ELECTROSTATICS

25/04/2023

{ Charge at rest w.r.t to the charged body }

In a charged body, $\#e^- \neq \#p$

Charge is acquired due to e^- transfer.

Neutral body $\neq$ Chargeless body.

Charge of
1. $e^- = -1.6 \times 10^{-19}$ C
2. $p = 1.6 \times 10^{-19}$ C

→ Properties of Charge

1. Quantised ⇒ $\boxed{Q = \pm ne}$
2. Conserved
3. Follows Additive law
(algebraic addⁿ)

Units : $\boxed{1\ C = 3 \times 10^{9}\ esu}$

c.g.s unit

COULOMB'S LAW

$F \propto |q_1 q_2|$

$F \propto \frac{1}{d^2}$

q_1 q_2

d

$\Downarrow$

$$\boxed{F = k\frac{|q_1 q_2|}{r^2}}$$

Electrostatic const.

$k = \frac{1}{4\pi\varepsilon_0} = 9\times10^9$

Permitivitty of free space

$= 1$

$q_1 q_2 < 0 \iff$ Attractive force

$q_1 q_2 > 0 \iff$ Repulsive force

NOTE: (1) Coulomb's law only holds for particles & spherical distribution of charge.

(2) Force acts along line joining the two charges.

→ Ppts of Coulombian force

(1) Action - Reaⁿ pair. $\Rightarrow \vec{F}_{12} + \vec{F}_{21} = 0 \Rightarrow \boxed{\vec{F}_{12} = -\vec{F}_{21}}$

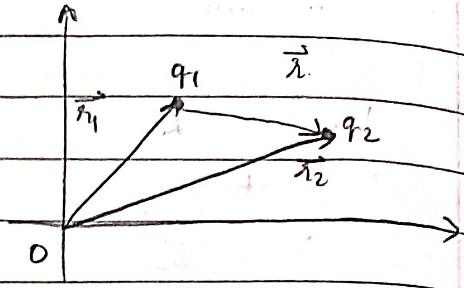
(Force on q_1 due to q_2)

(2) Cannot change motion of CM of sys. $\Rightarrow$ Momentum conserved

$$\vec{F}_{12} = \left(\frac{K q_1 q_2}{r^2}\right)(-\hat{r})$$

$$= \boxed{\left(\frac{k q_1 q_2}{r^2}\right)\hat{r}_{21}}$$

$$\boxed{\vec{r}_{21} = \vec{r}_1 - \vec{r}_2}$$



$$\vec{r} = \vec{r}_2 - \vec{r}_1$$

$$= \boxed{\left(\frac{k q_1 q_2}{r^3}\right)\vec{r}_{21}}$$

(3) Conservative force

⇒ Work done is <u>independant of path</u>

(4) Two-body interaction force.

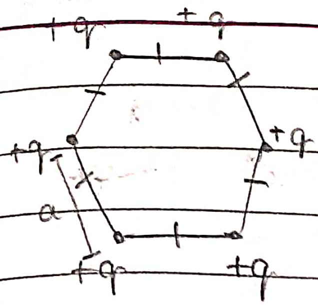
⇒ Force b/w 2 charged particles does <u>NOT</u> depend on presence of other charged particles.

→ <u>Law of Superposit.</u>

$$\vec{F}_{1(net)} = \vec{F}_{12} + \vec{F}_{13} + \dots + \vec{F}_{1N}$$

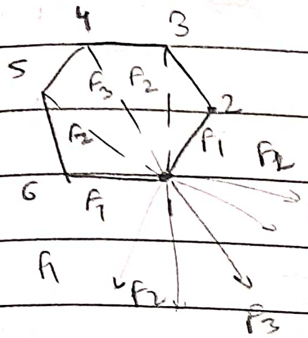
Net force on any charged particle is the <u>vector sum</u> of all forces acting on the particle.

Q.



Find mag. of net. elect. $\vec{F}$ acting on any one of charged particle.

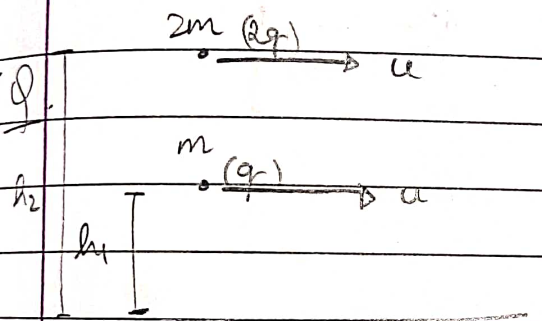
A.



$$\vec{F} = \langle F_1 \quad 0 \rangle + \langle -\frac{F_1}{2} \quad -\frac{\sqrt{3}}{2}F_1 \rangle + \langle 0 \; -F_2 \rangle + \langle \frac{\sqrt{3}}{2}F_2 \quad -\frac{F_2}{2} \rangle + \langle \frac{F_3}{2} \quad -\frac{\sqrt{3}}{2}F_3 \rangle$$

$$F = F_3 + 2F_2 \, c_{30^\circ} + 2F_3 \, c_{60^\circ}$$

*Q.



Projected simultaneously.
Lower particle hits ground at dist. d from initial vertical line.
Find height of 2nd particle at this inst.

A. ① $a_{CM} = -g \downarrow$

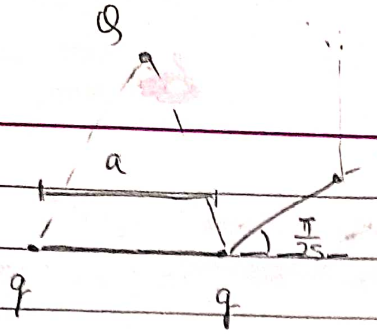
② $t = \left(\frac{d}{u}\right)$

③ $s = -\frac{1}{2}gt^2 \Rightarrow x'_{CM} - x_{CM} = -\frac{1}{2}gt^2$

$$\Rightarrow \frac{2m(h_2') + m(0)}{3m} - \frac{(2m)(h_2) + m(h_1)}{3m} = -\frac{1}{2}\frac{gd^2}{u^2}$$

$$\Rightarrow \boxed{h_2' = \frac{3}{2}\left[\frac{2h_2 + h_1}{3} - \frac{gd^2}{2u^2}\right]}$$

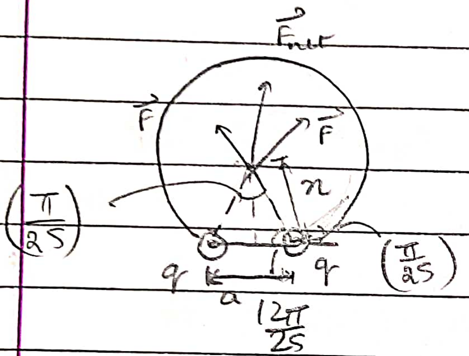
Q.



Find net force on central charge if 48 such charges are present

A if 80 were present ⇒ No force. (by symmetry)

$$\left\{ \text{since} \quad \# \text{side} = \frac{2\pi}{\pi/25} = 80 \right\}$$



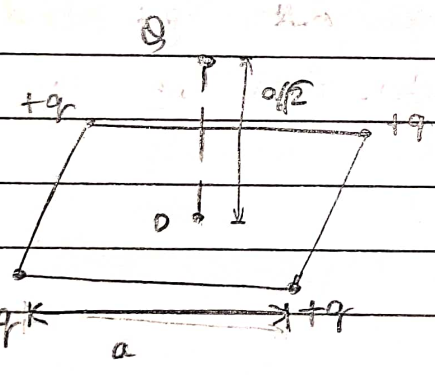
$$F_{net} = 2F\cos\left(\frac{\pi}{80}\right)$$

$$= \frac{2kQq}{a^2}\left(4s^2\left(\frac{\pi}{80}\right)\right)\left(c\frac{\pi}{80}\right)$$

$$= \boxed{\frac{8\,kQq}{a^2}\, s^2\left(\frac{\pi}{80}\right) c\left(\frac{\pi}{80}\right)}$$

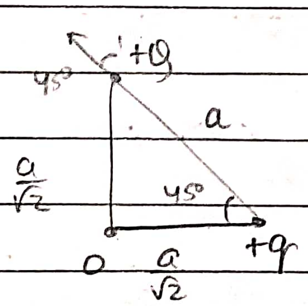
$$x = \frac{a}{2s\left(\frac{\pi}{80}\right)}$$

Q.



Find net force acting on particle.

A.

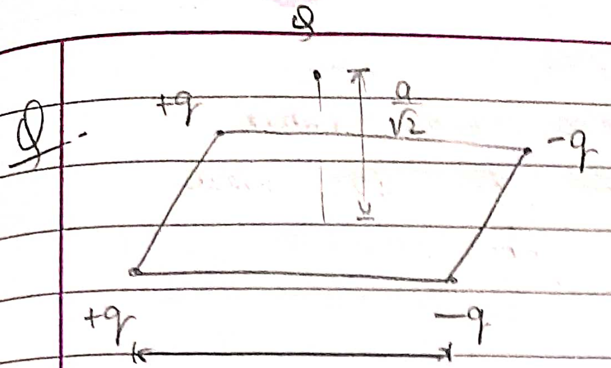


$$F_{net} = 4\left(\frac{F}{\sqrt{2}}\right)$$

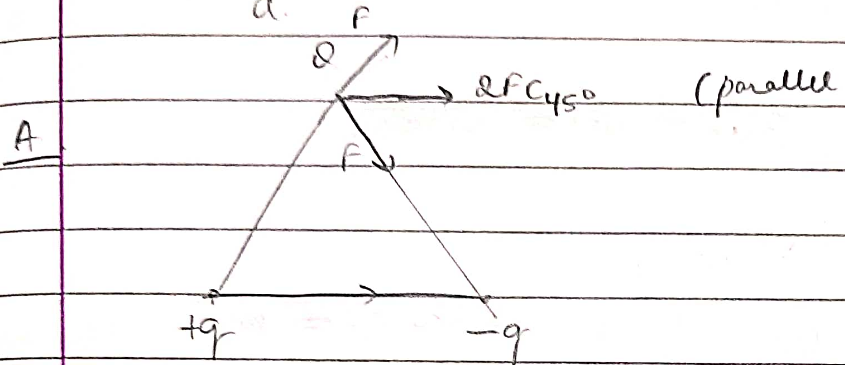
{Horizontal comps. cancel by sym.}

$$= \left(\frac{4}{\sqrt{2}}\right)\left(\frac{kQq}{a^2}\right)$$

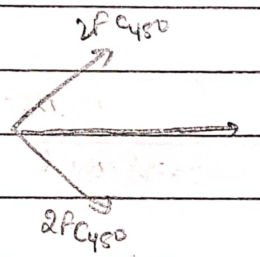
$$= \frac{2\sqrt{2}\,kQq}{a^2}$$

Q. 

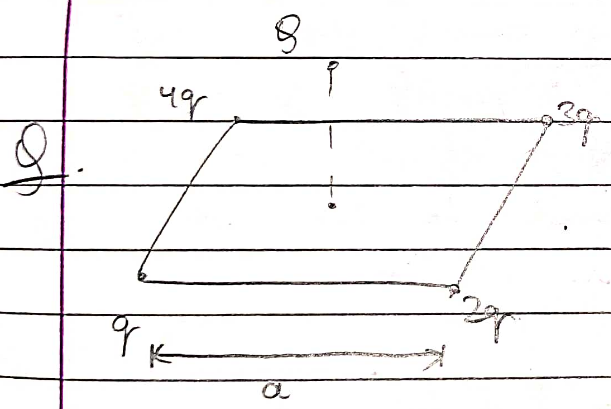
Find net force acting on particle.

A 

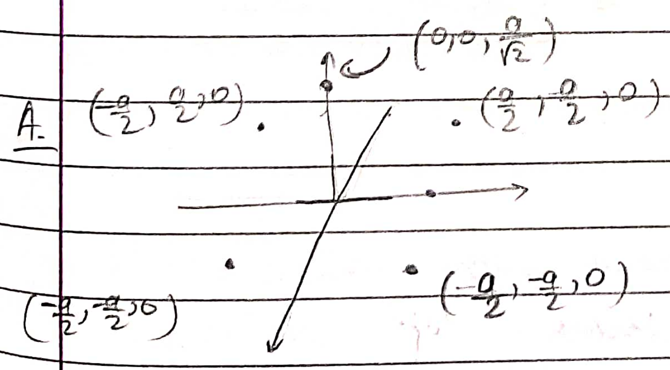
(parallel to diagonal 1)

Top view : 

$2(2F\cos45^\circ)\cos45^\circ = 2F = \frac{2kQq}{a^2}$

Q. 

Find net force acting on particle.

A. 

$$\vec{F}_{net} = \frac{kQ}{a^3}\Big[2q\left\langle \frac{-a}{2}\ \frac{a}{2}\ \frac{a}{\sqrt{2}} \right\rangle + 3q\left\langle \frac{-a}{2}\ \frac{-a}{2}\ \frac{a}{\sqrt{2}} \right\rangle + 4q\left\langle \frac{a}{2}\ \frac{-a}{2}\ \frac{a}{\sqrt{2}} \right\rangle + q\left\langle \frac{a}{2}\ \frac{a}{2}\ \frac{a}{\sqrt{2}} \right\rangle \Big]$$

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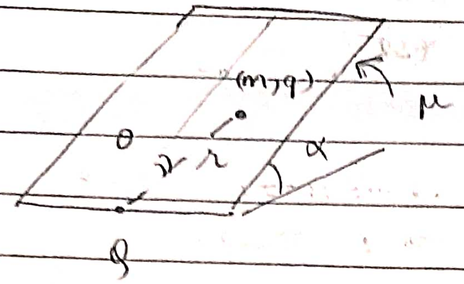
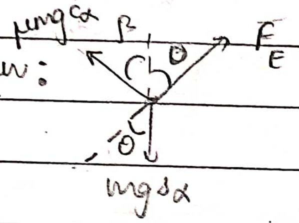
☆ Q.



Find max. value of θ for which eq. of charged particle is possible.

A. $N = mg\,c\alpha \Rightarrow f_{max} = \mu N = \mu mg\,c\alpha$

X Top view:



① $F_E\,c\theta + \mu mg\,c\alpha\,c\beta = mg\,s\alpha$

② $F_E\,s\theta = \mu mg\,c\alpha\,s\beta$.

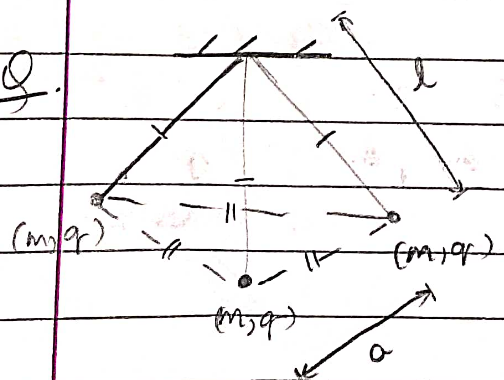
Consider comp. of $\vec{F_g}$ along & $\perp$ to $\vec{F_E}$.

We can only balance comp. of $\vec{F_g}$ along $\vec{F_E}$ by choosing suitable value of r.

☆ The comp. $\perp$ to $\vec{F_E}$ must be balanced by friction.

$\Rightarrow \mu mg\,c\alpha \geq mg\,s\alpha\,s\theta \Rightarrow \boxed{s\theta \leq \mu \cot(\alpha)}$

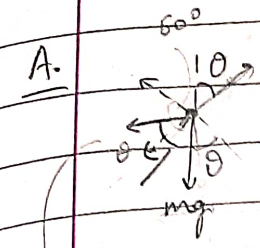
Q.

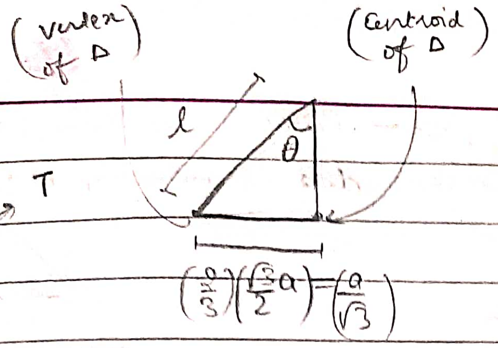


(3D shape)

Find q in terms of other qty.

A.





$\sin\theta = \left(\frac{a}{\sqrt{3}l}\right)$

$2F_E \cos 30^\circ = \sqrt{3}F_E$

$(\sqrt{3}F_E)(\sin\theta) = mg\cos\theta.$

$\Rightarrow (\sqrt{3})\left(\frac{kq^2}{a^2}\right)(\tan\theta) = mg.$

$\Rightarrow \boxed{q = \left(\frac{mga^3}{k\sqrt{9l^2 - 3a^2}}\right)^{1/2}}$

28/04/2023

ELECTRIC FIELD ($\vec{E}$)

Action at a distance

Effect of presence of q_1 is observed at q_2 via a field.

q_1 q_2

Charge (q_1) creates field → Charge (q_2) interacts with field → Experiences force

A charged body produces $\vec{E}$ around it, due to which, another charged body experiences force from the initial charge.

