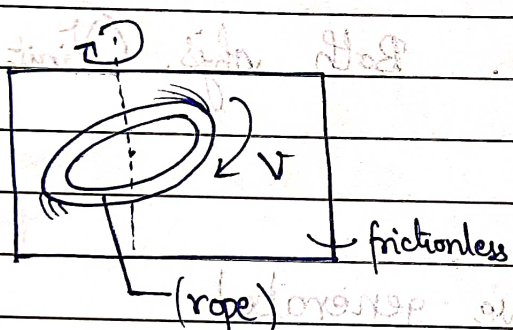
A) $v_0 = \sqrt{\frac{2g}{\mu}}$

$v_1 = \sqrt{\frac{8g}{\mu}}$

$$\left(\frac{v_0}{v_1}\right) = \frac{1}{2} = \left(\frac{\lambda \nu}{\lambda' \nu}\right) \Rightarrow \boxed{\lambda' = 2\lambda}$$

{ $\nu$ is propt of source ⇒ remains same }

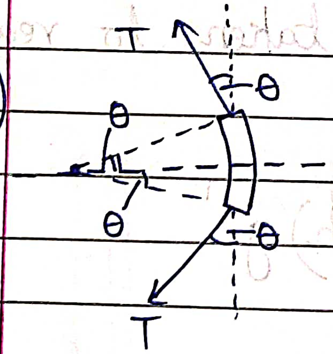
Q)




A rope is rotated on frictionless horiz. surface.

A wave is generated. Find speed of wave

A)




$$m = \mu \cdot R \cdot 2\theta$$

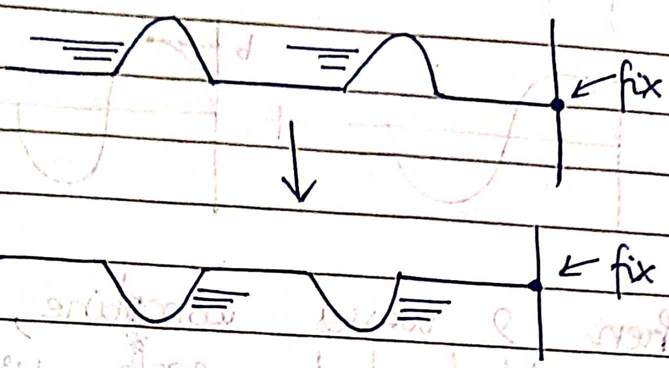
$$\frac{mv^2}{R} = 2T \sin(\theta)$$

$$\Rightarrow \frac{\mu R \cdot 2\theta \cdot v^2}{R} = 2T\theta$$

$$\Rightarrow \left(T/\mu\right) = v^2 \Rightarrow \boxed{v_{wave} = v}$$

# Reflection of Wave

From fixed end,



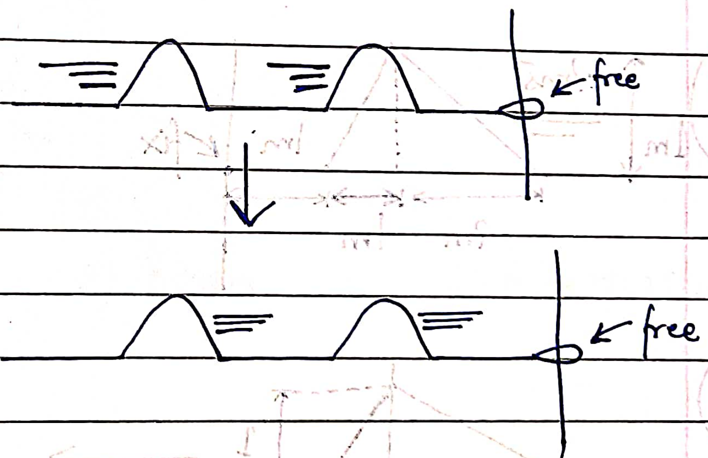

(as fix. pt. exert down force on wave as rxn force.)

$$y_i = A\sin(\omega t - kx) \longrightarrow y_f = A\sin(\omega t + kx + \pi)$$

(as wave ←) (as wave −ve)

$$\Rightarrow y_f = (-A)\sin(\omega t + kx)$$

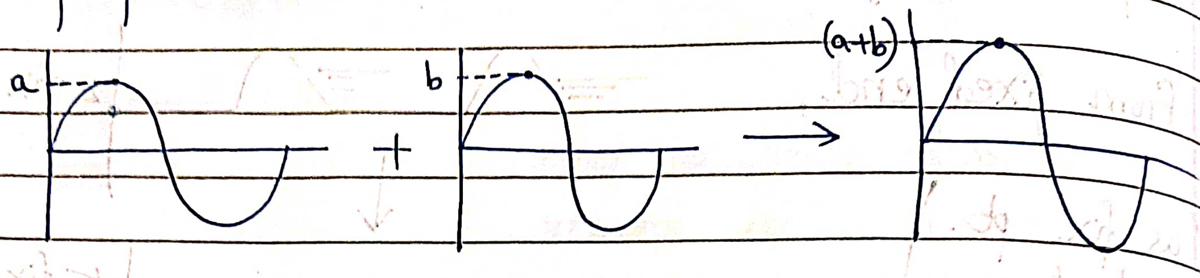
From free end,



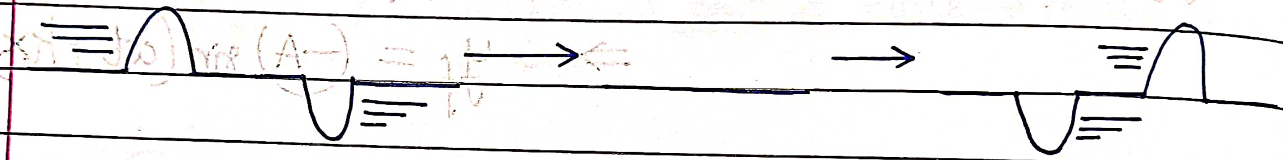

$$y_i = A\sin(\omega t - kx) \longrightarrow y_f = A\sin(\omega t + kx)$$

(as wave ←)

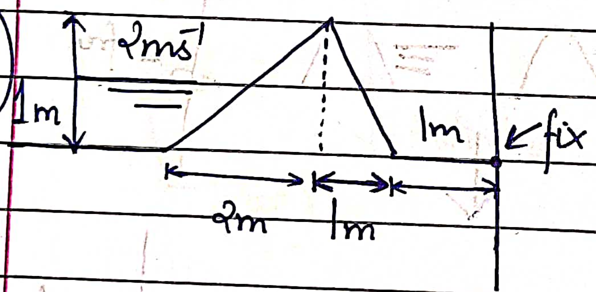
## Superposition




When 2 waves combine, the amplitude is added but each wave's individual prop^ts remain unaffected.

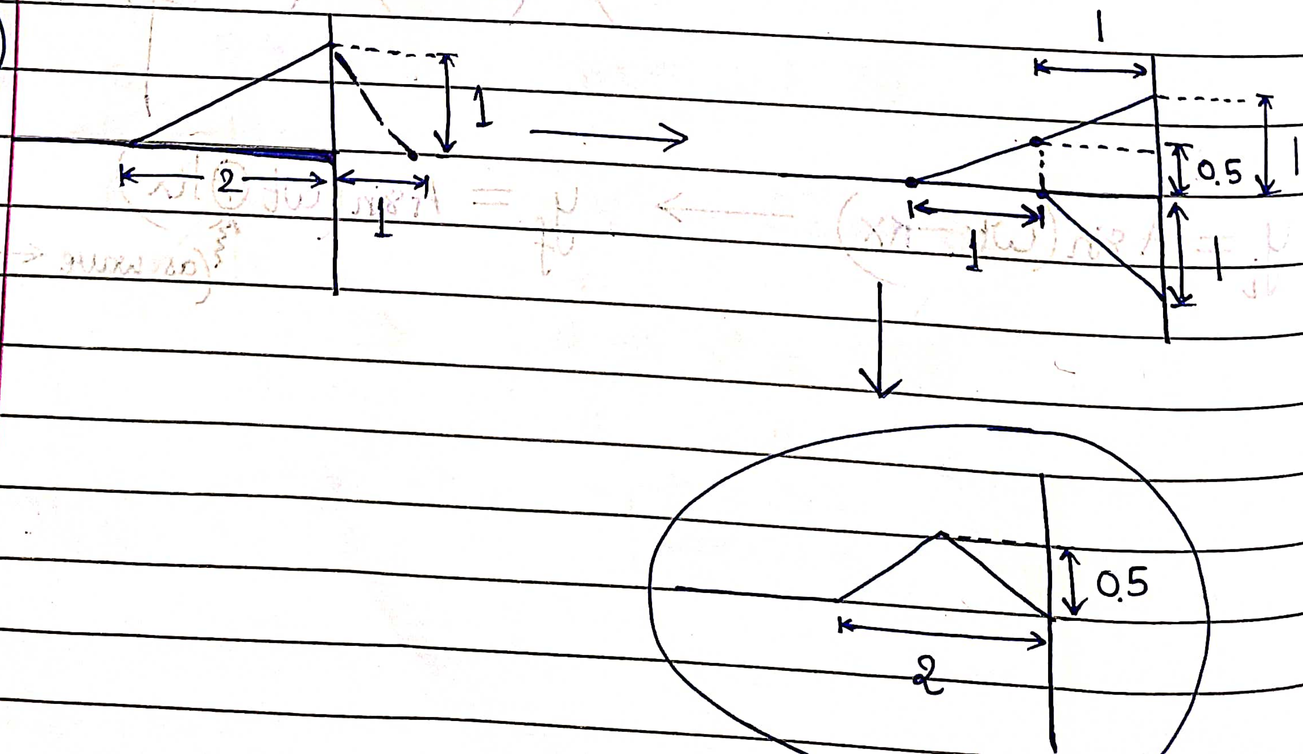


Q)