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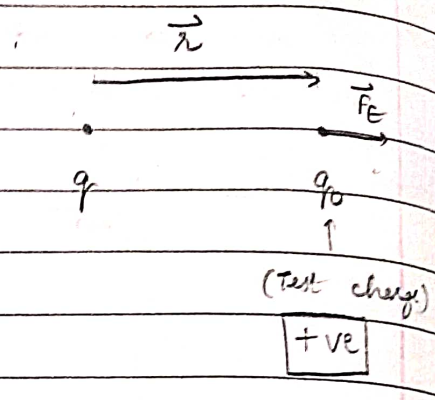
- Electric Field Intensity → Force experienced by unit test charge due to a given charge.

$$\boxed{\vec{E} = \lim_{q_0 \to 0} \left(\frac{\vec{F_E}}{q_0} \right)}$$



$$E = \frac{k q q_0}{q_0 r^2} \Rightarrow \boxed{E = \frac{kq}{r^2}}$$

NOTE: $q_0 \to 0$ as we want to keep $\vec{E}$ of q undisturbed while defining.

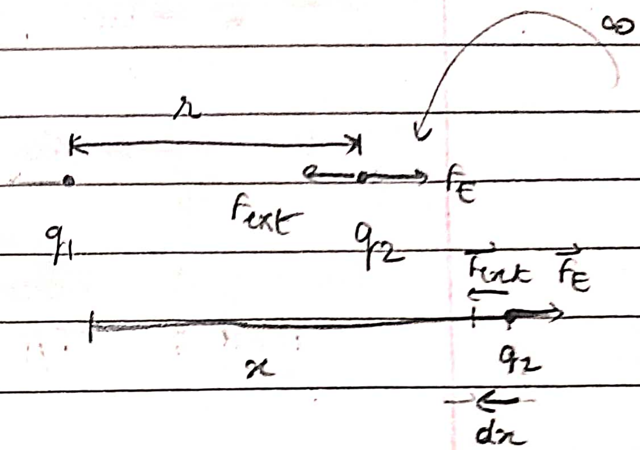
$$\boxed{\vec{E} = \left(\frac{kq}{r^2} \right) (\hat{r}) = \left(\frac{kq}{r^3} \right) (\vec{r})}$$

Law of superpos^n also holds for $\vec{E}$.

$$\vec{E}_{net} = \vec{E_1} + \vec{E_2} + \dots + \vec{E_n}$$

ELECTRIC POTENTIAL ENERGY (U) & ELECTRIC POTENTIAL (V)

U of a charged sys. is defined as $W_{F_{ext}}$ in slowly bringing charge particle from infinite sep to the given posi.



$$dW_{F_{ext}} = \vec{F}_{ext} \cdot d\vec{s}$$
$$= -\vec{F}_E \cdot d\vec{s} = \cancel{-} F_E\, ds\, \cancel{\cos\pi}$$
$$= -\frac{kq_1q_2}{x^2}\, dx$$

Here $ds = (-dx)$ since $x \downarrow$

$$W_{F_{ext}} = \int_{\infty}^{r} -\frac{kq_1q_2}{x^2}\, dx = kq_1q_2\left[\frac{1}{x}\right]_{\infty}^{r}$$
$$= \frac{kq_1q_2}{r}$$

if $q_1q_2 > 0 \Rightarrow U > 0$, $q_1q_2 < 0 \Rightarrow U < 0$

(similar charges) (F repulsion) (diff. charges) (F attraction)

When $U_\infty = 0$ & $U_{sys} < 0$

$\Rightarrow$ sys. is bounded

$\Rightarrow$ $F_{attraction}$ exists

Force & Potential Energy → Defined b/w 2 charges

Field & Potential → Defined for a pt. in space

Potential is defined as the work done in slowly bringing a unit test charge from ∞ to given posit^n.

$$V = \frac{W}{q_0} = \frac{kqq_0}{r(q_0)} = \frac{kq}{r}$$

Unit : J/C (V)
└ Volt

If a charge q is kept at a pt. with potential V,

$$\boxed{U = qV}$$

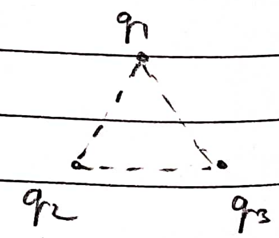
Similarly, if the field at that pt. is $\vec{E}$

$$\boxed{\vec{F} = q\vec{E}}$$

Law of superposit^n holds for V.

$$V_{net} = V_1 + V_2 + \dots + V_n$$

Also, $U_{sys} = U_{12} + U_{23} + U_{31}$



In general, $U_{sys} = \sum_{\substack{i=1 \\ j=i+1}}^{(n-1)} U_{ij}$

DATE (2 pairs at a) (1 pair at $\sqrt{2}a$)

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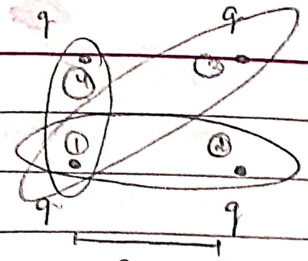
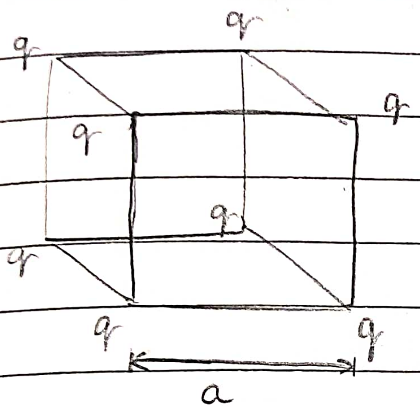
☆ eg.



$$U_{sys} = \underbrace{2}_{\left(\frac{N}{2}\right)\ (\#\text{particles})}\underbrace{\left[2\left(\frac{kq^2}{a}\right) + \left(\frac{kq^2}{\sqrt{2}a}\right)\right]}_{\text{P.E of } (N-1) \text{ pairs}}$$

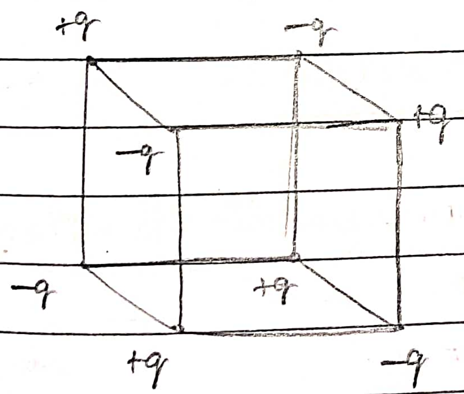
Q. Find total U_{sys}.



A $$U_{sys} = \left(\frac{8}{2}\right)\left[3\left(\frac{kq^2}{a}\right) + 3\left(\frac{kq^2}{\sqrt{2}a}\right) + \left(\frac{kq^2}{\sqrt{3}a}\right)\right]$$

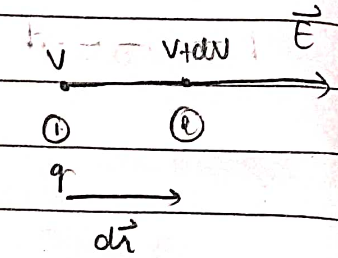
$$= \left(\frac{kq^2}{a}\right)\left[12 + \frac{12}{\sqrt{2}} + \frac{4}{\sqrt{3}}\right]$$

Q. Find total U_{sys}.



A $$U_{sys} = \left(\frac{8}{2}\right)\left[-3\left(\frac{kq^2}{a}\right) + 3\left(\frac{kq^2}{\sqrt{2}a}\right) - \left(\frac{kq^2}{\sqrt{3}a}\right)\right]$$

→ Rel^n b/w E & V



$$W_{F_E} = -\Delta U$$
$$= -(q(V+dV) - qV)$$
$$= -q\,dV$$

Also, $W_{F_E} = \vec{F_E}\cdot d\vec{r} = q\vec{E}\cdot d\vec{r}$

Equating, $-q\,dV = q\vec{E}\cdot d\vec{r}$

$$\Rightarrow \boxed{dV = -\vec{E}\cdot d\vec{r}}$$

if $d\vec{r}$ along $\vec{E}$ $\Rightarrow$ $dV = -E\,dr$.

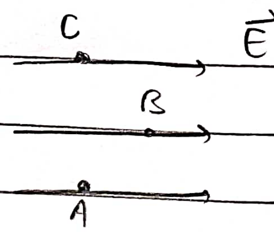
$$\Rightarrow \boxed{E = -\frac{dV}{dr}}$$

$$\boxed{E = -\left(\frac{dV}{dr}\right)(d\hat{r})}$$

NOTE: ① The potential (V) ↓ in the dir^n of field ($\vec{E}$) & ↑ in the dir^n opp to field.

② V is const. ⊥ to $\vec{E}$

i.e. $V_A = V_C > V_B$



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$$\vec{E} = \langle E_x \quad E_y \quad E_z \rangle$$
$$d\vec{r} = \langle dx \quad dy \quad dz \rangle$$

$$dV = -\vec{E}\cdot d\vec{r} = -(E_x dx + E_y dy + E_z dz)$$

if $dy = dz = 0 \Rightarrow dV = -E_x dx$

$$\Rightarrow E_x = -\left(\frac{dV}{dx}\right) \quad (y, z = \text{const.})$$

$$\equiv E_x = -\left(\frac{\partial V}{\partial x}\right)$$

$$\Rightarrow E_y = -\left(\frac{\partial V}{\partial y}\right), \quad E_z = -\left(\frac{\partial V}{\partial z}\right)$$

So, in general $\boxed{\vec{E} = -\left\langle \frac{\partial V}{\partial x} \quad \frac{\partial V}{\partial y} \quad \frac{\partial V}{\partial z} \right\rangle}$

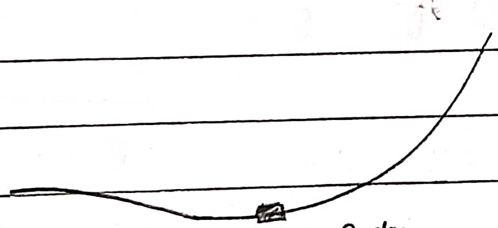
$$\boxed{\vec{F} = -\left\langle \frac{\partial U}{\partial x} \quad \frac{\partial U}{\partial y} \quad \frac{\partial U}{\partial z} \right\rangle}$$

CHARGE DISTRIUTIONS

Types of Charge Distribution :-

1. Linear

$$\begin{pmatrix}\text{Linear} \\ \text{charge density}\end{pmatrix} = \lambda$$



$dq = \lambda\, dx$

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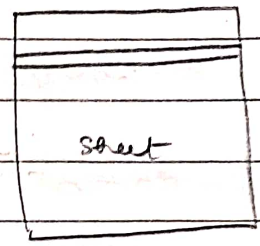
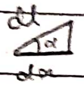
(2) Surface

(Integrating over linear element)

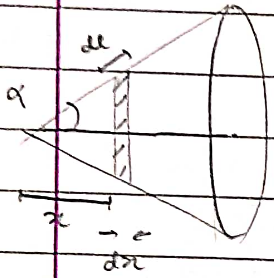
$\begin{pmatrix}\text{Surface charge} \\ \text{density}\end{pmatrix} = \sigma$

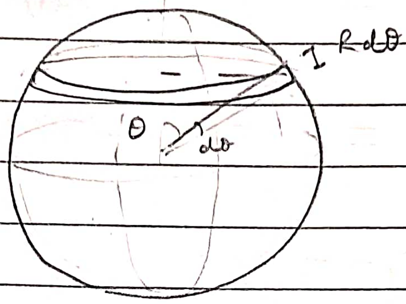


Hollow Cone

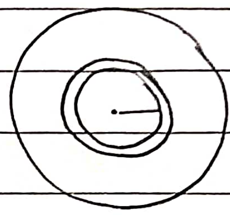


$dl = \frac{dx}{c\alpha}$





Hollow sphere



Disc

$dA = (2\pi x\, t\alpha)(dl)$
$= (2\pi x\, t\alpha)\left(\frac{dx}{c\alpha}\right)$

$dA = (2\pi R\, s\theta)(R\, d\theta)$

(3) Volumetric

$\begin{pmatrix}\text{Volumetric} \\ \text{charge density}\end{pmatrix} = \rho$

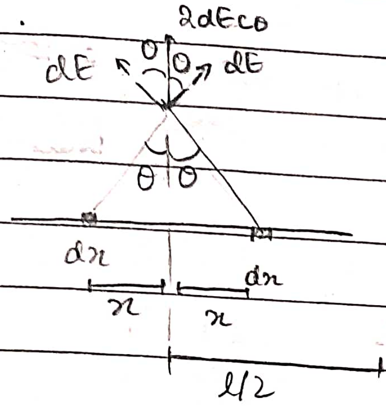
NOTE: (1) If field due to each elem in same dirn

$$\boxed{E = \int dE}$$

(2) Field at pt. lying on axis of symmetry.
$\Rightarrow$ Resultant field along axis of sym.

In such a case, we take elems. on each side of axis symmetrically.

eg.



$$E = \int_0^{l/2} 2\cos dE$$

$$= \int_{-l/2}^{l/2} dE \cos$$

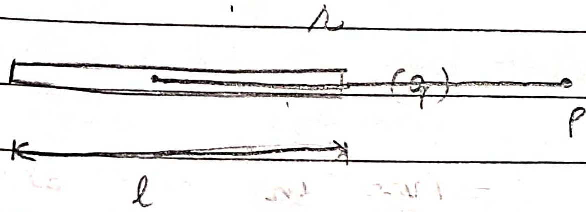
(3) If $\vec{E}$ due to elems in diff. dirn^{n}s.

$$\boxed{d\vec{E} = \langle dE_x \quad dE_y \quad dE_z \rangle}$$

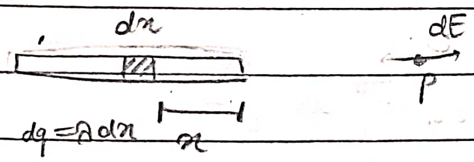
$$\vec{E} = \int d\vec{E} = \left\langle \int dE_x \quad \int dE_y \quad \int dE_z \right\rangle$$

$$= \langle E_x \quad E_y \quad E_z \rangle$$

Q. Find $\vec{E}$ & V at P.



A. (1)



$$dE = \frac{k\,dq}{(r-\frac{l}{2}+x)^2} = \frac{k\lambda}{(r-\frac{l}{2}+x)^2}dx.$$

$$\Rightarrow E = \int_0^l \frac{k\lambda}{(r-\frac{l}{2}+x)^2}dx = \left[\frac{-k\lambda}{(r-\frac{l}{2}+x)}\right]_0^l$$

$$= k\lambda\left(\frac{1}{r-\frac{l}{2}} - \frac{1}{r+\frac{l}{2}}\right) = \frac{k\lambda l}{r^2-\frac{l^2}{4}}$$

$$= \frac{4kq}{4r^2-l^2}$$

(2) $dV = \dfrac{k\lambda dx}{(r-\frac{l}{2}+x)}$

$$\Rightarrow V = \left[k\lambda \ln\left(r-\frac{l}{2}+x\right)\right]_0^l = \boxed{\frac{kq}{l}\ln\left(\frac{2r+l}{2r-l}\right)}$$

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if $r >> l \Rightarrow E = \frac{4kq}{4r^2} = \frac{kq}{r^2}$

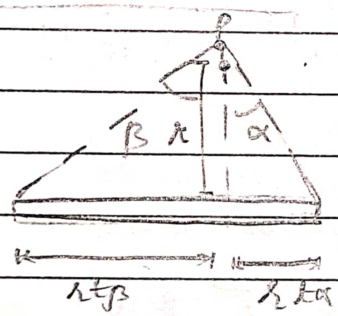
NOTE: Always check in options whether $r >> l$ gives E & V for pt. charge.

$$-V = \frac{kq}{l}\left[\ln\left(1+\frac{l}{2r}\right) - \ln\left(1-\frac{l}{2r}\right)\right]$$

$$= \frac{kq}{l}\left[\frac{l}{2r} - \left(\frac{-l}{2r}\right)\right] \qquad \left\{ \because \ln(1+x) \sim x \right\}$$

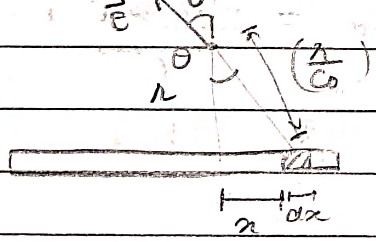
$$= \frac{kq}{r}$$

Q.