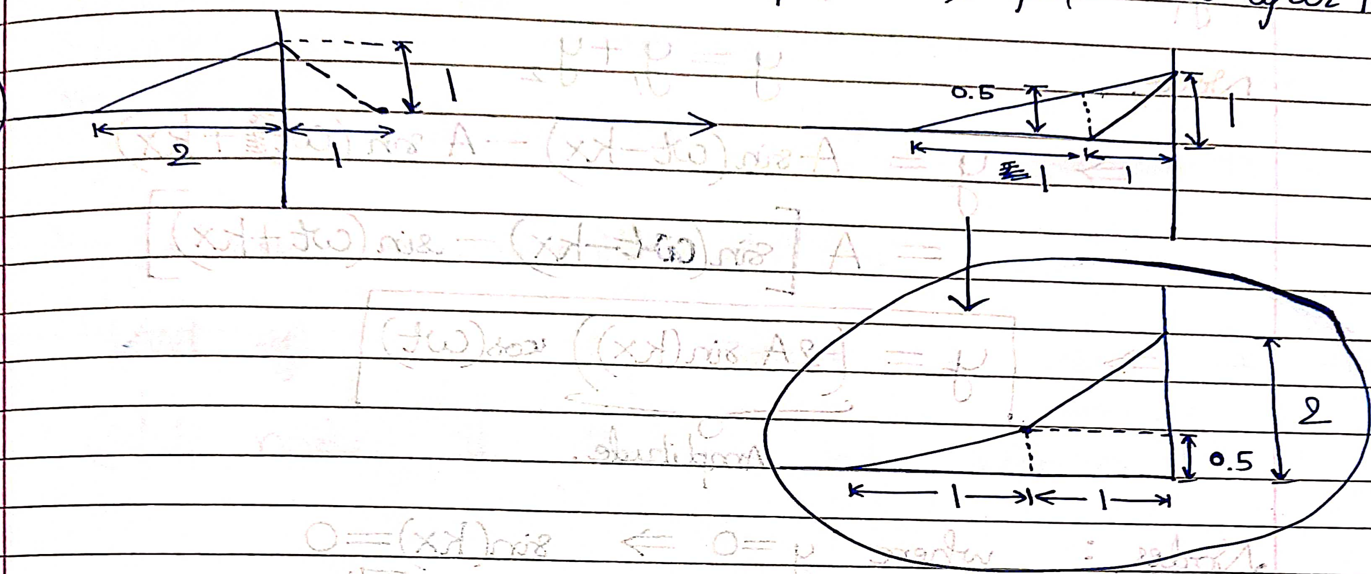



find wave after 1 s.

A)



Q) If in above Q it was free end, find wave after 1 s.

A) 


## Examples of Superposition –

1) Standing Waves 2) Beats 3) Interference

## Standing Waves

2 waves with same amplitude, & wavelength travelling in opp. dirx$^n$ with same speed with a phase diff. of $\pi$.

DATE
PAGE

$y_1 = A\sin(\omega t - kx)$ $\quad$ $y_2 = (-A)\sin(\omega t + kx)$

Now, $\quad y = y_1 + y_2$

$$\Rightarrow y = A\sin(\omega t - kx) - A\sin(\omega t + kx)$$

$$= A\left[\sin(\omega t - kx) - \sin(\omega t + kx)\right]$$

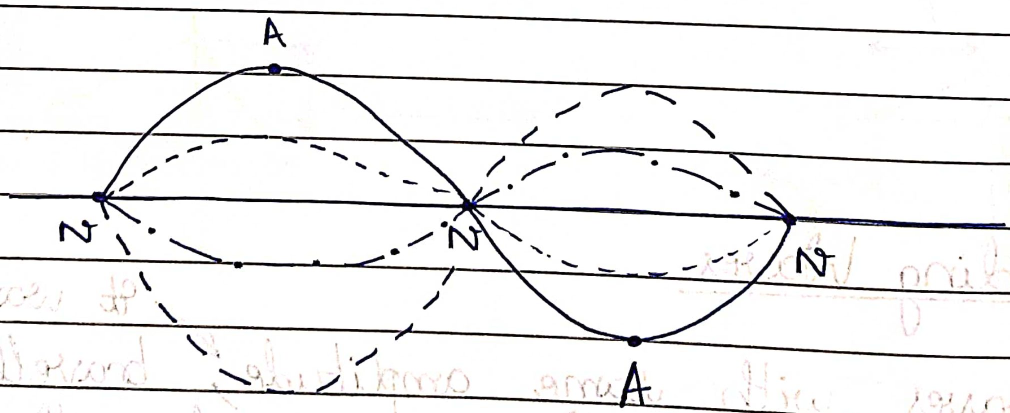
$$\Rightarrow \boxed{y = \underbrace{(-2A\sin(kx))}_{\text{Amplitude.}}\cos(\omega t)}$$

Nodes : where $y=0 \Rightarrow \sin(kx)=0$

$\Rightarrow x = n\pi/k$

$\Rightarrow \boxed{x = \frac{n\lambda}{2}}$ $\quad \{k = 2\pi/\lambda\}$

Antinodes : where $y$ can acheive max. possible value.




B/w any 2 nodes all particles vibrate with same phase but diff. amplitude.

After crossing a node, there is a phase change of $\pi$

Now, $v_p = \cancel{\frac{dy}{dt}} \left(\frac{\partial y}{\partial t}\right) = \overset{2\omega A}{\cancel{2A}} \sin(kx) \sin(\omega t)$

So $v_p = 0$ when $x = n\lambda/2$

$\Rightarrow$ Nodes permanently at rest.

~~(Vel. node & Pressure node coincide)~~

(Vel. node & Particle node coincide)

Pressure wave : $y = 2A \sin(\omega t)$

Pressure variation, $P = -B \frac{\partial y}{\partial x}$

$= B\left(2Ak \cos kx \cos \omega t\right)$

$P_{(node)} = 0$ @ $\cos kx = 0$

Therefore <u>P antinode & disp. node</u> coexist.

Q) $y = (-10)\sin(0.1x)\cos(5t)$. ( x in 'm' & y in 'cm' )

1) Find sep. b/w nodes & antinodes.

A) $(\text{Sep. b/w node \& antinode}) = \frac{\lambda}{4} = \frac{\pi}{2k} = 5\pi$ m

(as x in 'm' $\Rightarrow \lambda$ in 'm')

2) Find speed of wave.

A) $v = \omega/k = 5/0.1 = 50\ ms^{-1}$

3) Find amp. of particle at $x = 2.5\pi$.

A) $y\big|_{x=5\pi/2} = (-10)\sin\left(\frac{5\pi}{2}\cdot\frac{1}{10}\right)\cos(5t) = (-5\sqrt{2})\cos(5t)$

$\Rightarrow (\text{Amp.}) = 5\sqrt{2}$ cm

(as y in 'cm')

Examples of Standing Waves –

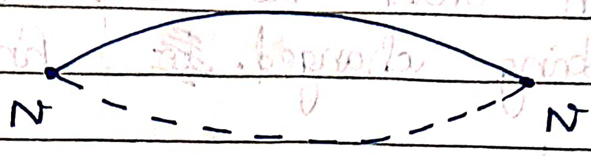
1) Vibrations in Stretched ~~String~~ String.

2) Vibrations in Air Column.

# Vibrations in Stretched String

(fundamental mode)

**First mode of vibration :**

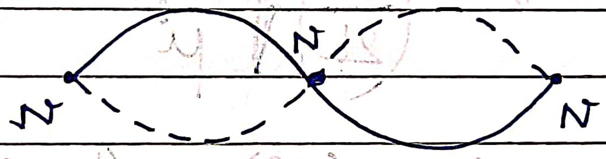



$L = \lambda/2 \Rightarrow \lambda = 2L$

$\nu = v/\lambda \Rightarrow (1/2L)\sqrt{T/\mu} = \nu$ (first harmonic = fundamental freq.)

**Harmonic :-** Set of all freq. emitted by an instrument.

**Second mode of vibration :**




$L = 2(\lambda/2) \Rightarrow \lambda = L$

$\nu = v/\lambda \Rightarrow \nu = (1/L)\sqrt{T/\mu}$ (Second harmonic first overtone)

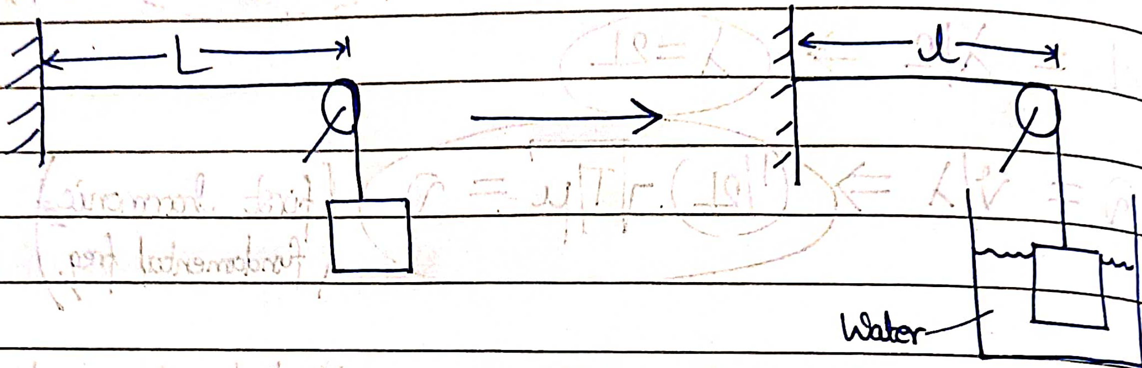
**Overtone :-** freq. higher than fundamental freq.

**nth mode of vibration :** $\boxed{\nu = \left(\dfrac{n}{2L}\right)\sqrt{\dfrac{T}{\mu}}}$

$(\#\text{ Harmonics}) = (\#\text{ Loops}) = n$

$(\#\text{ Overtones}) = (n-1)$

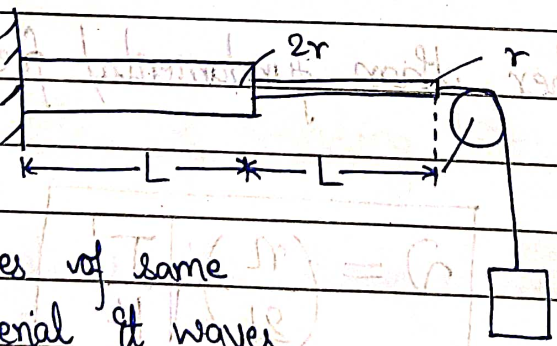
Q) A string is vibrating in fundamental mode. The mass is dipped in water. In order to keep $\nu$ const., length of string changed. Find rel. density of mass.




A) $$\nu = \left(\frac{1}{2L}\right)\sqrt{\frac{\rho V g}{\mu}} = \left(\frac{1}{2l}\right)\sqrt{\frac{(\rho-\rho_w)Vg}{\mu}}$$

$$\Rightarrow \left(\frac{l^2}{L^2}\right) = \left(1-\frac{\rho_w}{\rho}\right) \Rightarrow \boxed{\left(\text{Rel. Density}\right) = \left(\frac{L^2}{L^2-l^2}\right)}$$

Q)



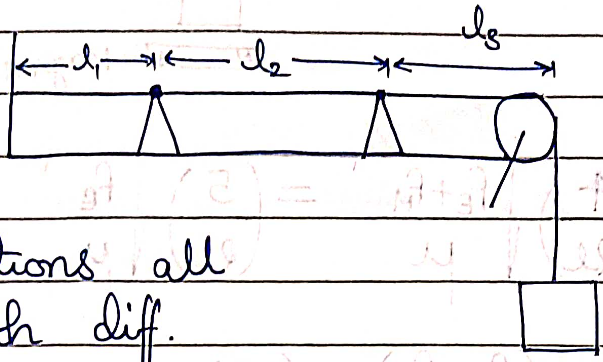

Find ratio no. of loops in which the 2 sections are vibrating.

Ropes of same material & waves of same freq.

A) $$\frac{\mu_1}{\mu_2} = \frac{m_1}{l}\cdot\frac{l}{m_2} = \frac{\rho A_1}{\rho A_2} = 4 \longrightarrow \boxed{n_1/n_2 = 2}$$

Now, $$\frac{\nu_1}{\nu_2} = \left(\frac{n_1}{2l}\right)\sqrt{\frac{T}{\mu_1}}\cdot\left(\frac{2l}{n_2}\right)\sqrt{\frac{\mu_2}{T}} = 1 \Rightarrow \left(\frac{n_1}{n_2}\right) = \sqrt{\frac{\mu_1}{\mu_2}}$$

Sonometer




The 3 sections all vibrate with diff. freq.

Q) $l = 110$ cm. The 3 sections of wire are all vibrating in fundamental mode with 1:2:3. Find dist. of bridges from ends.

A) $\nu_1 : \nu_2 : \nu_3 = 1:2:3 = \frac{1}{l_1} : \frac{1}{l_2} : \frac{1}{l_3}$

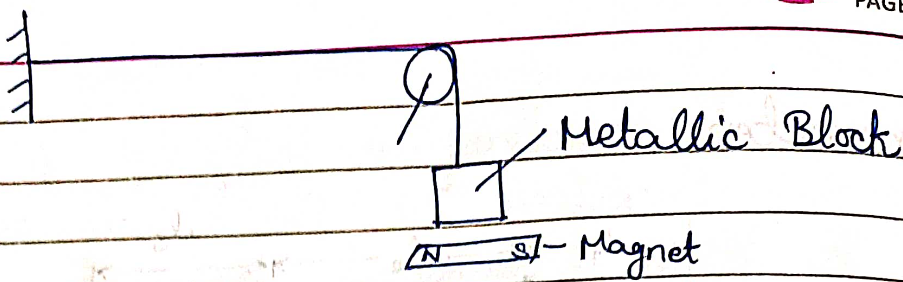
Let $l_1 = \lambda$, $l_2 = \lambda/2$, $l_3 = \lambda/3$

Now, $l_1 + l_2 + l_3 = 110 \Rightarrow \lambda + \frac{\lambda}{2} + \frac{\lambda}{3} = 110$

$\Rightarrow \quad \lambda = 60$

$\Rightarrow \quad \boxed{l_1 = 60}, \quad \boxed{l_2 = 30}, \quad \boxed{l_3 = 20}$

Q) Wire is vibrating in 5 loops. A magnet is brought below metallic block, then string vibrates in 4 loops. Find ratio of pull of magnet to that of Earth (freq. is kept const.)




A) $\nu = \left(\frac{4}{2l}\right)\sqrt{\frac{f_E+f_M}{\mu}} = \left(\frac{5}{2l}\right)\sqrt{\frac{f_E}{\mu}}$

$$\Rightarrow \left(\frac{f_E+f_M}{f_E}\right) = \left(\frac{25}{16}\right)$$

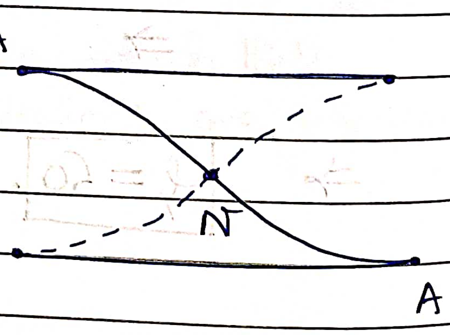
$$\Rightarrow \boxed{\left(\frac{f_M}{f_E}\right) = \left(\frac{9}{16}\right)}$$

## Vibrations in Air Column

Wave reflects as density of air outside is diff. from density of air inside.

### Open Pipe

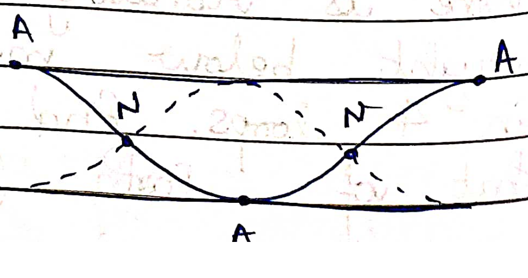
First mode of vibration:




$L = \lambda/2 \Rightarrow \lambda = 2L$

$\nu = v/\lambda \Rightarrow \nu = v/2L$

Second mode:



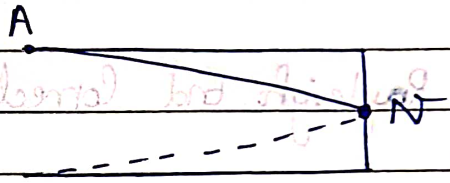
☆ All graphs shown here are disp. of particles graph.
for pressure graph, change node to antinode
& antinode to node !