Find E & V at P

A.



$$dE = \frac{k\,dq}{r^2\sec^2(\theta)} = \frac{k\lambda\, r\sec^2(\theta)\,d\theta}{r^2\sec^2(\theta)}$$

$$= \left(\frac{k\lambda}{r}\right) d\theta$$

$x = r\tan\theta$

$\Rightarrow dx = r\sec^2(\theta)\,d\theta$

$$E_y = \int dE\cos\theta = \int_{-\beta}^{\alpha}\left(\frac{k\lambda}{r}\right)\cos\theta\,d\theta$$

$$= \boxed{\left(\frac{k\lambda}{r}\right)(\sin\alpha + \sin\beta)}$$

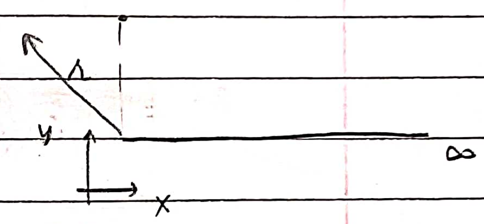
$$E_x = \int dE\sin\theta = \int_{-\beta}^{\alpha}\left(\frac{k\lambda}{r}\right)\sin\theta\,d\theta = \left(\frac{k\lambda}{r}\right)(\cos\beta - \cos\alpha)$$

$\vec{E} = \langle -E_x \quad E_y \rangle = \boxed{\left(\frac{k\lambda}{r}\right)\left\langle (C_\alpha - C_\beta) \quad (S_\alpha + S_\beta)\right\rangle}$

NOTE (1) $\alpha \rightarrow \angle$ to the side of +ve x-axis.

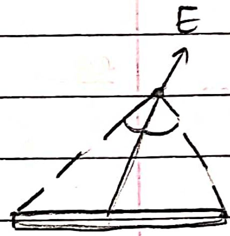
(2) for semi-infinite wire.

$\alpha = 90°, \quad \beta = 0°$

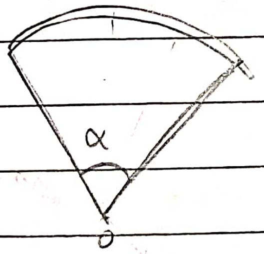


$$\boxed{\vec{E}_{si} = \left(\frac{k\lambda}{r}\right)\langle -1 \quad 1\rangle}$$

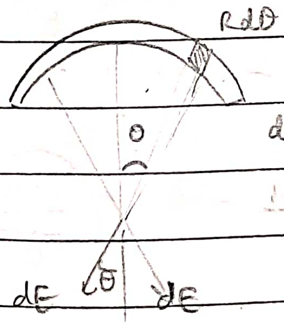
(3) $\vec{E}$ lies along angle bisector i.e



(See Next Pg)

Q. 

Find $\vec{E}$ at O.

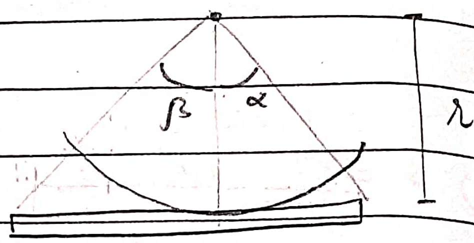
A. 

$dq = \lambda R d\theta$

$|\vec{dE}| = 2 dE \cos\theta = \frac{2k\cos\theta \lambda R d\theta}{R^2}$

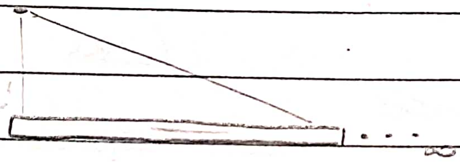
$\Rightarrow |\vec{E}| = \int_0^{\alpha/2} \frac{2k\lambda \cos\theta \, d\theta}{R} = \frac{2k\lambda \sin(\alpha/2)}{R}$

NOTE: The rod behaves as a sector of circle of radius (r) with $\vec{E}$ lying along angle bisector



& $\boxed{E = \left(\frac{2k\lambda}{r}\right) s\left(\frac{\alpha+\beta}{2}\right)}$

eg.



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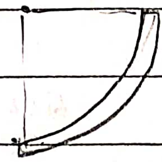
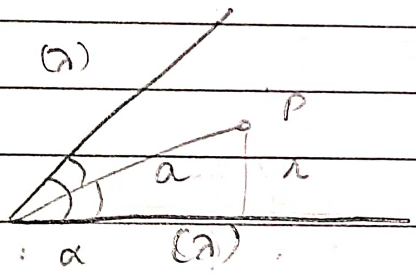
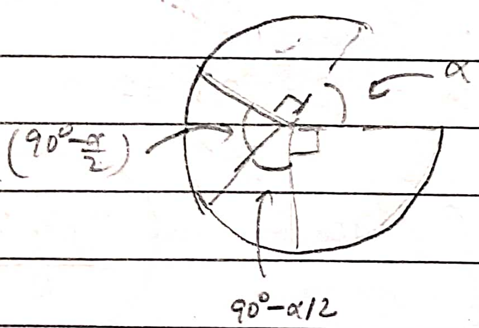


Q.



find $\vec{E_p}$

A.



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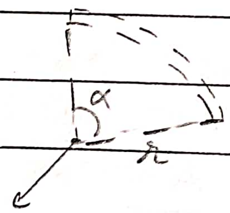
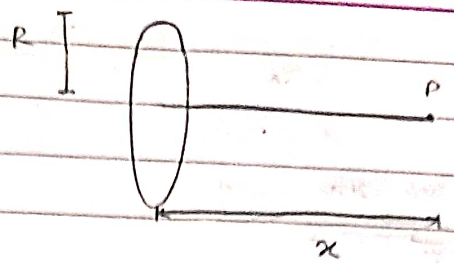


$E_p = \frac{2k\lambda}{r} s(\alpha/2) = \frac{2k\lambda}{a}$

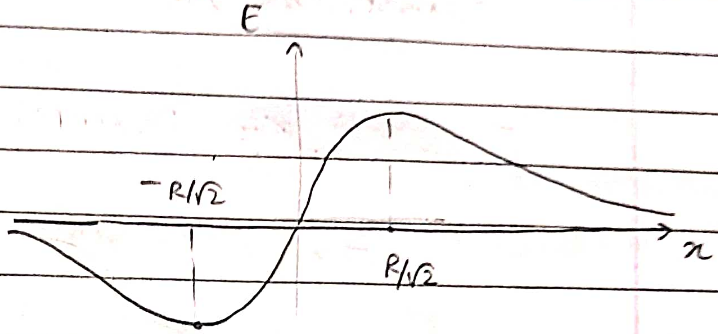
$r = a s(\alpha/2)$

→ Ring



$$\boxed{E = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{qx}{(x^2+R^2)^{3/2}}}$$

$$\boxed{V = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{q}{(x^2+R^2)^{1/2}}}$$



NOTE: If total charge q is not uniformly distributed,

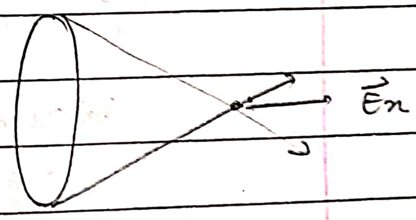
① still $\boxed{V = kqx/(x^2+R^2)^{1/2}}$

since every pt. is at equal dist. from P

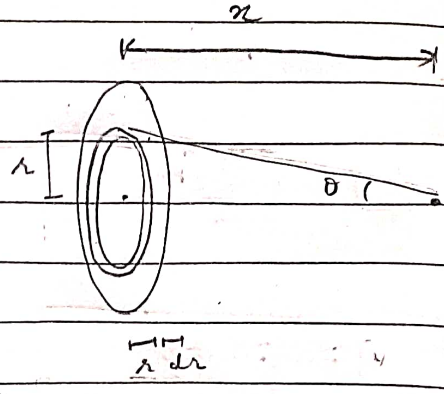
& V is a scalar

② $E_x = \frac{kqx}{(x^2+R^2)^{3/2}}$ ⟹ $E_{net} = \sqrt{E_x^2 + E_y^2}$

$E_y \neq 0$. ⟹ $\boxed{E_{net} > E_x}$



→ **Disc**



$$dV = \frac{k\,dq}{\sqrt{x^2+r^2}}$$

$$= \frac{k\sigma(2\pi r)(dr)}{\sqrt{x^2+r^2}}$$

$$= \frac{(2\pi\sigma k)\,\cancel{x}\tan\theta\,(x\sec^2(\theta)\,d\theta)}{\cancel{x}\sec(\theta)}$$

$$\left\{\begin{aligned} r &= x\tan\theta \\ dr &= x\sec^2(\theta)\,d\theta \end{aligned}\right\}$$

$$= 2\pi\sigma k x\,\sec(\theta)\tan\theta\,d\theta$$

$$V = \int_0^{\tan^{-1}(R/x)} dV = 2\pi\sigma k x\,\Big[\sec(\theta)\Big]_0^{\sec^{-1}\left(\frac{\sqrt{R^2+x^2}}{x}\right)}$$

$$= \boxed{\frac{\sigma}{2\epsilon_0}\left(\sqrt{R^2+x^2} - x\right)}$$

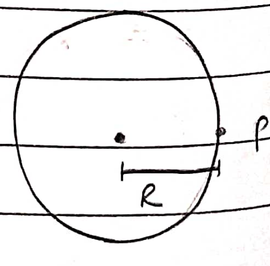
$$dE = \frac{kx\,dq}{(x^2+r^2)^{3/2}} = \frac{2\pi k\sigma x r\,dr}{(x^2+r^2)^{3/2}} = \frac{(2\pi k\sigma x)(x\tan\theta)\,(x\sec^2(\theta))\,d\theta}{x^3\sec^3(\theta)}$$

$$= (2\pi\sigma k)\sin\theta\,d\theta$$

$$\Rightarrow E = \int_0^{\cos^{-1}\left(\frac{x}{\sqrt{R^2+x^2}}\right)} (2\pi\sigma k)\sin\theta\,d\theta = \left(\frac{\sigma}{2\epsilon_0}\right)\Big[\cos\theta\Big]_{\cos^{-1}\left(\frac{x}{\sqrt{R^2+x^2}}\right)}^{0} = \boxed{\left(\frac{\sigma}{2\epsilon_0}\right)\left(\frac{x}{\sqrt{x^2+R^2}}\right)}$$

☆ Q. Find the V at rim of disc.

(σ)



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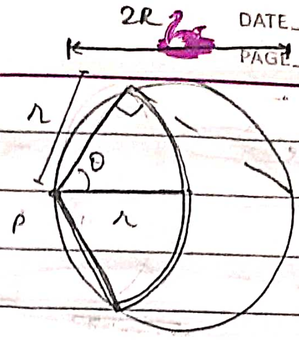
A. $dV = \frac{k\,dq}{r}$

$= \frac{(2k\sigma)(\cancel{r}\theta\,dr)}{\cancel{r}}$

$= (2k\sigma)\ \theta\,dr$



★ $r = 2R\cos\theta$

$dr = -2R\sin\theta\,d\theta$

$\Rightarrow V = (2k\sigma)\int_{\pi/2}^{0} -2R\sin\theta\,\theta\,d\theta$

$= -(4kR\sigma)\int_{\pi/2}^{0} \theta\sin\theta\,d\theta$

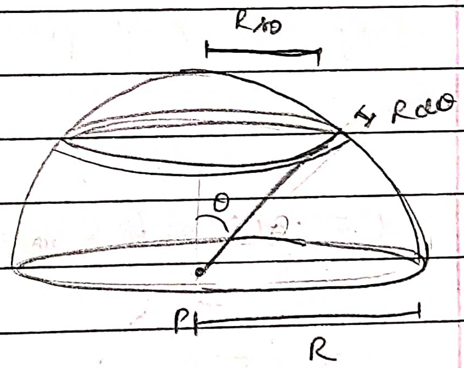
$u = \theta \Rightarrow du = d\theta$

$dv = \sin\theta\,d\theta \Rightarrow v = -\cos\theta$

$= -(4kR\sigma)\left[[-\theta\cos\theta]_{\pi/2}^{0} + \int_{\pi/2}^{0}\cos\theta\,d\theta\right]$

$= -(4kR\sigma)(-1) = \frac{\sigma R}{\pi\epsilon_0}$

→ Hemisphere (Shell)



$dE = \frac{k(2\pi R\sin\theta)(\sigma)(R\,d\theta)(R\cos\theta)}{R^3} \leftarrow (R^2\sin^2\theta + R^2\cos^2\theta)^{3/2}$

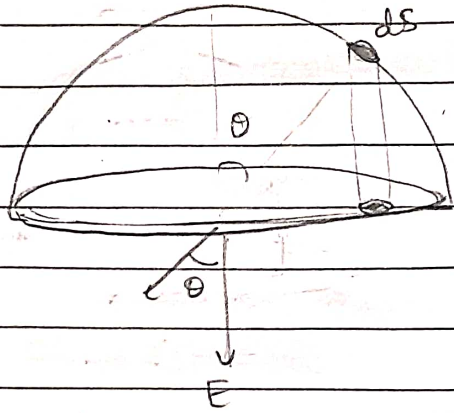
$= \frac{\sigma}{2\epsilon_0}\sin\theta\cos\theta\,d\theta$

$E = \frac{\sigma}{4\epsilon_0}\int_0^{\pi/2}\sin 2\theta\,d\theta \rightarrow E = \frac{\sigma}{8\epsilon_0}(1-(-1)) = \frac{\sigma}{4\epsilon_0}$

$dV = \frac{k(2\pi R\sin\theta)(\sigma)(R\,d\theta)}{R} = \left(\frac{\sigma R}{2\epsilon_0}\right)\sin\theta\,d\theta$

$\Rightarrow V = \left(\frac{\sigma R}{2\epsilon_0}\right)\int_0^{\pi/2}\sin\theta\,d\theta = \frac{\sigma R}{2\epsilon_0}$

☆



$$E = \int dE\, C_\theta$$

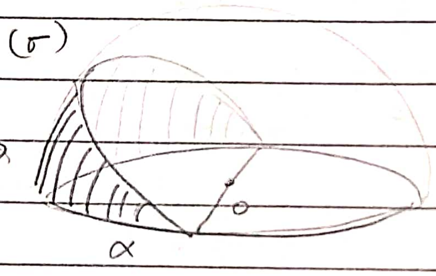
$$= \int \frac{k\sigma}{R^2} \underbrace{dS\, C_\theta}_{\text{Projection of } dS \text{ on x-y plane.}}$$

$$= \frac{k\sigma}{R^2} \underbrace{\int dS\, C_\theta}_{\pi R^2}$$

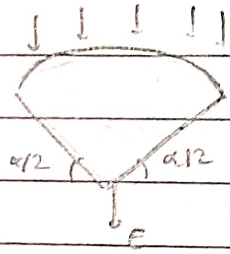
$$= \frac{\sigma}{4\epsilon_0}$$

☆ **Q.** (σ) Find $\vec{E}$ at centre

(cut out)

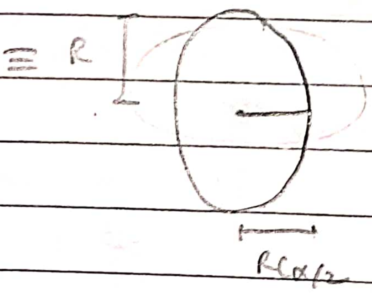


A. Proj. on x-y plane of



(Front View)

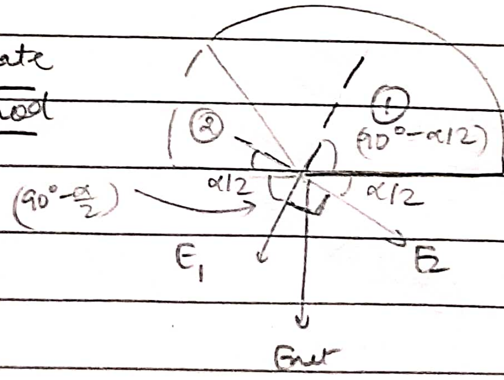
$$E = \left(\frac{\sigma}{4\epsilon_0 \pi R^2}\right)\left(\pi R^2 C_{\alpha/2}\right) \equiv$$



(Top view)

$$= \frac{\sigma\, C_{\alpha/2}}{4\epsilon_0}$$

Alternate Method



1. $\sqrt{E_1^2 + E_2^2} = \frac{\sigma}{4\epsilon_0}$

2. $E_1\, S_{\alpha/2} = E_2\, C_{\alpha/2}$
$\Rightarrow E_2 = E_1\, t_{\alpha/2}$