$L = \lambda$ ; $\nu = v/\lambda \Rightarrow \nu = v/L$

nth mode: $\boxed{\nu = \left(\frac{nv}{2L}\right)}$

## Closed Pipe

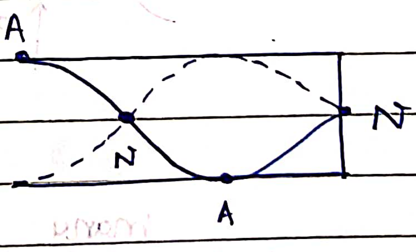
first mode:




$L = \lambda/4 \Rightarrow \lambda = 4L$

$\nu = v/\lambda \Rightarrow \nu = v/4L$

2nd mode:
(3rd harmonic / 1st overtone)




$L = 3\lambda/4 \Rightarrow \lambda = 4L/3$

$\nu = v/\lambda \Rightarrow \nu = 3v/4L$

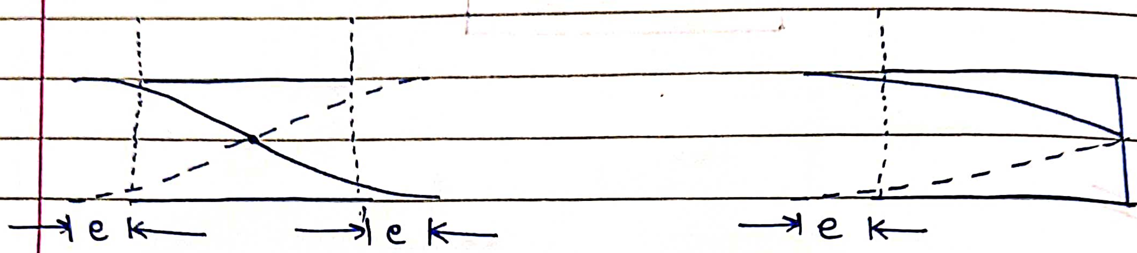
(n+1)th mode: $\boxed{\nu = (2n+1)\, v/4L}$

(# Harmonics) = (2n+1) (#Overtones) = n

DATE
PAGE

## End Correction

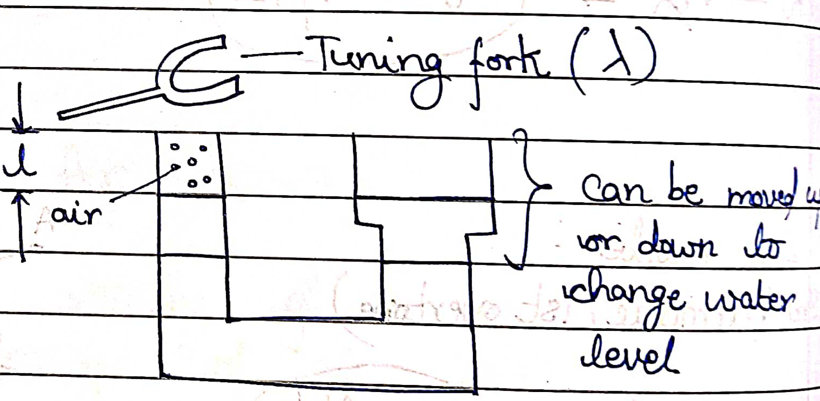
In closed & open pipe, antinodes form a little bit outside the pipe.




**Rayleigh End Correction:** $\boxed{e = (0.3)\underbrace{D}_{\text{diameter of pipe}}}$

**Absolute Method:**

(Resonance Tube)




We find many lengths of air column when resonance happens (loud sound heard).

These resonating lengths must bear 1 : 3 : 5 : ... if no end correction.

If end correction, then.

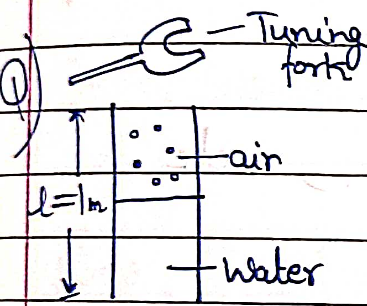
$$(l_1 + e) = \frac{\lambda}{4}, \quad (l_2 + e) = \frac{3\lambda}{4}, \quad (l_3 + e) = \frac{5\lambda}{4}, \ldots$$

Now, $\left(3\lambda/4 - \lambda/4\right) = (l_2 + e) - (l_1 + e)$

$\Rightarrow \lambda = 2(l_2 - l_1)$

This can be used to find vel. of wave as freq. of tuning for is known.

Now, $\boxed{e = \dfrac{(l_2 - 3l_1)}{2}}$

1) 


$(\nu = 500\ \text{Hz})$

Find ~~max~~ no. of harmonics in which tube can be vibrated.

(Speed of wave = $330\ \text{ms}^{-1}$)

A) $\lambda = v/\nu = 330/500 \Rightarrow \lambda = 0.66 \Rightarrow \lambda \approx 2/3$

~~Harmonics~~: $\lambda/4,\ 3\lambda/4,\ 5\lambda/4,\ 7\lambda/4, \ldots$

Resonating Lengths: $1/6,\ 1/2,\ 5/6,\ 7/6$

✓ ✓ ✓ ✗

Only 1st, 3rd & 5th harmonics possible.

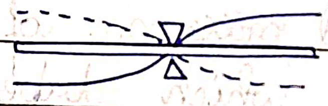
$\Rightarrow$ 3 harmonics

## Vibrations in Thin Rod

(Rod fix at both ends) = (Stretched String)

(Rod fix at one end) = (Closed Pipe)

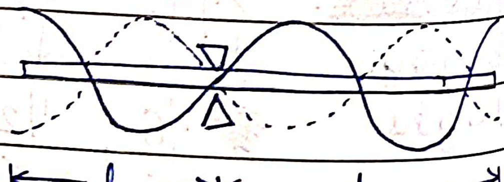
For free at both ends, we need to clamp it somewhere.

Eg:  ,  , etc.

If clamped in the middle (symmetric) ⇒ No problem.

" " NOT " " " (unsymmetric) ⇒ Problem!

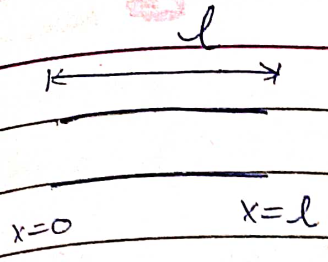
Let $\lambda$ be wavelength of vibration.




So, $(odd_1)(\lambda/4) = l_1$

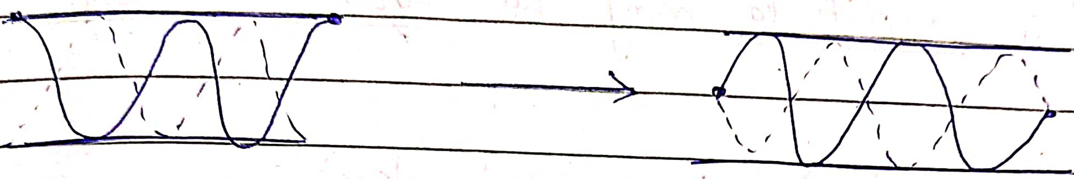
$(odd_2)(\lambda/4) = l_2$

Find $\lambda$ for which $(odd_1)$ & $(odd_2)$ are min.




Q) vibrating in 3rd overtone. Pressure amplitude $= P_0$.

Find value of pressure at $x = l/6$.

A) 

Disp. Pressure

Since node at $x=0 \Rightarrow$ $P = P_0 \sin(kx)$ ✓
$P = P_0 \cos(kx)$ ✗

Now, 3rd overtone $\Rightarrow$ 4th mode.

$\Rightarrow 4\left(\frac{\lambda}{2}\right) = l \Rightarrow \lambda = \frac{l}{2} \Rightarrow k = \frac{4\pi}{l}$

So, $P = P_0 \sin(kx) = P_0 \sin\left(\frac{4\pi}{l} \cdot \frac{l}{6}\right) = P_0 \sin\left(\frac{2\pi}{3}\right)$

$\Rightarrow \boxed{P = P_0 \sqrt{3}/2}$

196

DATE
PAGE

## Beats

2 waves travelling in same dir$x^n$, with slight diff. in freq$^n$ $(|\nu_1 - \nu_2| \leq 10 \text{ Hz})$.

Their amplitude may be equal or diff.

$$y_1 = A \sin(\omega_1 t - kx)$$

$$y_2 = A \sin(\omega_2 t - kx)$$

$$\Rightarrow \quad y = y_1 + y_2$$

$$= A\left(\sin(\omega_1 t - kx) + \sin(\omega_2 t - kx)\right)$$

$$= \cancel{A\,2\sin}\ 2A \sin\left(\left(\frac{\omega_1+\omega_2}{2}\right)t - kx\right) \cos\left((\omega_1-\omega_2)t\right)$$

$$\Rightarrow \boxed{y = \underbrace{\left[2A\cos\left((\omega_1-\omega_2)t\right)\right]}_{\text{Amplitude}} \sin\left(\left(\frac{\omega_1+\omega_2}{2}\right)t - kx\right)}$$

Since, $(\text{Intensity}) \propto (\text{Amplitude})^2$

$\Rightarrow$ I max. if $\cos(\omega t) = \pm 1$

$$\Rightarrow t = 0, \frac{\pi}{(\omega_1-\omega_2)}, \frac{2\pi}{(\omega_1-\omega_2)}, \ldots$$

(for max. intensity)

Since $\omega = 2\pi\nu$ $\Rightarrow$ $\boxed{t = \left(\dfrac{n}{\nu_1 - \nu_2}\right)}$

$$\left(\text{Time b/w successive max.}\right) = \left(\frac{1}{\nu_1 - \nu_2}\right)$$

$\Rightarrow$ $\left(\#\text{max. in } 1s\right) = \left(\nu_1 - \nu_2\right)$ $\longrightarrow$ (Beat freq.)