(Components ⊥ to resultant are same)

$E_2 \sec(\alpha/2) = \frac{\sigma}{4\epsilon_0} \Rightarrow \boxed{E_2 = \frac{\sigma\, C}{4\epsilon_0}}$

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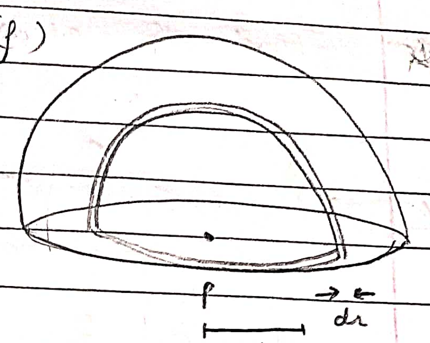
→ Hemisphere (Solid)



$$dE = \frac{\sigma}{4\epsilon_0} = \frac{\rho\, dr}{4\epsilon_0}$$

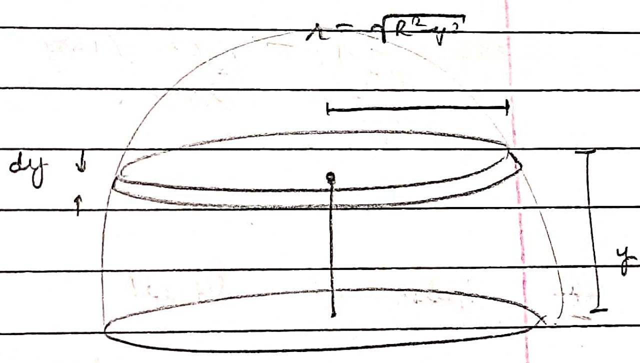
$$\Rightarrow E = \int_0^R \frac{\rho\, dr}{4\epsilon_0} = \frac{\rho R}{4\epsilon_0}$$

$q = q$

★ $(\cancel{2\pi r^2})(\sigma) = (\cancel{2\pi r^2})(dr)(\rho)$

$\Rightarrow \boxed{\sigma = \rho\, dr}$

OR



$$dE = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{y}{R}\right)$$

$$= \left(\frac{\rho}{2\epsilon_0}\right)\left(1 - \frac{y}{R}\right) dy$$

$$E = \left(\frac{\rho}{2\epsilon_0}\right)\int_0^R \left(1 - \frac{y}{R}\right) dy$$

$$= \left(\frac{\rho}{2\epsilon_0}\right)\left(R - \frac{R}{2}\right)$$

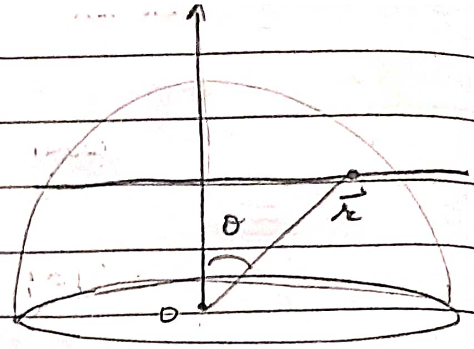
$$= \frac{\rho}{4\epsilon_0}$$

$q = q$

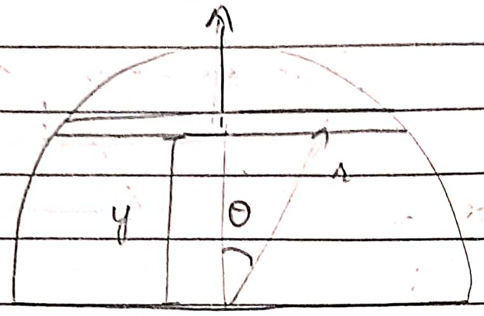
★ $(\pi r^2)(\sigma) = \rho(\pi r^2)(dy)$

$\Rightarrow \sigma = \rho\, dy$

Q. Find E at O.



A.



$$\rho = \vec{a} \cdot \vec{r}$$

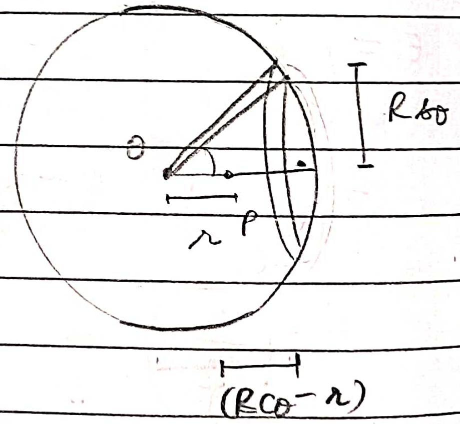
★ For all pts on disc, ρ is same.

$$\rho = ar\cos\theta = ay$$

so $$dE = \left(\frac{\rho}{2\epsilon_0}\right)\left(1 - \frac{y}{R}\right) dy = \left(\frac{a}{2\epsilon_0}\right)\left(y - \frac{y^2}{R}\right) dy$$

→ Spherical Shell

$$dE = \frac{k\,(2\pi R\sin\theta)(R\,d\theta)\,\sigma\,(R\cos\theta - r)}{\left(R^2\sin^2\theta + (R\cos\theta - r)^2\right)^{3/2}}$$



$$= \left(\frac{\sigma R^2}{2\epsilon_0}\right) \frac{(R\cos\theta - r)\sin\theta\, d\theta}{(R^2 + r^2 - 2Rr\cos\theta)^{3/2}}$$

$$= \left(\frac{\sigma R^2}{2\epsilon_0}\right)\left(\frac{1}{u^3}\right)\left(\frac{R^2 - r^2 - u^2}{2r}\right)\left(\frac{u}{Rr}du\right)$$

$$= \left(\frac{\sigma R}{4\epsilon_0 r^2}\right)\left[\left(R^2 - r^2\right)\left(\frac{1}{u^2}\right) - \frac{1}{R}\right] du$$

★ $R^2 + r^2 - 2Rr\cos\theta = u^2$

$\Rightarrow 2Rr\sin\theta\, d\theta = 2u\, du$

$\Rightarrow \sin\theta\, d\theta = \frac{u}{Rr} du$

$R\cos\theta = \left(\frac{R^2 + r^2 - u^2}{2r}\right)$

$\theta: 0 \to \pi$

$u: (R-r)^2 \to (R+r)^2$

$$E = \left(\frac{\sigma R}{4\epsilon_0 r^2}\right)\left[\left(R^2 - r^2\right)\left(\frac{1}{u}\right) + u\right]_{(R-r)^2}^{(R+r)^2}$$

$$= \left(\frac{\sigma R}{4\epsilon_0 r^2}\right) \frac{2R}{r(R^2 - r^2)} \qquad = 2R$$

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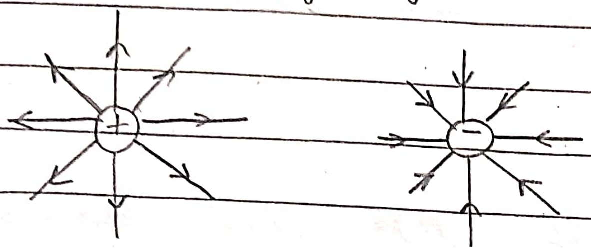
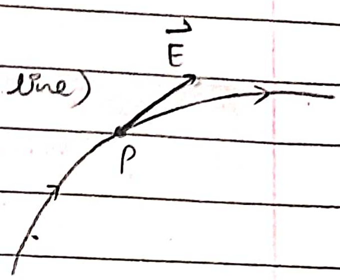
ELECTRIC FIELD LINES & FLUX

- **Field lines** – give dirⁿ of $\vec{E}$ in space
  (along tangent at a pt. on the line)

eg.





→ **Characteristics of $\vec{E}$ lines** :-

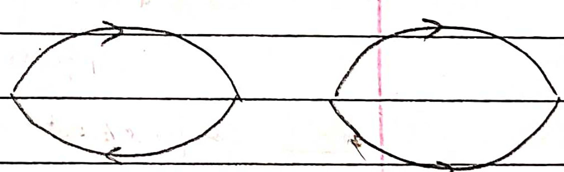
(due to a charge) ⟶ Electrostatic field
(~~NOT induced electric field~~)

(1) Starts at +ve charge (source) & terminates at –ve charge (sink)

(2) Non – intersecting

(3) Can't join two similar charges

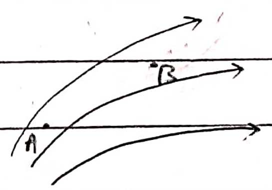
(4) Can't be closed curves.
⇒ $\vec{E}$ is conservative.



Closed (NOT possible) Open (Possible)

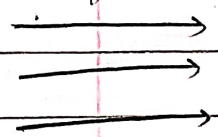
(5) Concentration of field lines qualitatively describe $\vec{E}$ intensity.

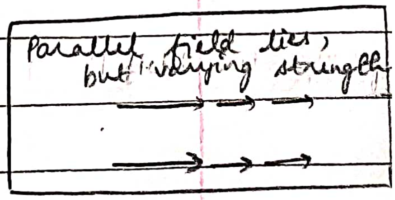
$\boxed{(\text{Strength}) \propto (\text{Density})}$

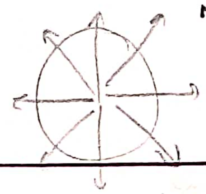


$E_A > E_B$

uniform field





eg 

$$\frac{N}{4\pi r^2} = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q}{r^2}\right) \leftarrow E$$

Field lines / Normal Area

$$\Rightarrow \boxed{N = \frac{q}{\epsilon_0}}$$

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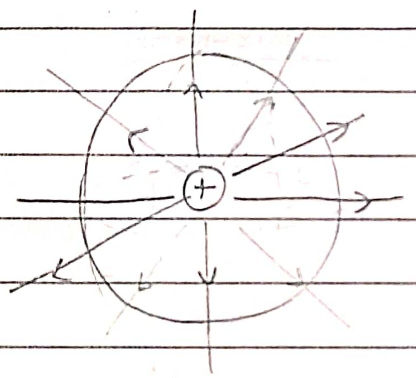
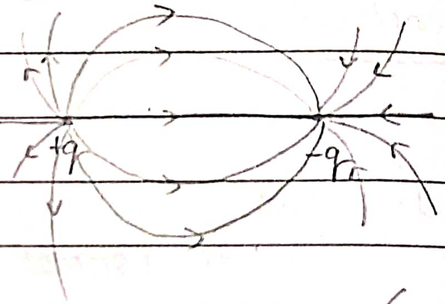
(6) $\vec{E}$ strength is the # field lines passing through unit normal area.

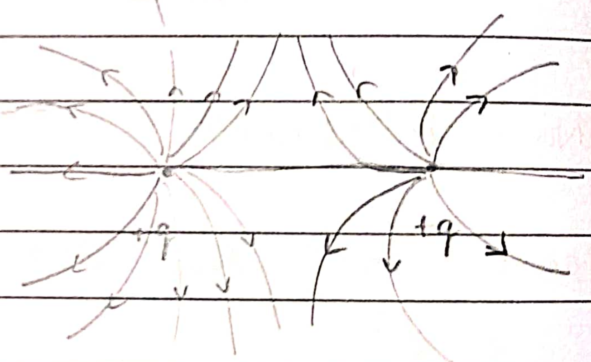
So, we draw, $\boxed{N \propto |q|}$

(# field lines)

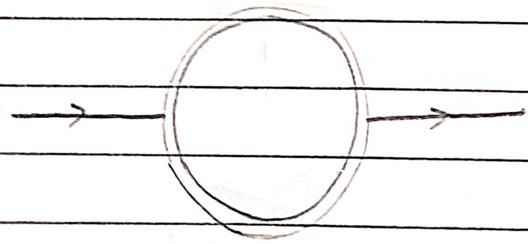
(7) The distribution of field lines around a pt. charge is symmetrical.

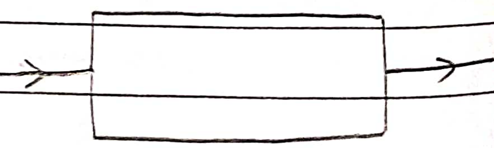






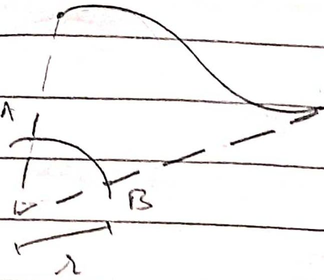
(8) Not present inside conductor.
They emerge & terminate at surface.

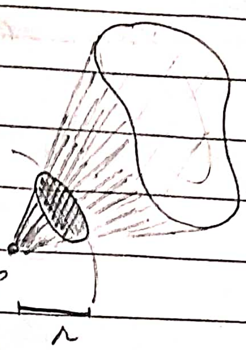




NOTE:

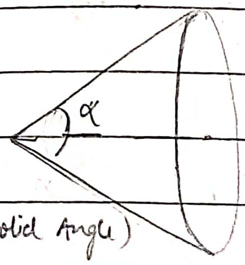
Plane Angle | **Solid Angle**



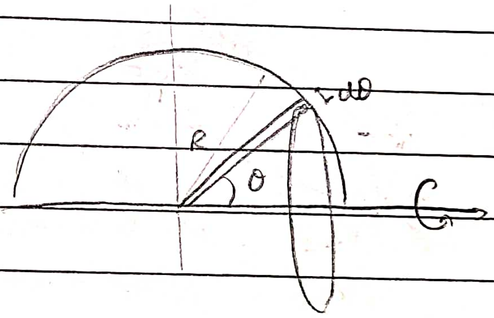


$$\theta = \frac{\overset{\frown}{AB}}{r} \quad (\text{in rad})$$

$$\Omega = \frac{\text{Area}}{r^2} \quad (\text{in steradian})$$



(Solid Angle)



$$dA = (2\pi R s\theta)(R\,d\theta)$$

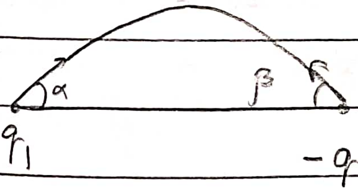
$$A = \int_0^{\alpha/2} 2\pi R^2 s\theta\, d\theta = \underline{2\pi R^2(1-c\alpha)}$$

$$S.A = \frac{2\pi R^2(1-c\alpha)}{R^2}$$

$$= \boxed{2\pi(1-c\alpha)}$$

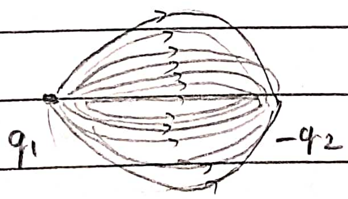
$$\Rightarrow (\#\text{ field lines}) = \left(\frac{q}{\epsilon_0}\right)(S.A) = \boxed{\frac{2\pi q}{\epsilon_0}(1-c\alpha)}$$

Q.

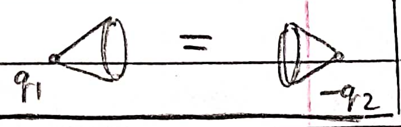


Find $\left|\frac{q_1}{q_2}\right|$

A. In reality (3D)



$\Rightarrow$ ★ # field lines in



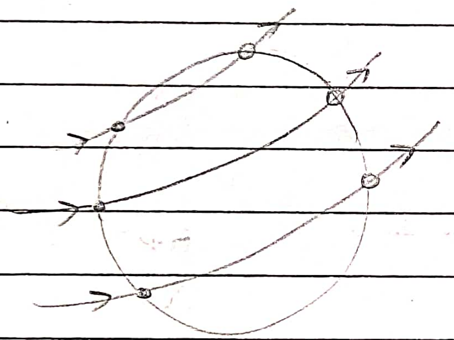
$$\Rightarrow \frac{2\pi}{\epsilon_0} q_1(1-c\alpha) = \frac{2\pi}{\epsilon_0} q_2(1-c\beta) \Rightarrow \boxed{\frac{q_1}{q_2} = \frac{s^2\beta/2}{s^2\alpha/2}}$$

- **Flux** — # field lines passing through a surface

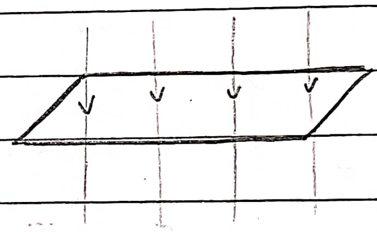
$$\boxed{d\varphi = \vec{E} \cdot d\vec{S}}$$

Area vector
(⊥ to surface at a given pt.)