Now, I min. if $\cos(\dots) = 0$

$$\Rightarrow \quad t = \frac{\pi}{2(\omega_1 - \omega_2)}, \frac{3\pi}{2(\omega_1 - \omega_2)}, \frac{5\pi}{2(\omega_1 - \omega_2)}, \dots$$

$\Rightarrow$ $\boxed{t = \dfrac{(2n+1)}{2(\nu_1 - \nu_2)}}$

(for min. intensity)

$$\left(\text{Time b/w successive min.}\right) = \left(\frac{1}{\nu_1 - \nu_2}\right)$$

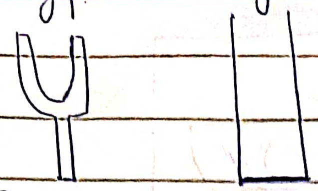
$\Rightarrow$ $\left(\#\text{min. in } 1s\right) = \left(\nu_1 - \nu_2\right)$ $\longrightarrow$ (Beat freq.)

Beat freq. — ~~#Max~~ (#Max. or #Min. in 1s)

DATE
PAGE

Tuning fork    Organ Pipe

Q)



Beat freq. = 5 Hz.

$\nu = 100$ Hz

On loading tuning fork, beat freq. becomes 4 Hz.

A) Let $\nu_{pipe}$ of organ pipe. ← be $\nu$

Now, $|\nu - 100| = 5 \Rightarrow \nu = 105$ or $95$

If tuning fork loaded $\Rightarrow \nu_{fork} \downarrow$

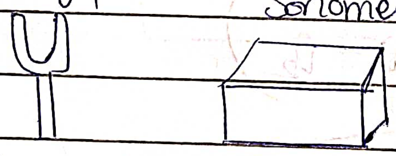
If $\nu_{pipe} = 105$ & " $\Rightarrow$ (Beat freq. $\uparrow$) ✗

If $\nu_{pipe} = 95$ & " $\Rightarrow$ (Beat freq. $\downarrow$) ✓

$\Rightarrow$ $\nu_{pipe} = 95$ Hz

Tuning fork    Sonometer

Q)



(Beat freq.) = 4 Hz

$\nu = 100$ Hz

If tension in sonometer inc. then beat freq. becomes 5 Hz

Find $\nu_{orig.}$ of sonometer

A) $|\nu - 100| = 4 \Rightarrow \nu = 104$ or $96$

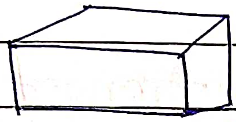
If Tension inc. $\Rightarrow$ $\nu_{sono.} \uparrow$

$\left(\nu_{Sonometer} \propto \sqrt{T}\right)$ $\Rightarrow$ $\nu_{orig.} =$ (104)

as if $\nu_{orig} = \cancel{104}$ 96 $\Rightarrow$ ($\nu_{sono} \uparrow \Rightarrow$ Beat freq. reduced)

Tuning fork

Sonometer

Q)  

Beat freq. $= 4$.

$\nu = 100 Hz$

On inc. tension, beat freq. remain same.

Find $\nu_{orig.}$ of sonometer.

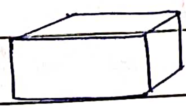
A) $|\nu - 100| = 4 \Rightarrow \nu = 104$ or $96$

Now, $T\uparrow \Rightarrow \nu \uparrow \Rightarrow$ $\nu_{orig} = 96$

& $\nu_{new} = 104$

Tuning fork

Sonometer

Q)  

Beat freq. $= 5$ Hz

when length of sonometer wire $= 95$ cm & $105$ cm.

Find $\nu_{Tuning\ fork}$.

DATE
PAGE

A) $$5 = \left(\frac{n}{2\cdot 95}\sqrt{\frac{T}{\mu}} - \nu_{fork}\right) = \left(\nu_{fork} - \frac{n}{2\cdot 105}\sqrt{\frac{T}{\mu}}\right)$$

$$\Rightarrow \left(5\cdot 95 + 5\cdot 105\right) = \left(\nu_{fork}\right)\left(105 - 95\right)$$

$$\Rightarrow \nu_{fork} = 100 \text{ Hz}$$

Q) 65 tuning forks are arranged in a row, in order of dec. freq. ~~Beat~~ Beat freq. b/w any 2 succesive forks is 4 Hz. freq. of 1st is octave of last.

Find freq. of 1st fork.

first is

A) (Octave of last) $\Rightarrow$ (freq. of 1st) = 2(freq. of last)

Now, freq: $\nu$, $(\nu - 4)$, ..., $(\nu - 64\cdot 4)$

fork: 1st, 2nd, ..., 65th.

Now, $\nu = 2(\nu - 64\cdot 4)$.

$$\Rightarrow \boxed{\nu = 512 \text{ Hz}}$$

## Interference

2 waves travelling in same dirxn with same freq., wavelength & same or nearly same amplitude

$$y_1 = A_1 \sin(\omega t - kx)$$

$$y_2 = B \sin(\omega t - kx + \phi)$$

$$\Rightarrow \quad y = A_1 \sin(\omega t - kx) + B \sin(\omega t - kx + \phi)$$

$$= \left[A_1 + B\cos(\phi)\right] \sin(\omega t - kx) + B\sin(\phi)\cos(\omega t - kx)$$

Now, let $\left[A_1 + B\cos(\phi)\right] = A\cos(\theta)$

& $B\sin(\phi) = A\sin(\theta)$

$$\Rightarrow \quad \boxed{y = \underbrace{A}_{\text{amplitude}} \sin(\omega t - kx + \theta)}$$

where $\boxed{A = \sqrt{A_1^2 + B^2 + 2A_1 B\cos(\phi)}}$

Now, $\underset{(\text{Intensity})}{I} \propto \underset{(\text{Amplitude})}{A^2} = A_1^2 + B^2 + 2A_1 B\cos(\phi)$

$$\Rightarrow \quad \boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\phi)}$$

DATE

PAGE

for $I_{max} \Rightarrow \phi = 2n\pi \Rightarrow I_{max} = \left(\sqrt{I_1}+\sqrt{I_2}\right)^2$

for $I_{min} \Rightarrow \phi = (2n-1)\pi \Rightarrow I_{min} = \left(\sqrt{I_1}-\sqrt{I_2}\right)^2$

Now for the two waves,

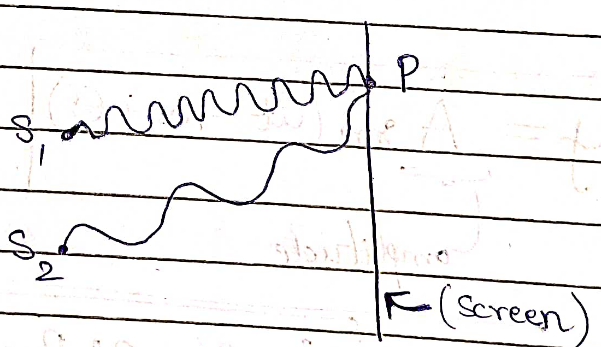
$$\boxed{(\text{Phase diff.}) = \left(\frac{2\pi}{\lambda}\right)(\text{Path diff.})}$$

~~for (Path diff.)max ⇒ (Phase diff.)~~

for $I_{max} \Rightarrow \phi = 2n\pi \Rightarrow \boxed{(\text{Path diff.}) = n\lambda}$

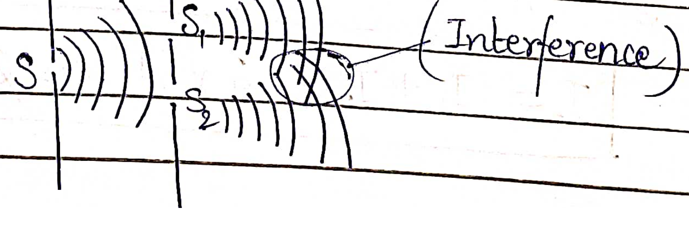
For $I_{min} \Rightarrow \phi = (2n-1)\pi \Rightarrow \boxed{(\text{Path diff.}) = \left(n-\frac{1}{2}\right)\lambda}$

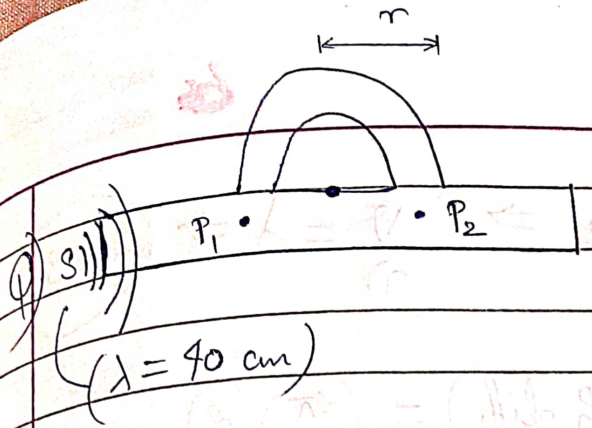
Now,



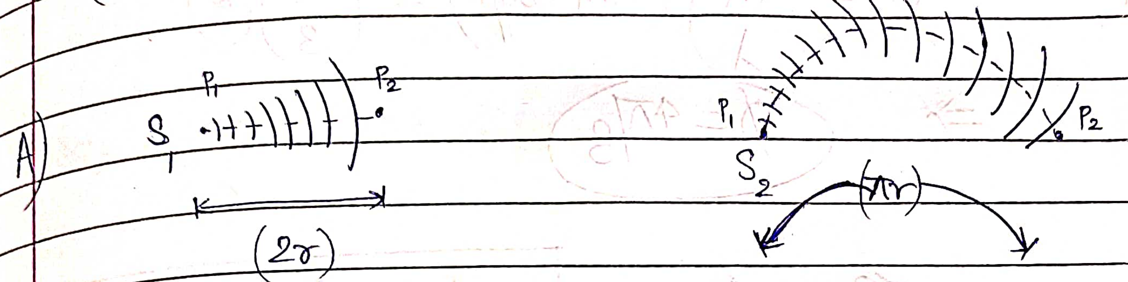

★ (Can apply even if $\Delta\phi_{init} \neq 0$ of waves.) $\rightsquigarrow$ $\boxed{(\text{Path diff.}) = (S_1P - S_2P)}$

Also,



Q) 


At detector, Intensity is min.
find $r_{min}$.

A) 

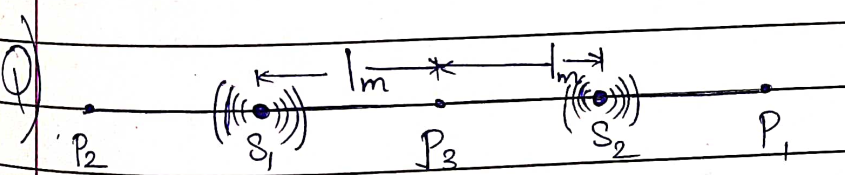

$$(\text{Path diff.}) = (\pi - 2)\, r.$$

Now, $I_{min} \Rightarrow \phi = (2n-1)\pi.$

Also, $$(\text{Path diff}) = \left(\frac{\lambda}{2\pi}\right)(\cancel{\text{Path}}\ \text{Phase diff.})$$

$$\Rightarrow (\pi - 2)\, r = \left(\frac{40 \text{ cm}}{2\cancel{\pi}}\right)(2n-1)\cancel{\pi}$$

$$\Rightarrow r = \frac{20(2n-1)}{(\pi-2)} \text{ cm} \Rightarrow \boxed{r_{min} = \left(\frac{20}{\pi-2}\right) \text{cm}}$$

Q) 


$y_1 = (0.03) \sin(\pi t)$
$y_2 = (0.02) \sin(\pi t)$
$v = 1.5 \text{ m s}^{-1}$

find ~~intensity~~ amplitude at $P_1$, $P_2$ & $P_3$.

A) for $P_1$ & $P_2$, $(\text{Path diff.}) = 2 \text{ m}$
$\left(|S_1P_1 - S_2P_1| \;\&\; |P_2S_1 - P_2S_2|\right)$

DATE
PAGE

Now, $\nu = \frac{\omega}{2\pi} = 1/2 \Rightarrow \frac{v}{\nu} = \lambda \Rightarrow \lambda = 3$

Now, $\phi = \left(\frac{2\pi}{\lambda}\right)(\text{Path diff.}) = \left(\frac{2\pi}{3}\right)(2)$

$\Rightarrow \phi = 4\pi/3$

Now, $A_{P_1 \& P_2} = \sqrt{(0.02)^2 + (0.03)^2 + 2(0.02)(0.03)\cos\left(\frac{4\pi}{3}\right)}$

$\Rightarrow A_{P_1 \& P_2} =$

for $P_3$, $(\text{Path diff.}) = 0$ $\quad \left(|S_1P_3 - S_2P_3| = 0\right)$

$\Rightarrow \phi = 0$

$\Rightarrow \boxed{A_{P_3} = 0.05}$

☆ Q) In above Q, if $y_1 = 0.03 \sin(\pi t + \pi/3)$, find phase diff. at $P_1$

A) ☆ (Jo wave pehle pahunchi, uska phase jyada hoga. at reaching pt.)

Initially, $\left(\phi_{wave\,1} - \phi_{wave\,2}\right) = \pi/3$

Afterwards we know, $\phi_{wave\,2} > \phi_{wave\,1}$

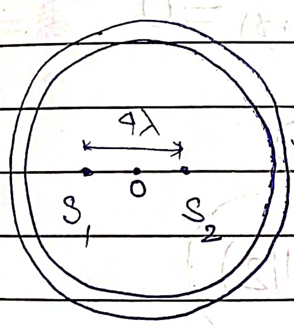
Now, $\left(\text{Net phase diff.}\right) = \left(\phi_{wave\,2} - \phi_{wave\,1}\right)$

$$= (\Delta\phi)_{\text{due to path diff.}} + (\Delta\phi)_{init}$$

$$= \left(\frac{2\pi}{\lambda}\right)\left(\text{Path diff.}\right) + \left(\frac{-\pi}{3}\right)$$

$$= \left(\frac{2\pi}{3}\right)(2) - \pi/3 \Rightarrow \boxed{(\Delta\phi)_{net} = \pi}$$

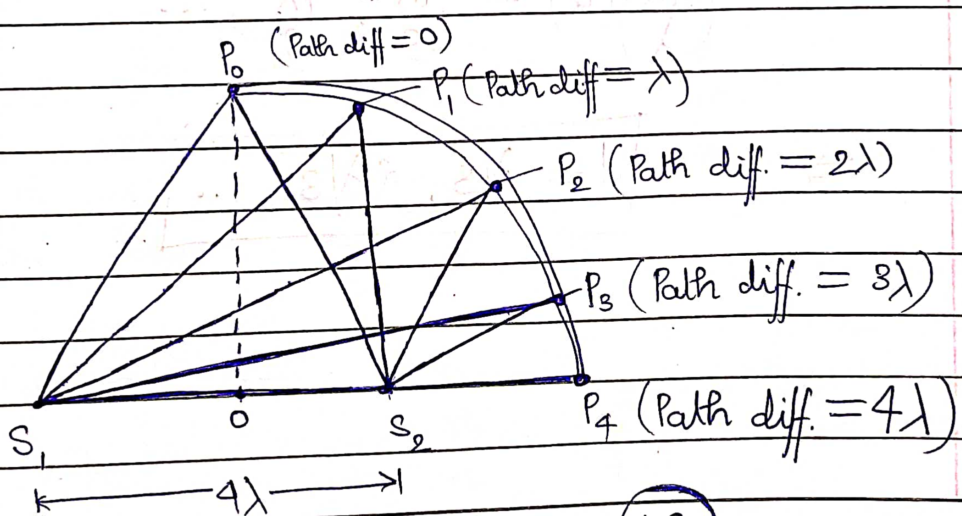
Q) ☆




find no. of maxima obtained on screen.

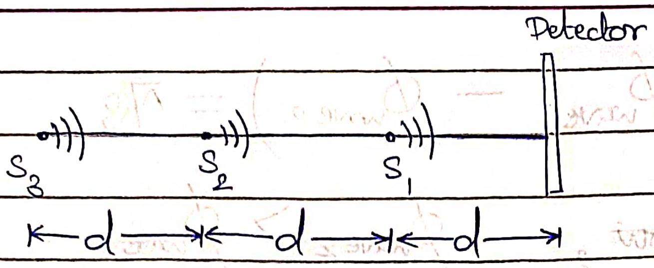
A) For $I_{max}$, $\left(\text{Path diff.}\right) = n\lambda$

Now,




$\Rightarrow$ 16 maxima

Q)




find 'd' in terms of 'λ', if intensity at detector = 0.

A) Let $y_2 = A\sin(\omega t - kx)$ $\quad \left\{\phi = \left(\frac{2\pi}{\lambda}\right)d\right\}$

if $y_1 = A\sin(\omega t - kx + \phi)$

$\Rightarrow y_3 = A\sin(\omega t - kx - \phi)$

We need, $y_1 + y_2 + y_3 = 0$

$$\Rightarrow s_{(\omega t - kx - \phi)} + s_{(\omega t - kx)} + s_{(\omega t - kx + \phi)} = 0$$

$$\Rightarrow 2 s_{(\omega t - kx)} c_\phi + s_{(\omega t - kx)} = 0$$

$$\Rightarrow (2c_\phi + 1) = 0 \Rightarrow c_\phi = (-1/2)$$

$$\Rightarrow \phi = \frac{2\pi}{3}, \frac{4\pi}{3}, \ldots$$

$$\Rightarrow \left(\frac{2\pi}{\lambda}\right)(d) = \frac{2\pi}{3}, \frac{4\pi}{3}, \ldots$$

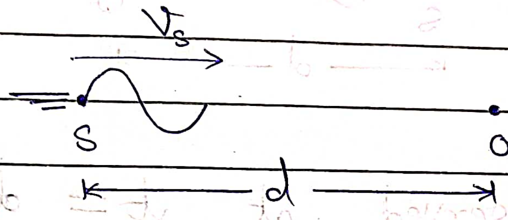
$$\Rightarrow \boxed{d = \lambda/3,\ 2\lambda/3,\ \ldots}$$

# Doppler Effect

When there is rel. motion b/w source & observer, then freq. emitted by source & freq. observed by observer are diff.

C1 : If observer at rest, but source is moving.