Closed surface



Open surface



-o — $d\varphi < 0$ (field line entering surface)

o — $d\varphi > 0$ (field line exiting surface)

$$\varphi = \oint_S \vec{E} \cdot d\vec{S}$$

(Represents closed surface)

$$\varphi = \int_S \vec{E} \cdot d\vec{S}$$

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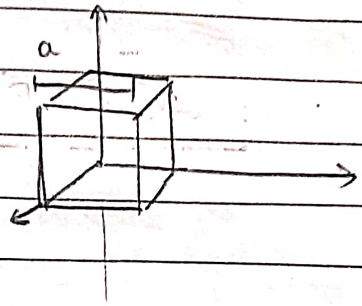
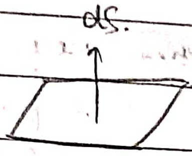
Q. In space $\vec{E} = E_0 \langle x \quad y \quad z \rangle$; x, y, z are coordinates.

Consider a cube of side $= a$, & one vertex at origin.
Find φ_{net}.



A. Consider the top face



$d\vec{S} = \langle 0 \quad dS \quad 0 \rangle$

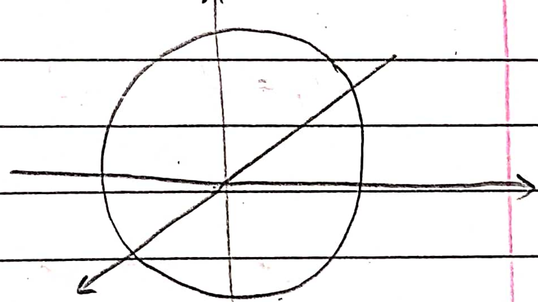
$d\varphi = \vec{E} \cdot d\vec{S} = E_0 y \cdot dS.$
$= E_0 a \cdot dS$ (since $y = a$ $\forall$ pts of top face)

$\Rightarrow \varphi = \int E_0 a \, dS = E_0 a \int dS = E_0 a (a^2) = \underline{E_0 a^3}$

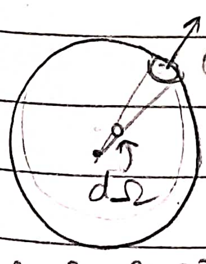
Similarly for faces, φ :
Top $- E_0 a^3$
Bottom $- 0$
Right $- E_0 a^3$
Left $- 0$
Front $- E_0 a^3$
Back $- 0$

$\varphi_{net} = \underline{3 E_0 a^3}$

Q. $\vec{E} = \vec{E_0} \langle x \quad y \quad z \rangle$.
Spherical shell of radius 'r'
centred at origin
Find φ



A



(x, y, z) $d\vec{S} = (d\Omega r^2) \dfrac{\langle x \quad y \quad z \rangle}{\sqrt{x^2 + y^2 + z^2}}$

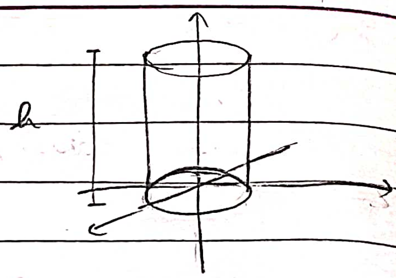
$d\varphi = \vec{E} \cdot d\vec{S} = E_0 r^2 \dfrac{x^2 + y^2 + z^2}{\sqrt{x^2 + y^2 + z^2}} d\Omega$
$= E_0 r^3 d\Omega$

$x^2 + y^2 + z^2 = R^2$

$\varphi = \oint_S E_0 r^3 d\Omega = E_0 r^3 \oint_S d\Omega = 4\pi E_0 r^3$

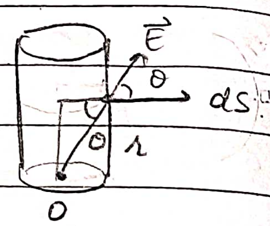
Q. $\vec{E} = E_0 \langle x \quad y \quad z \rangle$

Cylinder closed at both ends.



A. (1) $d\vec{s} = \dfrac{\langle x \quad y \quad 0 \rangle}{\sqrt{x^2+y^2}} dS$

$= \langle x \quad y \quad 0 \rangle \left(\dfrac{dS}{R}\right)$



$d\varphi = \vec{E} \cdot dS = \left(\dfrac{E_0}{R}\right)\left(\dfrac{x^2+y^2}{\sqrt{x^2+y^2}}\right)(dS)$

$= E_0 \, dS.$

$\varphi = \int E_0 \, dS = E_0 \int dS = \underline{E_0 (2\pi R h)}$

(2) Top face $\quad d\vec{S} = \langle 0 \quad 0 \quad dS \rangle$

$\varphi_T = \int \vec{E} \cdot d\vec{S} = \int_{top} E_0 z \, dS = E_0 h \int_{top} dS$

$= \underline{E_0 \pi h r^2}$

(3) Bottom face $\quad z = 0 \Rightarrow \underline{E_z = 0} \Rightarrow \underline{\varphi_B = 0}$

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GAUSS LAW

The flux through any closed surface is

$$\boxed{\frac{q_{en}}{\epsilon_0} = \oint_S \vec{E} \cdot d\vec{s}}$$

due to all the charges (~~outside & inside~~ the body)

NOTE: ① $\boxed{\frac{q_{en}}{\epsilon_0} = \oint_S \vec{E}_{\substack{\text{enclosed} \\ \text{charges}}} \cdot d\vec{s}}$ is also correct,

but it is not the statement of Gauss law.

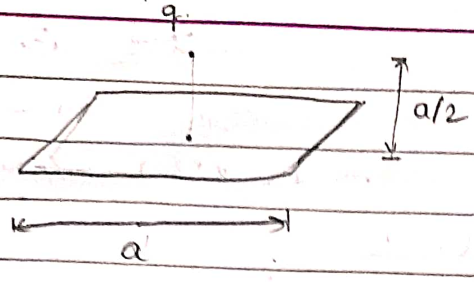
05/05/2023

② If pt. charge present at gaussian surface, Gauss's law not valid.

However, if continuous charge present on surface, it is valid.

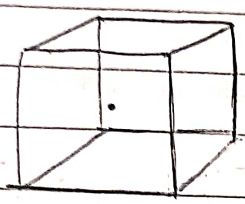
→ Finding φ

Q.



Find flux through plate.

A.

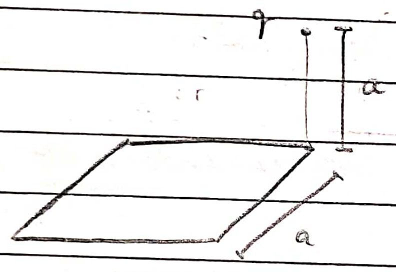


Symmetrically enclosed q.

$$\varphi_{Total} = \frac{q}{\epsilon_0}$$

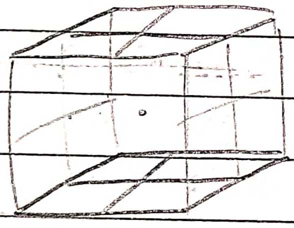
$$\varphi_{plate} = \boxed{\left(\frac{1}{6}\right)\left(\frac{q}{\epsilon_0}\right)}$$

Q



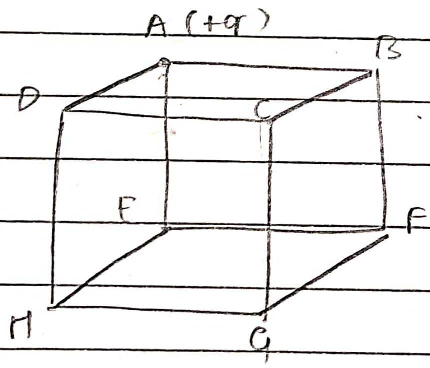
Find flux through plate.

A.



$$\varphi_{Total} = \frac{q}{\epsilon_0}$$

Q.



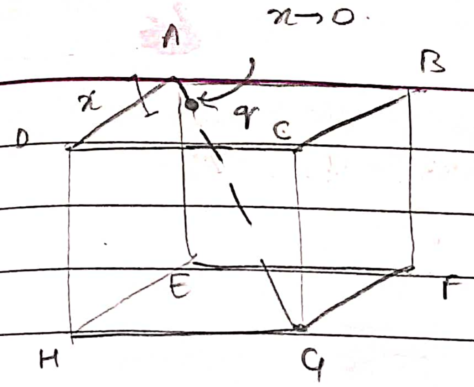
Find flux through each individual face.

A

$$\varphi_{AEHD} = \varphi_{ABCD} = \varphi_{ABFE} = 0$$

$$\varphi_{Rest} = \left(\frac{1}{24}\right)\left(\frac{q}{\epsilon_0}\right)$$

(Same as Q above)



Q. Charge shifted from A by a very small dist. x along main diagonal (AG).

Find flux through each face

A. Since $x \to 0$,

$\varphi_{Rest} = \left(\frac{q}{24\epsilon_0}\right)$ (in prev. Q)

$\Downarrow$

$\varphi_{others} = \varphi_{Total} - 3\varphi_{Rest}$

$= \frac{q}{\epsilon_0} - \frac{3q}{24\epsilon_0}$

$= \frac{7q}{8\epsilon_0}$

$\Rightarrow \underbrace{\varphi_{ABCD} + \varphi_{AEHD} + \varphi_{ABFE}}_{\text{(equal, by symmetry abt diagonal)}} = \frac{7q}{8\epsilon_0}$

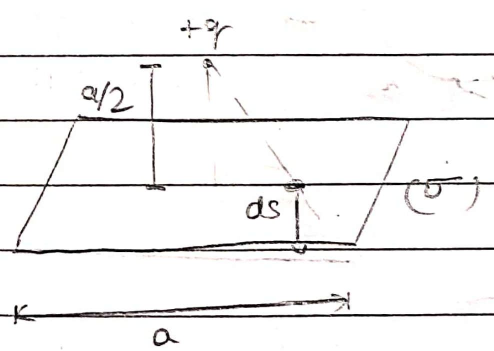
$\Rightarrow \boxed{\varphi_{ABCD} = \frac{7q}{24\epsilon_0}}$

Q. In the above Q., if q shifted outside cube.

A. $\varphi_{Rest} = \frac{q}{24\epsilon_0} \Rightarrow \varphi_{others} = \varphi_{Total} - 3\varphi_{Rest} = 0 - \frac{3q}{24\epsilon_0}$

$= -\frac{q}{8\epsilon_0}$

$\boxed{\varphi_{Each} = \frac{-q}{24\epsilon_0}}$



Q. Find net force exerted by charge q on the plate.

A. $\varphi = \int \vec{E} \cdot \vec{ds} = \int E\, dS \cos\theta = \frac{q}{6\epsilon_0}$

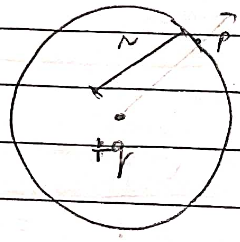
$F = \int dF \cos\theta = \int E\sigma\, dS \cos\theta = \sigma \int E\, dS \cos\theta = \frac{q\sigma}{6\epsilon_0}$

→ **Finding $\vec{E}$**

• **Spherical symmetry**

→ Pt. charge
→ spherically sym. charge distribution

eg (i)



(Pt charge)

$$(\vec{E} \parallel \vec{ds})$$

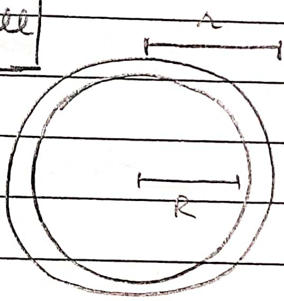
$$\varphi = \oint_S \vec{E} \cdot \vec{ds} = \oint_S E\, ds$$

$$\Rightarrow \frac{q}{\epsilon_0} = E \oint_S ds = (4\pi r^2)(E)$$

$$\Rightarrow \boxed{E = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q}{r^2}\right)}$$

Uniformly charged spherical shell

(ii)



(Shell)

1. $r < R$, $E = 0$.

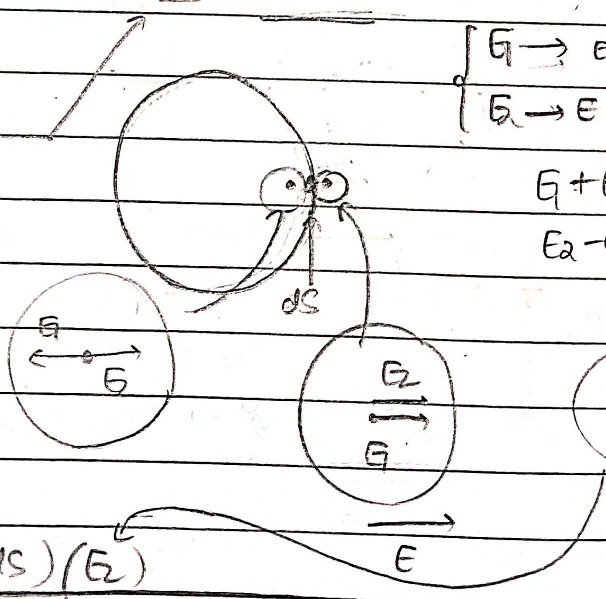
2. $r > 0$, $E = \left(\frac{kq}{r^2}\right)$

3. $r = R$

(Gauss's law not applicable)

$E_1 \to$ E due to ds
$E_2 \to$ E due to remaining sphere

$$E_1 + E_2 = E$$
$$E_2 - E_1 = 0.$$
$$\Downarrow$$
$$E_2 = \frac{E}{2}$$



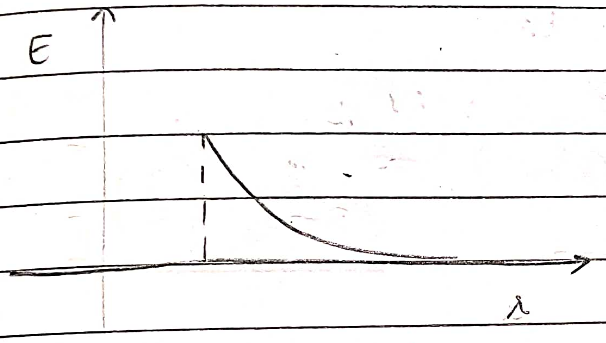
$\Rightarrow$ (Force on ds due to rest of the sphere) $= (\sigma\, ds)(E_2)$

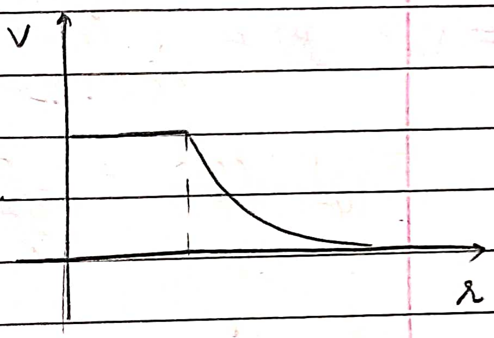
$$= \boxed{\left(\frac{q}{4\pi R^2}\right)\left(\frac{1}{8\pi\epsilon_0}\frac{q}{R^2}\right)(ds)}$$

(Electrostatic Pressure)

$$P = \frac{dF}{dS} = \boxed{\frac{q^2}{32\pi^2\epsilon_0 R^4}}$$

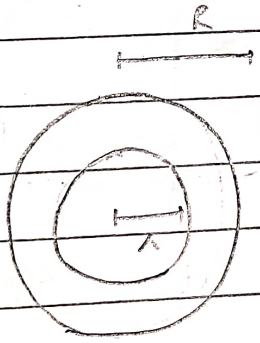
$$V = \begin{cases} \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r}\right) & ; \quad r \geq R \\ \frac{1}{4\pi\epsilon_0}\left(\frac{q}{R}\right) & ; \quad r \leq R \end{cases}$$





Uniformly charged sphere

(iii)



$$E = \begin{cases} \left(\frac{kq}{r^2}\right) & ; \quad r \geq R \\ \frac{\rho r}{3\epsilon_0} & ; \quad r < R \end{cases}$$

$$(4\pi r^2)(E_r) = \frac{q}{\epsilon_0} = \frac{\left(\frac{4}{3}\pi r^3\right)(\rho)}{\epsilon_0}$$

$$\Rightarrow E_r = \frac{\rho r}{3\epsilon_0}$$

$$dV = -\vec{E}\cdot d\vec{r}$$
$$= -\frac{\rho r}{3\epsilon_0}\,dr$$

$$V = \begin{cases} \left(\frac{kq}{r}\right) ; & r \geq R \\ \left(\frac{kq}{2R}\right)\left[3 - \frac{r^2}{R^2}\right] ; & r \leq R \end{cases}$$

$$\int_{V_r}^{V_s} dV = \int_r^R -\frac{\rho r}{3\epsilon_0}\,dr$$

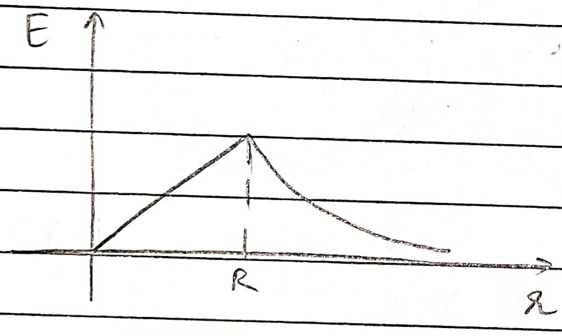
$$V_s - V_r = -\frac{\rho}{6\epsilon_0}\left[r^2\right]_r^R$$

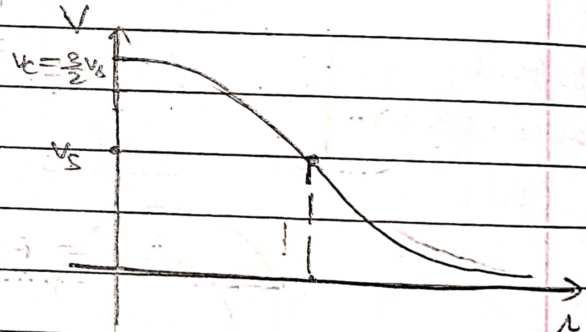
☆ $\boxed{V_c = \frac{3}{2} V_s}$

$$= -\frac{\rho}{6\epsilon_0}(R^2 - r^2)$$

$$\Rightarrow V_r = V_s + \frac{\rho}{6\epsilon_0}(R^2 - r^2)$$

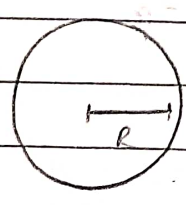
$$= \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q}{R}\right) + \frac{q}{\left(\frac{4\pi R^3}{3}\right)(6\epsilon_0)}(R^2 - r^2) = \underline{\frac{1}{(8\pi\epsilon_0)}\left(\frac{q}{R}\right)\left[3 - \frac{r^2}{R^2}\right]}$$





$$\text{slope} = \left(\frac{dV}{dr}\right)$$
$$= -E$$

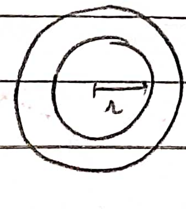
Q.



$$\rho(r) = \rho_0\left(1 + \frac{r}{R}\right)$$ r → dist from centre.