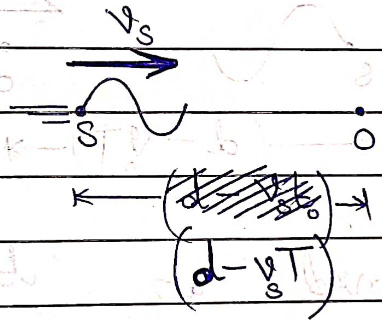
$t = 0$ ;




{ $c$ = Speed of wave }

Observer observes 1st wave at $t_0 = d/c$.

$t = T$ ; (Time period of wave)




Observer observes next wave at $t_1 = \frac{(d - v_s T)}{c} + T$

$$\left(\text{Time b/w succesive wave pulses}\right) = (t_1 - t_0)$$

$$= \left(\frac{d - v_s T}{c}\right) + T - \left(\frac{d}{c}\right)$$

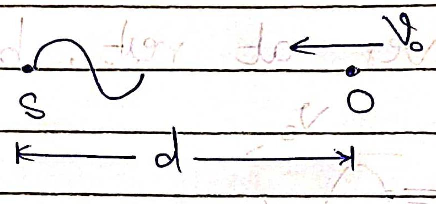
$$= T - \frac{v_s T}{c} = T\left(1 - \frac{v_s}{c}\right)$$

$$\Rightarrow \left(\text{Observed freq.}\right) = \frac{1}{T\left(1 - \frac{v_s}{c}\right)} = \frac{1/T}{\left(1 - \frac{v_s}{c}\right)}$$

$$\Rightarrow \boxed{\nu' = (\nu)\left(\frac{c}{c - v_s}\right)}$$

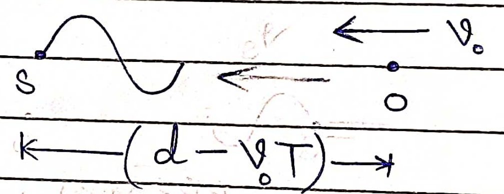
C2: If observer moving, but source at rest.

$t = 0$;




1st wave observed at $t_0 = d/(v_o + c)$

$t = T$;




2nd wave observed at $t_1 = \frac{(d - v_o T)}{(v_o + c)} + T$

$$\left(\text{Time b/w successive waves}\right) = (t_1 - t_0)$$

$$= T + \left(\frac{d - v_o T}{v_o + c}\right) - \left(\frac{d}{v_o + c}\right)$$

$$= T - \left(\frac{v_o T}{v_o + c}\right) = (T)\left(1 - \frac{v_o}{(v_o + c)}\right)$$

$$= \left(\frac{Tc}{v_o + c}\right)$$

$$\Rightarrow (\text{Observed freq.}) = \frac{(v_o + c)}{(cT)} = (1/T)\left(\frac{v_o + c}{c}\right)$$

$$\Rightarrow \quad \boxed{\nu' = (\nu)\left(\frac{V_o + v}{v}\right)}$$

In general,
$$\boxed{\nu' = \left(\frac{v + V_o}{v - V_s}\right)\nu}$$

Sign Convention :

If Source towards Observer $\Rightarrow$ $V_s > 0$.

" " away from " $\Rightarrow$ $V_s < 0$.

If Observer towards Source $\Rightarrow$ $V_o > 0$.

" " away from " $\Rightarrow$ $V_o < 0$.

o $\bullet\!\longrightarrow$ $\longleftarrow\!\bullet$ s $\quad$ $V_o > 0$, $V_s > 0$

$\longleftarrow\!\bullet$ o $\quad$ s $\bullet\!\longrightarrow$ $\quad$ $V_o < 0$, $V_s < 0$

o $\bullet\!\longrightarrow$ $\quad$ s $\bullet\!\longrightarrow$ $\quad$ $V_o > 0$, $V_s < 0$

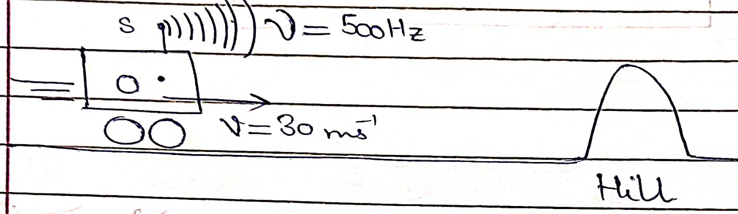
$\longleftarrow\!\bullet$ o $\quad$ $\longleftarrow\!\bullet$ s $\quad$ $V_o < 0$, $V_s > 0$

$\nu' > \nu \Rightarrow$ Gap <u>decreasing</u>.

$\nu' = \nu \Rightarrow$ Gap <u>same</u>.

$\nu' < \nu \Rightarrow$ Gap <u>increasing</u>.

Q)




Wave reflects & comes back.

find $\nu_{obs}$ ($v = 330\ ms^{-1}$)

A) ☆ Hill pe jo freq. aayegi, wahi reflect hooyegi.

1) Pehle observer ko hill pe rakho to find kaunsi freq. reflect hogi.

S $\xrightarrow{}$ $30\,ms^{-1}$     O

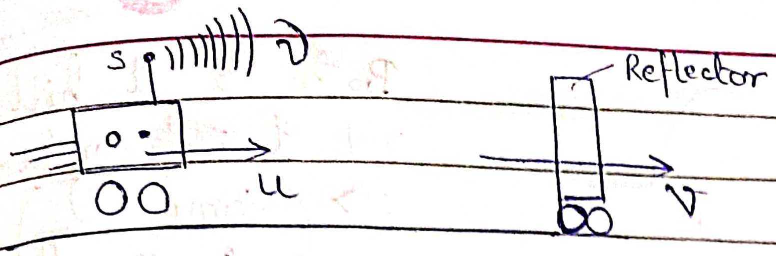
$$\underset{\text{hill}}{\nu'} = \left(\frac{v}{v-30}\right)\nu$$

2) Yahi freq. reflect hogi.

O $\xrightarrow{}$ $30\,ms^{-1}$     $\xleftarrow{\nu'}$ S

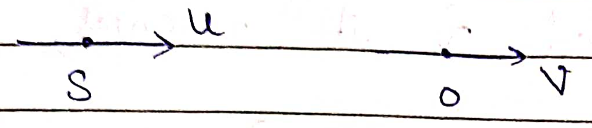
$$\underset{\text{car}}{\nu''} = \left(\frac{v+30}{v}\right)\nu'$$

$$\Rightarrow \nu'' = \left(\frac{v+30}{v}\right)\left(\frac{v}{v-30}\right)\nu \Rightarrow \boxed{\nu'' = 600\,Hz}$$

Q) 

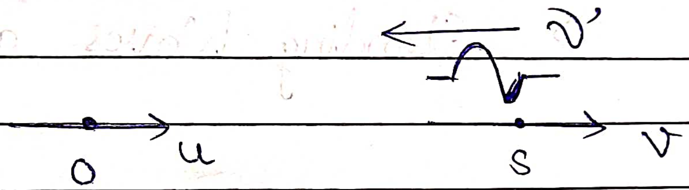

find $\nu_{obs}$.

A) first,




$$\nu' = \left(\frac{c+(-v)}{c-(+u)}\right)\nu = \left(\frac{c-v}{c-u}\right)\nu$$

Then,

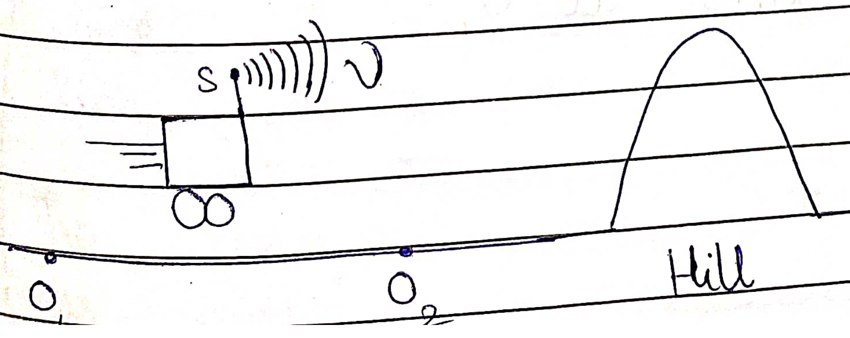



$$\nu'' = \left(\frac{c+(+u)}{c-(-v)}\right)\nu' = \left(\frac{c+u}{c+v}\right)\nu'$$

$$\Rightarrow \quad \nu'' = \left(\frac{c+u}{c+v}\right)\left(\frac{c-v}{c-u}\right)\nu$$

$$\Rightarrow \quad \boxed{\nu'' = \left(\frac{c+u}{c-u}\right)\left(\frac{c-v}{c+v}\right)\nu}$$

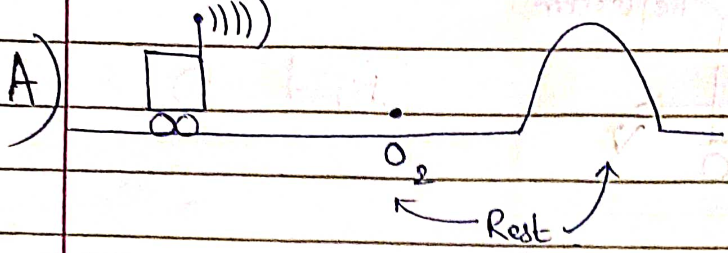
Q) ☆




What happens at $O_1$ & $O_2$

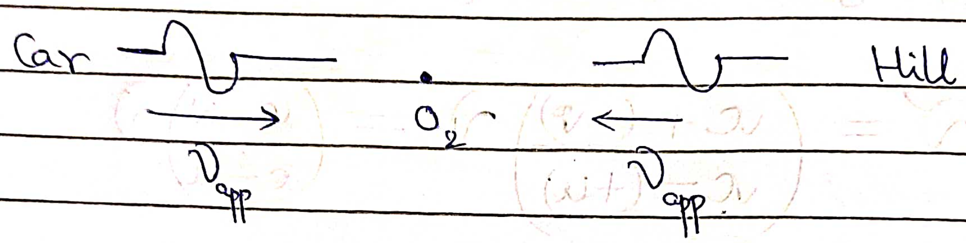
21

DATE
PAGE

A) 

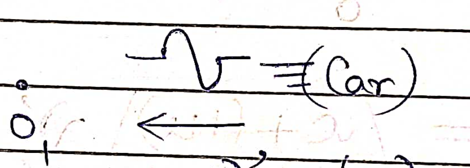

(Both $O_2$ & hill at rest & ~~rest~~ car towards them) $\Rightarrow$ (Same $\nu_{app}$ for them)

Since hill reflect $\nu$ it receives,




$\Rightarrow$ Standing Waves at $O_2$

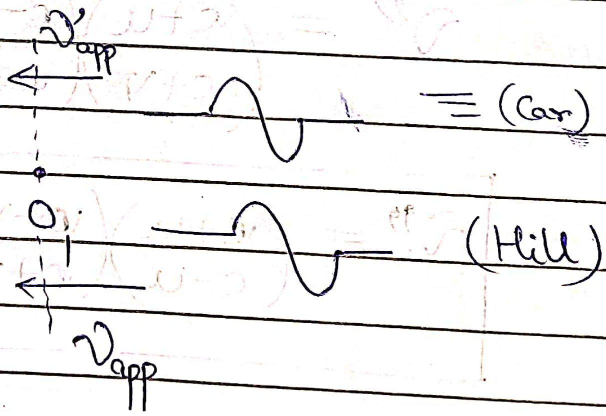
Now,




$\nu'_{app} \neq \nu_{app}$

as even though $O_1$ at rest, car moving away from it

So,




$\Rightarrow$ Beats at $O_1$

$\{v << c\}$

Q) S •—→ $v$     • O (Rest.)

% dec. in freq. as observed by observer is 0.1 %, while source crosses him.

Find ~~..~~ speed of source.

A) S •—→ $v$     • O

$$\nu_1 = \left(\frac{c}{c-(+v)}\right)\nu$$

O •     S •—→ $v$

$$\nu_2 = \left(\frac{c}{c-(-v)}\right)\nu$$

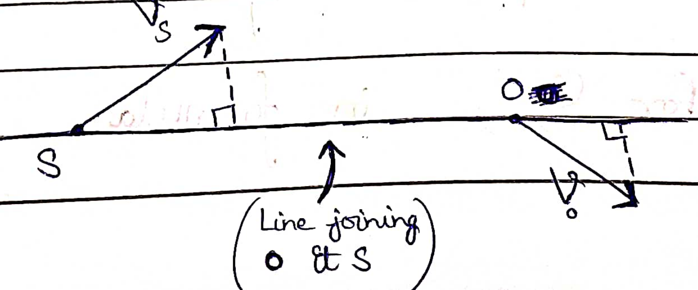
$$\Rightarrow \left(\frac{\nu_2}{\nu_1}\right) = \left(\frac{c-v}{c+v}\right)$$

$$\Rightarrow \left[1-\left(\frac{\nu_2}{\nu_1}\right)\right] = \left(\frac{2v}{c+v}\right) = \frac{1}{1000} \approx \left(\frac{2v}{c}\right)$$

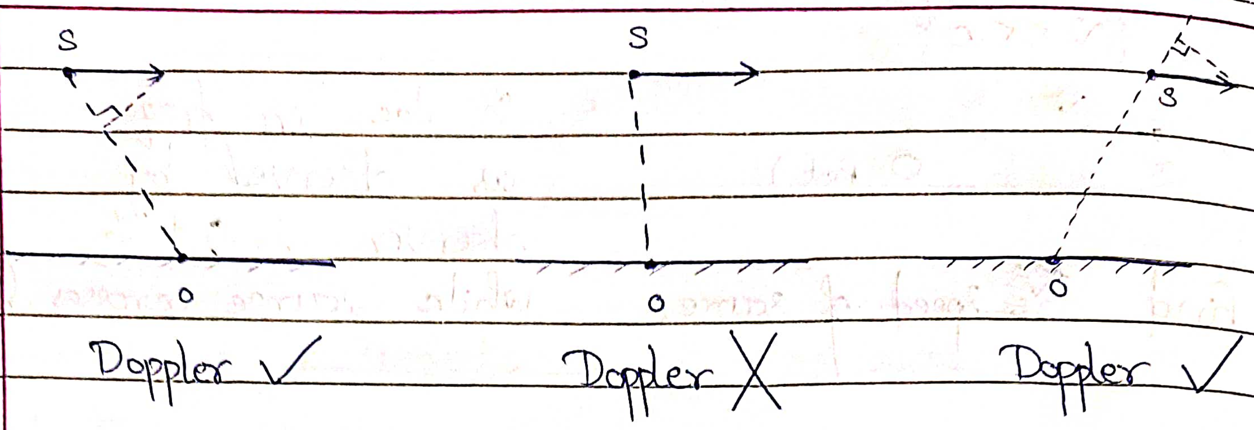
$$\Rightarrow \boxed{v = \left(\frac{c}{2000}\right)}$$

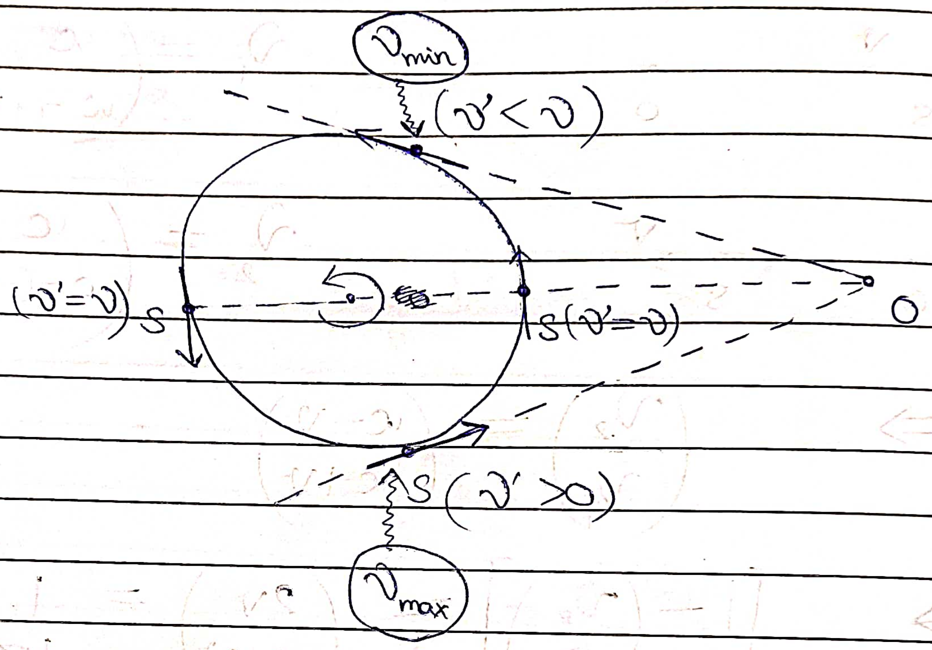
☆ If Source & Observer NOT in one line, take comp. of their vel. along line joining source & observer.

Then, solve as usual !



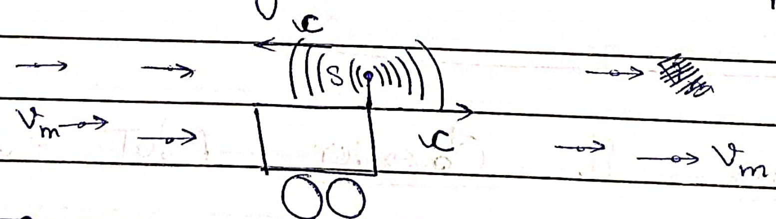
DATE
PAGE

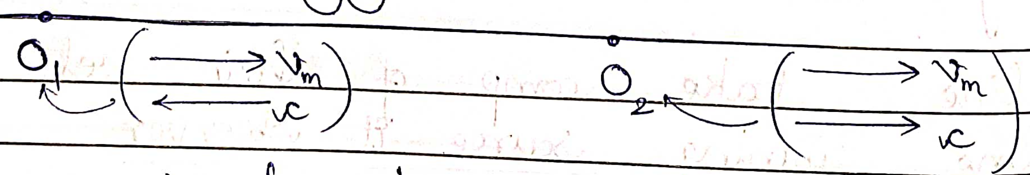



## Effect of Moving Medium on Doppler Effect –

If medium moving with vel. '$v_m$', then




For $O_1$, in formula $c \rightarrow (c - v_m)$

For $O_2$, in formula $c \rightarrow (c + v_m)$