Find $\vec{E}$ at pt inside & outside sphere.

A.



By G.L, $$\oint E\cdot dS = \frac{q}{\epsilon_0} = \frac{\int \rho(r)\,dV}{\epsilon_0}$$

$$\Rightarrow (E)(4\pi r^2) = \frac{1}{\epsilon_0}\int (4\pi r^2)(\rho r)(dr)$$

$$\Rightarrow (4\pi r^2)(E) = \frac{1}{\epsilon_0}\int_0^r (4\pi\rho_0)\left(r^2+\frac{r^3}{R}\right) dr$$

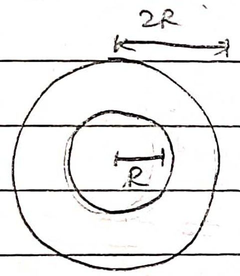
$$= \left(\frac{4\pi\rho_0}{\epsilon_0}\right)\left(\frac{r^3}{3}+\frac{r^4}{4R}\right)_0^r = \left(\frac{4\pi\rho_0}{\epsilon_0}\right)\left[\frac{r^3}{3}+\frac{r^4}{4R}\right]$$

$$\Rightarrow \boxed{E_r = \left(\frac{\rho_0}{\epsilon_0}\right)\left[\frac{r}{3}+\frac{r^2}{4R}\right]} \qquad \underline{r \le R}$$

For $r \geq R$,

$$q = \int_0^R \rho(r)(4\pi r^2)(dr) = (4\pi\rho_0)\left(\frac{7R^3}{12}\right) = \frac{7\pi\rho_0 R^3}{3}$$

$$E = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{7\pi\rho_0 R^3}{3r^2}\right) = \frac{7\rho_0 R^3}{12r^2}$$



Q. Thick spherical shell.
Find E at pts of shell $\underline{r \in [R, 2R]}$

A. $\rho = \frac{q}{\frac{4}{3}\pi(8R^3-R^3)} = \frac{3q}{28\pi R^3}$

By G.L,

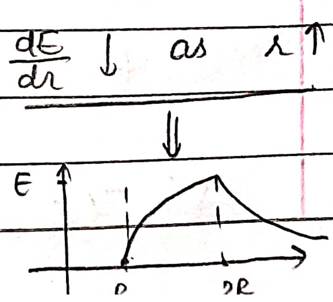
$$(E)(4\pi r^2) = \frac{1}{\epsilon_0}\int_R^r (4\pi r^2)\left(\frac{3q}{28\pi R^3}\right)(dr)$$

$$\Rightarrow \cancel{(4\pi r^2)}(E) = \left(\frac{\cancel{3}q}{28\pi\epsilon_0 R^3}\right)\frac{\cancel{(4\pi)}}{\cancel{3}}\left[r^3\right]_R^r$$

$$\Rightarrow \boxed{E = \frac{q}{28\pi\epsilon_0 r^2}\left[\left(\frac{r}{R}\right)^3-1\right]}$$

Graph $\frac{dE}{dr} = (\sim)\left(\frac{1}{R^3}+\frac{2}{r^3}\right) \rightarrow \underline{\frac{dE}{dr}\downarrow \text{ as } r\uparrow}$

$\Downarrow$



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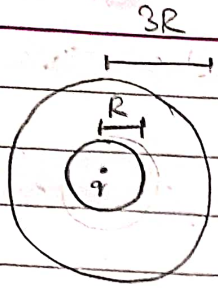
Q



$$\rho(r) = \frac{\rho_0}{r} \qquad r \in [R, 3R].$$

find mag. of q in terms of other qty. s.t. $\vec{E}$ has const. mag. for $r \in [R, 3R]$

A. By G.L

$$(E_r)(4\pi r^2) = \left(\frac{1}{\epsilon_0}\right)\int_R^r \rho(r)\, 4\pi r^2\, dr$$

$$= \left(\frac{1}{\epsilon_0}\right)\int_R^r \rho_0 (4\pi)\, r\, dr$$

$$\Rightarrow (\cancel{4\pi} r^2)(E_r) = \left(\frac{1}{\epsilon_0}\right)(\cancel{4\pi}\rho_0)\left(\frac{1}{2}\right)(r^2 - R^2)$$

$$\Rightarrow E_r = \frac{\rho_0}{2\epsilon_0}\left(1 - \frac{R^2}{r^2}\right)$$

Acc. to Q,

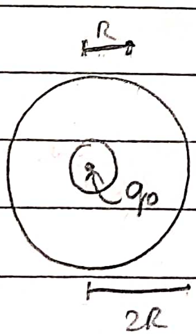
$$E_{net} = E_q + E_r = \text{const}.$$

$$\Rightarrow \boxed{E_q = \frac{-\rho_0 R^2}{2\epsilon_0 r^2}}$$

$$\Rightarrow \left(\frac{1}{2\cancel{4\pi\epsilon_0}}\right)\left(\frac{q}{\cancel{r^2}}\right) = -\frac{\rho_0 R^2}{\cancel{2\epsilon_0 r^2}}$$

$$\Rightarrow \boxed{q = -2\pi \rho_0 R^2}$$

Q



Find $\rho(r)$ for which E is const for $r \in [R, 2R]$

A. By G.L

$$(E_x)(4\pi x^2) = \frac{\left[q_0 + \int_R^x \rho(x)(4\pi x^2)\, dx\right]}{\epsilon_0}$$

$$\left\{ \frac{d}{dx}\left(\int_{L(x)}^{U(x)} f(x)\, dx \right) = f(U(x))\, U'(x) - f(L(x))\, L'(x) \right\}$$

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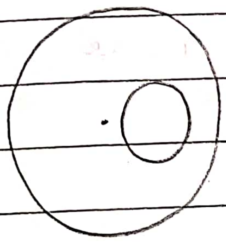
Differentiating wrt x, $(\cancel{4\pi} E)(2\cancel{r}) = 0 + \frac{1}{\epsilon_0}(\cancel{4\pi})\left[r^{\cancel{2}}\rho(r)(1) - R^2\rho(R)(0) \right]$

$$\Rightarrow \rho(r) = \frac{2E\epsilon_0}{r} = \frac{q_0}{2\pi R^2 r}$$

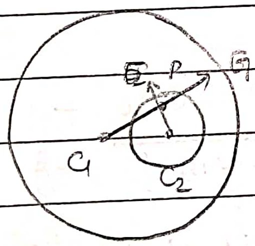
Since for $r = R$, $E = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q_0}{R^2}\right)$

★ Q



Field inside a spherical cavity formed within a uniformly charged sphere of radius R & ρ.

A.



$$\vec{E_1} = \left(\frac{\rho}{3\epsilon_0}\right)(\vec{C_1P})$$

$$\vec{E_2} = \left(\frac{\rho}{3\epsilon_0}\right)(\vec{C_2P})$$

$$\vec{E_1} - \vec{E_2} = \left(\frac{\rho}{3\epsilon_0}\right)(\vec{C_1P} - \vec{C_2P}) = \left(\frac{\rho}{3\epsilon_0}\right)(\vec{C_1C_2})$$

$\hookrightarrow \vec{C_1P} + \vec{PC_2}$

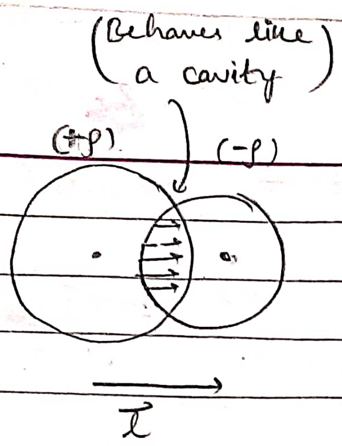
Therefore, $\vec{E}$ is const. in both mag. & dirn

$$\boxed{\vec{E} = \left(\frac{\rho}{3\epsilon_0}\right)(\vec{l})}$$

$\vec{l}$: (Sphere centre to cavity centre)

* Independent of size of cavity

NOTE:



$$\boxed{\vec{E} = \left(\frac{\rho}{3\varepsilon_0}\right)(\vec{l})}$$

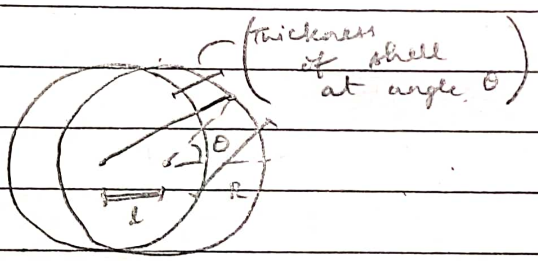
☆ Q

(Shell)

$\sigma = \sigma_0 \cos\theta$

Prove that E inside shell is const.

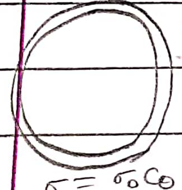
A. Consider 2 spheres of radius 'R' each having $+\rho$ & $-\rho$ density intersecting with their centres, l dist. apart.



$$= \sqrt{l^2 + R^2 + 2lR\cos\theta} - R$$
$$= R\sqrt{1 + \frac{2l\cos\theta}{R} + \frac{l^2}{R^2}} - R$$
$$= R\left(1 + \left(\frac{1}{2}\right)\frac{2l\cos\theta}{R}\right) - R = \underline{l\cos\theta}$$

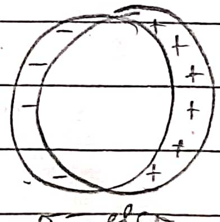
Now, $\sigma_\theta = \rho\,(\text{Thickness})_\theta = \rho l\cos\theta \Rightarrow \boxed{\sigma_0 = \rho l}$

☆ $\boxed{\left(\text{Shell with uniform thickness but varying } \rho\right) = \left(\text{Shell with uniform } \rho \text{ but varying thickness}\right)}$



$\sigma = \sigma_0\cos\theta$

$\equiv$



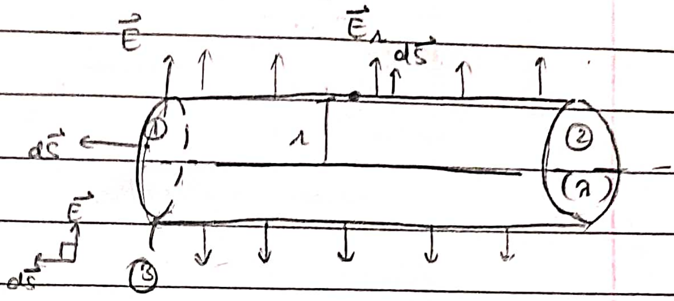
$\sigma = \rho l\cos\theta$

$$\Rightarrow \vec{E}_{\text{inside shell}} = \vec{E}_{\text{cavity}} = \frac{\rho l}{3\varepsilon_0} = \frac{\sigma_0}{3\varepsilon_0}$$

Since cavity $\Rightarrow \vec{E}$ uniform

- Cylinderical symmetry

Infinitely long wire



eg:- (i) $\varphi_1 = \varphi_2 = 0$

$(\vec{E} \perp d\vec{s})$

$$\varphi_3 = \oint_S \vec{E}\,d\vec{s} = \int E\,dS$$

$$= E\int dS$$

$$\Rightarrow \frac{q_{en}}{\epsilon_0} = E(2\pi r l) \Rightarrow E = \frac{\lambda l}{2\pi r l \epsilon_0} = \boxed{\left(\frac{1}{2\pi\epsilon_0}\right)\left(\frac{\lambda}{r}\right)}$$

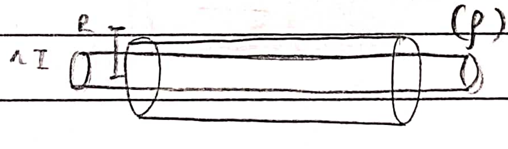
Long cylinderical shell

(ii) $E_r = \begin{cases} 0, & r < R \\ \dfrac{2\pi R\sigma}{2\pi\epsilon_0 r}, & r \geq R \end{cases}$

$$\varphi = \oint E\cdot dS = \frac{q_{en}}{\epsilon_0} \Rightarrow E(2\pi r \cancel{l}) = \frac{\sigma(2\pi R\cancel{l})}{\epsilon_0}$$

$$\Rightarrow \boxed{E = \frac{(2\pi R\sigma)}{2\pi\epsilon_0 r} = \frac{\lambda}{2\pi\epsilon_0 r}}$$

uniformly charged solid cylinder



(iii) $E_r = \begin{cases} \dfrac{\rho\vec{r}}{2\epsilon_0}, & r < R \\ \dfrac{\rho(\pi R^2)}{2\pi r\epsilon_0}, & r \geq R \end{cases}$

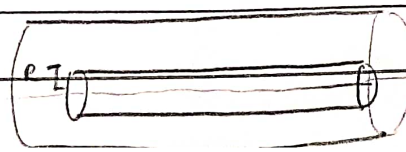
$$\oint_S E\,dS = \frac{q_{en}}{\epsilon_0}$$

$$\Rightarrow (E)(2\pi r\cancel{l}) = \frac{(\rho)(\pi r^2)(\cancel{l})}{\epsilon_0}$$

$$\Rightarrow E = \frac{\rho r}{2\epsilon_0}$$

$$\oint_S E\,dS = \frac{q_{en}}{\epsilon_0}$$

$$\Rightarrow (E)(2\pi r\cancel{l}) = \frac{\rho(\pi R^2)(\cancel{l})}{\epsilon_0}$$



$$\Rightarrow \boxed{E = \frac{\pi R^2\rho}{2\pi r\epsilon_0} = \frac{\lambda}{2\pi r\epsilon_0}}$$

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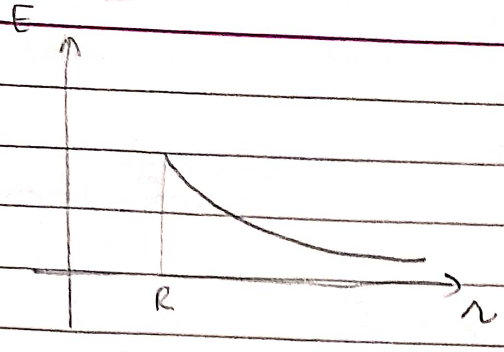
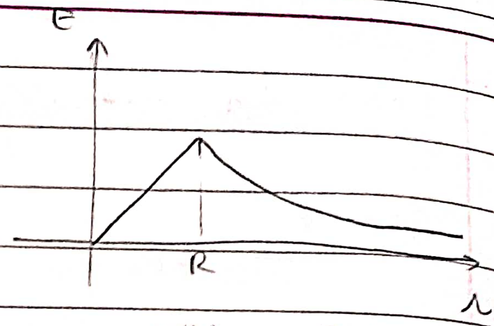


Hollow cylinder



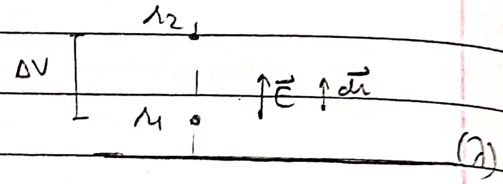
Solid cylinder

Potential diff

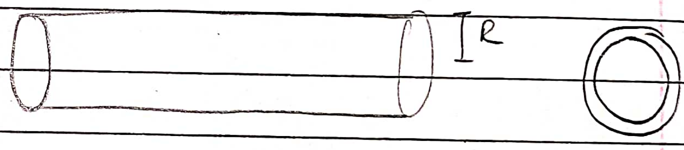
(iv) $dV = -E \cdot dr$

$\Rightarrow V_2 - V_1 = \int_{r_1}^{r_2} -E\, dr = \frac{-\lambda}{2\pi\epsilon_0} \int_{r_1}^{r_2} \frac{dr}{r}$

$\Rightarrow \boxed{\Delta V = \frac{-\lambda}{2\pi\epsilon_0} \ln\left(\frac{r_2}{r_1}\right)}$



Q. $\rho(r) = \rho_0 r$



A. ① $r < R$ By G.L., $\oint E \cdot ds = \frac{q_{en}}{\epsilon_0} \Rightarrow (E_r)(2\pi r l) = \frac{2\pi\rho_0 l r^3}{3\epsilon_0}$

$q_{en} = \int dq = \int_0^r (2\pi r l)\rho(r)\, dr$

$= 2\pi\rho_0 l \int_0^r r^2\, dr = \left(\frac{2\pi\rho_0 l r^3}{3}\right)$

$\Rightarrow \boxed{E_r = \left(\frac{\rho_0 r^2}{3\epsilon_0}\right)}$

② $r \geq R$ By G.L (~) $\Rightarrow (E_r)(2\pi r l) = \frac{2\pi\rho_0 l R^3}{3\epsilon_0}$

$\Rightarrow \boxed{E_r = \frac{\rho_0 R^3}{3\epsilon_0 r}}$

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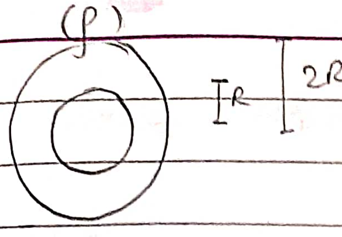
Q. 

Infinitely long cylinder

A. ① $r < R \Rightarrow \underline{E = 0}$

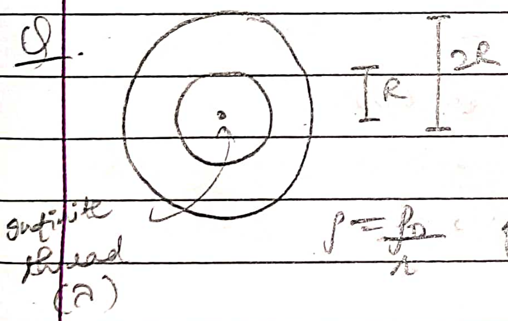
② $r \in [R, 2R]$ By G.L, $\oint E\,dS = \frac{q_{en}}{\epsilon_0}$

$$q_{en} = \int_R^r (\rho)(2\pi r l)(dr) = \underline{\pi \rho l (r^2 - R^2)}$$

$$\Rightarrow (E_r)(2\pi r l) = \frac{\pi \rho l (r^2 - R^2)}{\epsilon_0}$$

$$\Rightarrow \boxed{E_r = \frac{\rho}{2\epsilon_0 r}(r^2 - R^2)}$$

③ $r \geq 2R \Rightarrow (E)(2\pi r l) = 3\pi \rho l R^2 \Rightarrow \boxed{E = \frac{3\rho R^2}{2\epsilon_0 r}}$

Q. 

Infinitely long cylinder. Find λ s.t E_r ($r \in [R, 2R]$) has const. mag.

$\rho = \frac{\rho_0}{r}$ for $r \in [R, 2R]$

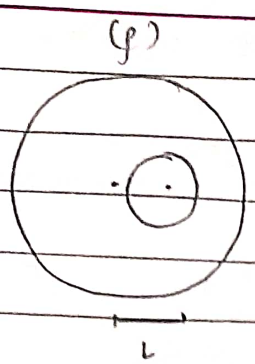
A. By G.L $\oint E\,dS = \frac{q_{en}}{\epsilon_0} \Rightarrow E_r(2\pi r l) = \frac{l(\lambda + 2\pi\rho_0(r - R))}{\epsilon_0}$

$$q_{en} = \lambda l + \int dq = \lambda l + \int_R^r \left(\frac{\rho_0}{\cancel{r}}\right)(2\pi \cancel{r} l)(dr) = \lambda l + 2\pi\rho_0(r - R)l$$

$$\Rightarrow E_r = \underbrace{\left(\frac{\lambda}{2\pi} - \rho_0 R\right)}_{0}\left(\frac{1}{\epsilon_0 r}\right) + \frac{\rho_0}{\epsilon_0}$$

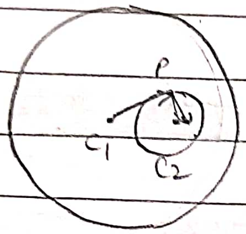
$$\Rightarrow \boxed{\lambda = 2\pi R \rho_0}$$

Q.



Find $\vec{E}$ inside cavity.

A.

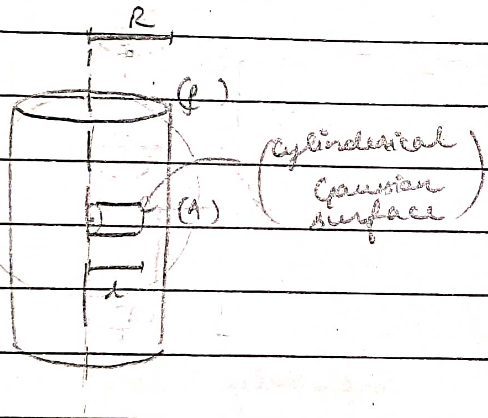


$$\vec{E_1} = \frac{\rho}{2\epsilon_0}\overrightarrow{C_1P}$$

$$\vec{E_2} = \frac{\rho}{2\epsilon_0}\overrightarrow{PC_2}$$

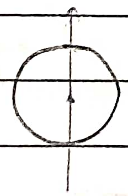
$$\vec{E} = \vec{E_1} + \vec{E_2} =$$

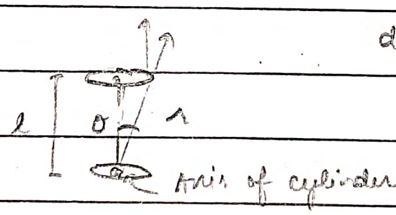
Q.



Axis ⊥.
Find φ through curved surface of this gaussian surface.

A.





Back view

Top view

$$d\varphi = (E\cos\theta)(dS) = \left(\frac{\rho r\cos\theta}{2\epsilon_0}\right)dS = \left(\frac{\rho l}{2\epsilon_0}\right)(dS)$$

$$\varphi_{flat(1)} = \frac{\rho l}{2\epsilon_0}\int(dS) = \left(\frac{\rho l}{2\epsilon_0}\right)(\pi R^2)$$

$$\varphi_{flat(2)} = 0$$

By G.L, $\varphi_{total} = \frac{q_{en}}{\epsilon_0}$ $\Rightarrow$ $\varphi_{curved} = \frac{\rho\pi R^2 l}{\epsilon_0} - \frac{\rho\pi R^2 l}{2\epsilon_0}$

$$= \boxed{\frac{\rho\pi R^2 l}{2\epsilon_0}}$$

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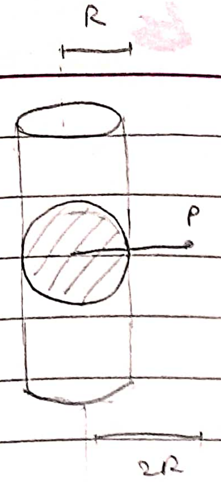


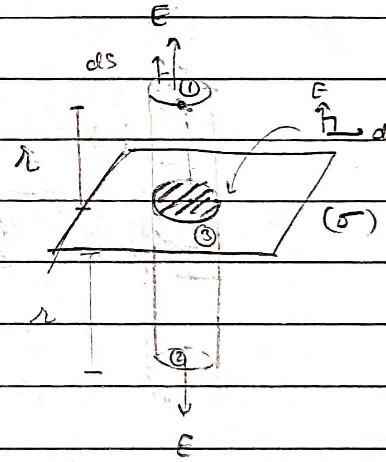
Q. Find $\vec{E}_P$

A. $$E_P = E_{P/cyl} - E_{P/sphere} = \frac{\pi \rho R^2}{(2\pi\epsilon_0)(2R)} - \frac{1}{4\pi\epsilon_0}\frac{\overbrace{\left(\frac{4}{3}\pi R^3\right)\rho}^{Q_S}}{(2R)^2}$$

- Plane Symmetry

Large Plane Sheet

eg (i) $\varphi_3 = 0$



$$\varphi_1 = \varphi_2 = EA \Rightarrow \varphi_{net} = 2EA$$

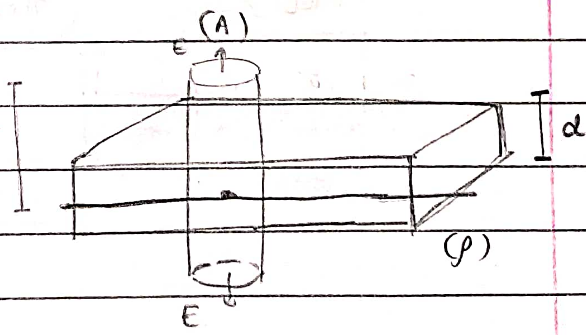
$$q_{en} = \sigma A \quad \Rightarrow \varphi_{net} = \frac{q_{en}}{\epsilon_0}$$

$$\Rightarrow 2EA = \frac{\sigma A}{\epsilon_0}$$

$$\Rightarrow \boxed{E = \frac{\sigma}{2\epsilon_0}}$$

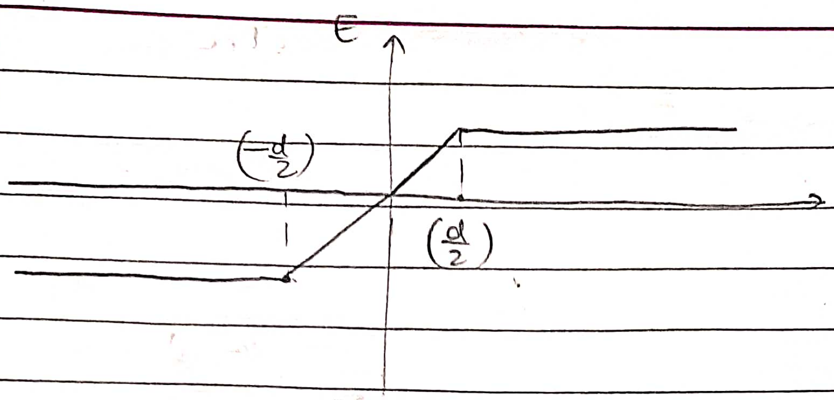
Thick sheet

(ii) $$E = \begin{cases} \dfrac{\rho x}{\epsilon_0}, & x < \dfrac{d}{2} \\ \dfrac{\rho d}{2\epsilon_0}, & x \geq \dfrac{d}{2} \end{cases}$$

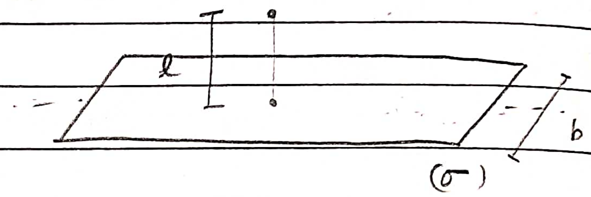


① $x < \frac{d}{2}$, $\oint E\,ds = \frac{q_{en}}{\epsilon_0} \Rightarrow 2EA = \frac{(\rho 2x)A}{\epsilon_0} \Rightarrow \boxed{E = \frac{\rho x}{\epsilon_0}}$

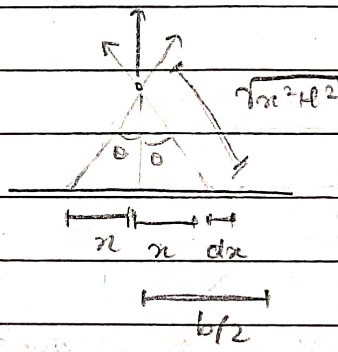
② $x \geq \frac{d}{2}$, $\oint E\,ds = \frac{q_{en}}{\epsilon_0} \Rightarrow 2EA = \frac{(\rho d)A}{\epsilon_0} \Rightarrow \boxed{E = \frac{\rho d}{2\epsilon_0}}$



Q Infinitely long strip of width 'b'



A. $dE_y = 2\cos\theta \, dE$



$$= (2\cos\theta)\left(\frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}\right)$$

$$= \left(\frac{\cos\theta}{\pi\epsilon_0}\right)\frac{(\sigma\, dx)}{\sqrt{x^2+l^2}} = \frac{\sigma\, l\, dx}{\pi\epsilon(x^2+l^2)}$$

$$= \left(\frac{\sigma}{\pi\epsilon_0}\right)\left(\frac{l^2\sec^2(\theta)\, d\theta}{l^2\sec^2(\theta)}\right)$$

$$\left\{\begin{array}{l} \lambda = \sigma\, dx \\ r = \sqrt{x^2+l^2} \end{array}\right\}$$

$$= \frac{\sigma}{\pi\epsilon_0}(d\theta)$$

$$\left\{\begin{array}{l} x = l\tan\theta \\ dx = l\sec^2(\theta)\, d\theta \end{array}\right\}$$

$$E = \left(\frac{\sigma}{\pi\epsilon_0}\right)\int_0^{\alpha} d\theta = \frac{\sigma\alpha}{\pi\epsilon_0}$$

$$= \boxed{\frac{\sigma}{\pi\epsilon_0}\tan^{-1}\left(\frac{b}{2l}\right)}$$

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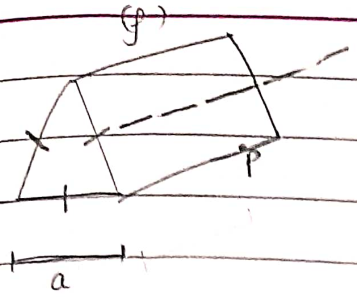
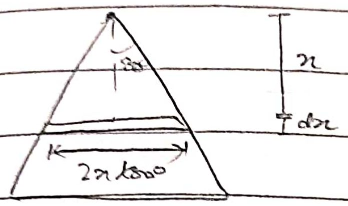
Q 

Find E at edge.

A. 

$$dE = \frac{\sigma}{\pi\epsilon_0}\tan^{-1}\left(\frac{2x\tan 30^\circ}{2x}\right) = \frac{\rho}{\pi\epsilon_0}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)dx$$

$$E = \left(\frac{\rho}{\pi\epsilon_0}\right)\left(\frac{\pi}{6}\right)\int_0^{\sqrt{3}a/2} dx = \boxed{\frac{\rho}{6\epsilon_0}\left(\frac{\sqrt{3}a}{2}\right)}$$

$\sigma = \rho\, dx$

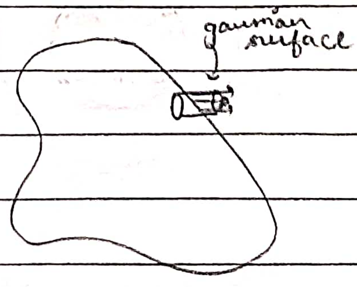
11/05/2023

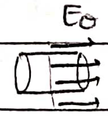
CONDUCTORS

1. For a conductor, any charge contained will be at surface.
2. $\vec{E}$ inside conductor is 0
3. E in the vicinity of cond. is $\perp$ to surface
4. (Consequence of 3) Cond. is an eqV surface
5. $E_{outside}$ (vicinity)

By G.L, $E_0 A + (0)(A) = \frac{\sigma A}{\epsilon_0}$

$$\Rightarrow \boxed{E_0 = \frac{\sigma}{\epsilon_0}}$$





NOTE: This E is due to whole body & outside charges (if present)

$\vec{E}$ in vicinity

Charge present outside changes σ, but would still be (σ/ϵ_0)

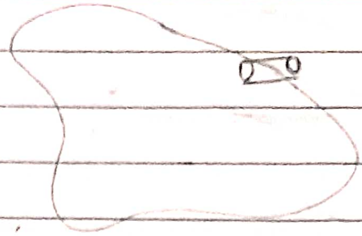
[E_1 — E due to charge present on gaussian surface
E_2 — E due to rest of the body]

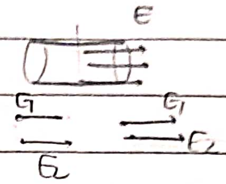
$E = E_1 + E_2$

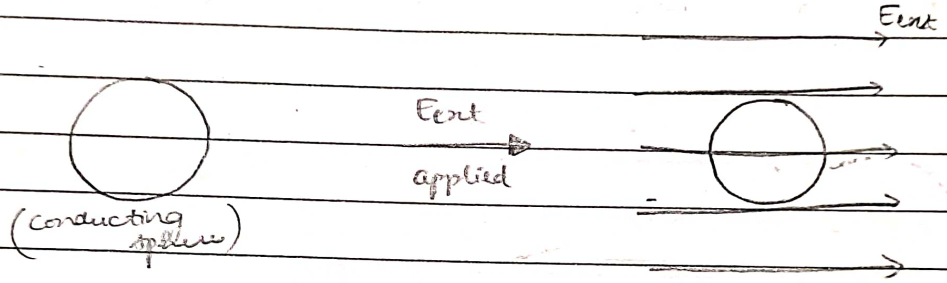
$E_1 - E_2 = 0$

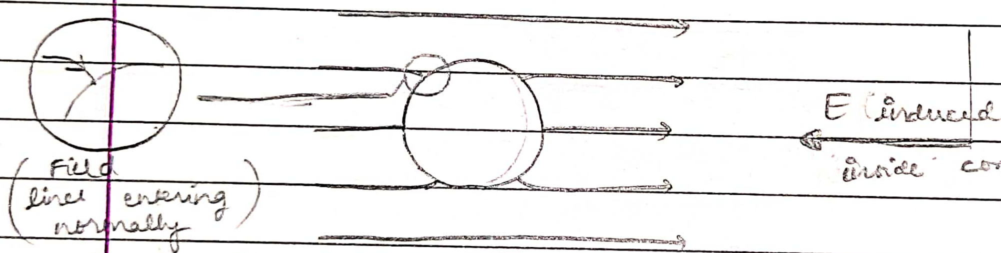
$\Rightarrow E_1 = \frac{E}{2}$

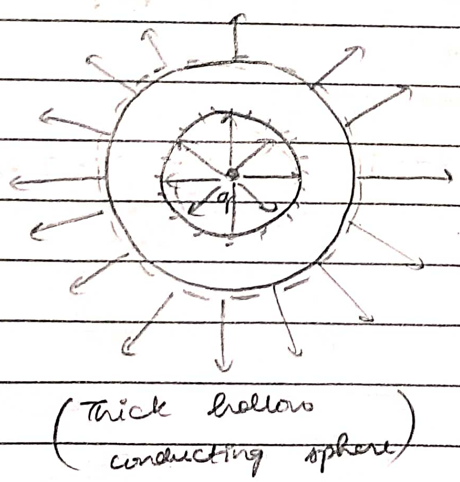
$\Rightarrow \boxed{E_1 = \frac{\sigma}{2\epsilon_0}}$











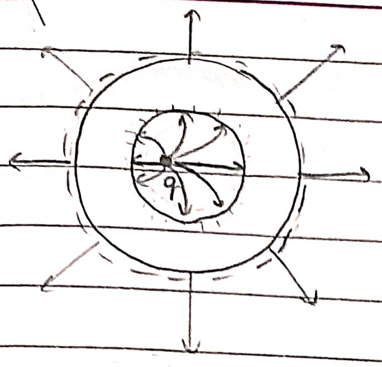
−q induced at inner surface
+q induced at outer surface

Proof: Take G.S of radius $r \in [R_{in}, R_{out}]$

$E = 0 \Rightarrow q_{en} = 0 \Rightarrow q + q_{is} = 0$

$\Rightarrow \boxed{q_{is} = -q}$

(Field lines stay same on shifting of charge)

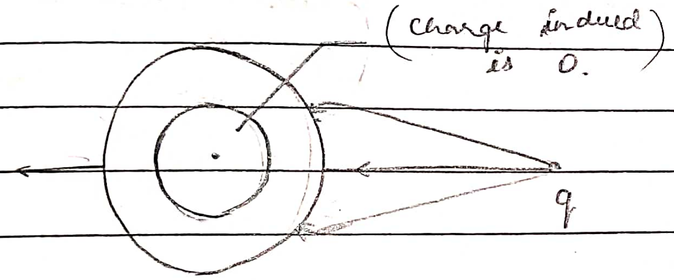


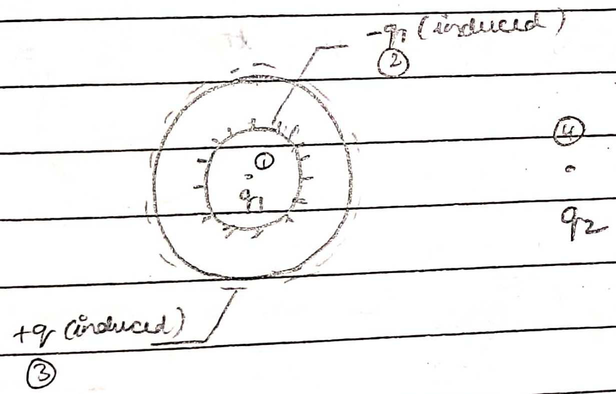
Still $-q$ & $+q$ induced at IS & OS respectively

So, σ not uniform inside but σ uniform outside

Same holds true for any cond. & its cavity.

So, for all pts. outside the cond., it will behave as if all q is at centre.





For any pt. in the thick shell,

$\vec{E_1}+\vec{E_2}+\vec{E_3}+\vec{E_4}=0$

($\because$ pt. inside conductor)

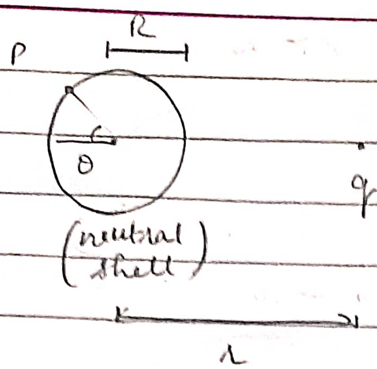
If no charge were outside, $\vec{E_1}+\vec{E_2}+\vec{E_3}=0$

$\Downarrow$

σ_{OS} would be uniform $\Rightarrow \vec{E_3}=0$ $\Bigg\}\Rightarrow \vec{E_1}+\vec{E_2}=0$

q outside (q_2) does not affect σ_{IS} $\Rightarrow \vec{E_1}+\vec{E_2}=0$ even if $\exists\, q_2$

$\Rightarrow \boxed{\vec{E_3}+\vec{E_4}=0}$

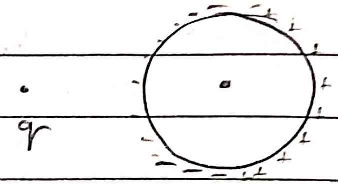
Q. 

q placed.
find V_P

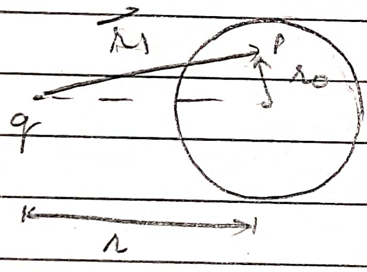
A. Since shell, $V_P = V_{centre}$. (since $\vec{E}_{inside} = 0$)

$$V_{centre} = \frac{kq}{r} + \frac{kq_{in}}{R}$$

$$= \frac{kq}{r} \quad [\because q_{in} = 0]$$



(induced charge redistributed s.t $q_{in} = 0$, since initially neutral)

Q. 

(i) E_P due to q_{in} only
(ii) V_P due to q_{in} only.

A. (i) $\vec{E}_P = 0 \Rightarrow \vec{E}_{P/q} + \vec{E}_{P/q_{in}} = 0 \Rightarrow \vec{E}_{P/q_{in}} = -\vec{E}_{P/q}$

$$= \boxed{\frac{kq}{r_1^2}(-\hat{r}_1)}$$

(ii) $V_P = V_{P/q} + V_{P/q_{in}} \Rightarrow \frac{kq}{r} = \frac{kq}{r_1} + V_{P/q_{in}}$

$$\Rightarrow V_{P/q_{in}} = kq\left(\frac{1}{r} - \frac{1}{r_1}\right)$$

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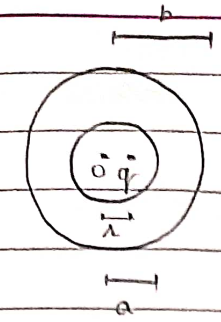
Q. 

(neutral thick shell)

q placed at r from centre.

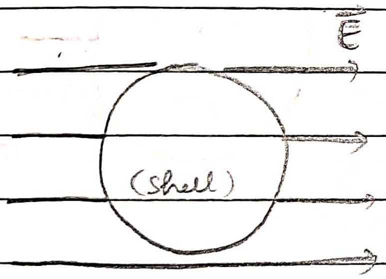
(i) Find V_{centre}

(ii) If P at r' $(r' > b)$, find V_P & E_P

A (i) $V_{centre} = V_{c/q} + V_{c/q_{is}} + V_{c/q_{os}} = \boxed{kq\left[\frac{1}{r} - \frac{1}{a} + \frac{1}{b}\right]}$

(ii) $V_P = \frac{kq}{r'}$ (shell behaves as if q at centre)

$E_P = \frac{kq}{(r')^2}$

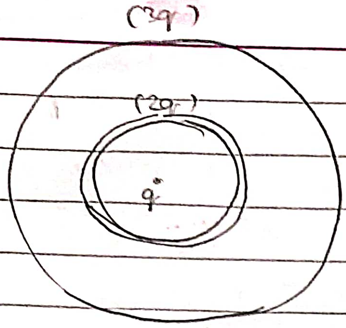
Q 

Find $\sigma_{induced}$.

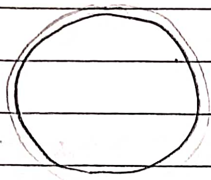
A. $\sigma = \sigma_0 \cos\theta$ (by Irodov Q)

$E = \frac{\sigma_0}{3\epsilon_0} \Rightarrow \underline{\sigma_0 = 3\epsilon_0 E}$

Q. Two shells. Find q_{in} at all surfaces.



A. By G.L.



$q_{en} = 0$

$q + q_{is(1)} = 0$

$\underline{q_{is(1)} = -q}$

$$q_{os(1)} = q_1 - q_{is(1)} = 2q - (-q) = \underline{3q}$$

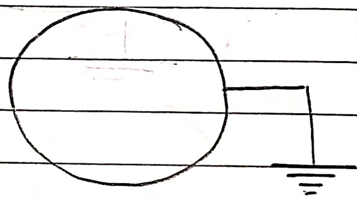
Similarly, $q_{is(2)} = -3q \Rightarrow q_{os(2)} = 3q - (-3q)$

$= \underline{6q}$.

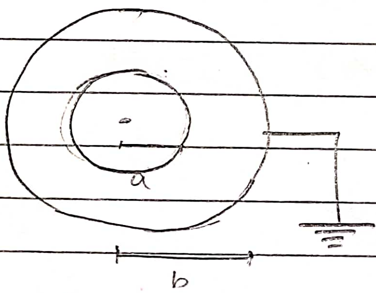
NOTE: Cond. is grounded (earthing)

$\Downarrow$

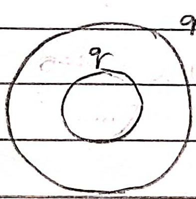
$\underline{V = 0}$



Q Two shells. Find q appearing on outer shell



A. Let q' come



$V_{os(2)} = 0 \Rightarrow V_{os(2)/q'} + V_{os(2)/q} = 0$

$\Rightarrow k\left(\frac{q'}{b} + \frac{q}{b}\right) = 0$

$\Rightarrow \underline{q' = -q}$

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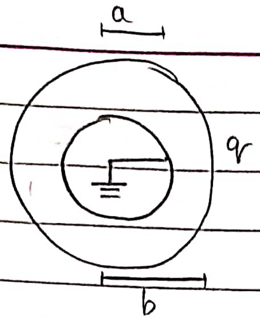
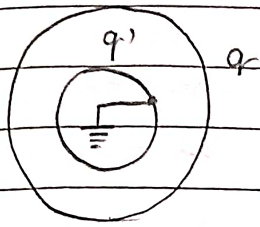
Q. If instead,



find charge appearing on grounded surface.

A

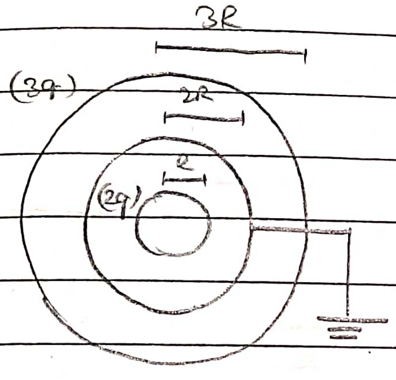


$V_{is(1)} = 0 \Rightarrow V_{is(1)}/q' + V_{is(1)}/q = 0$

$\Rightarrow k\left(\frac{q'}{a} + \frac{q}{b}\right) = 0$

$\Rightarrow \boxed{q' = -\left(\frac{a}{b}\right) q}$

Q.

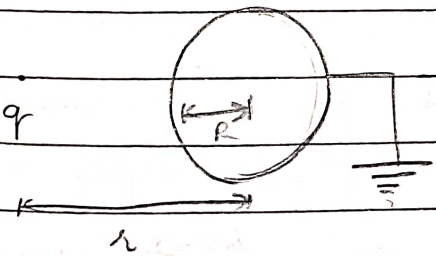


Find $q_{os(2)}$

A. $V_{os(2)} = 0 \Rightarrow V_{os(2)}/(3q) + V_{os(2)}/q' + V_{os(2)}/(2q) = 0$

$\Rightarrow k\left(\frac{3q}{3R} + \frac{q'}{2R} + \frac{2q}{2R}\right) = 0 \Rightarrow \underline{q' = -4q}$

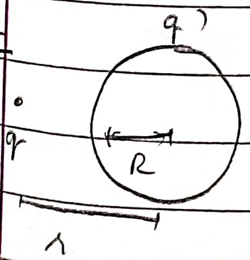
Q.



Find

(q' not uniformly distributed)

A.



$V_s = 0 \Rightarrow V_{s/q} + V_{s/q'} = 0$

$\Rightarrow V_{s/q} + V_{centre/q'} = 0$ ($V_s = V_{centre}$ ($\vec{E}$ inside zero))

$\Rightarrow k\left(\frac{q}{r} + \frac{q'}{R}\right) = 0 \Rightarrow \underline{q' = -\frac{Rq}{r}}$

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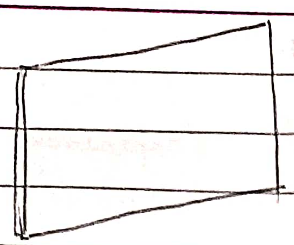
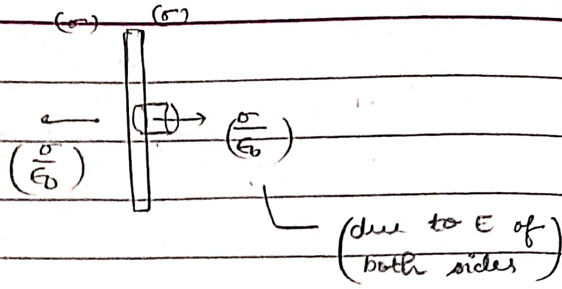
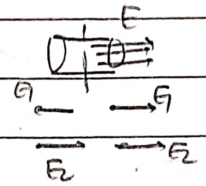


Thin conducting sheet



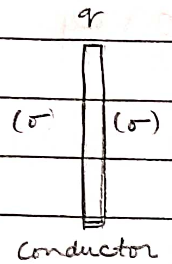


$E_1 + E_2 = E$

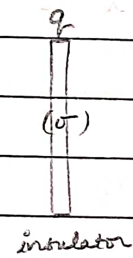
$E_1 - E_2 = 0$

$\Rightarrow E_1 = \frac{E}{2} = \left(\frac{\sigma}{2\epsilon_0}\right)$

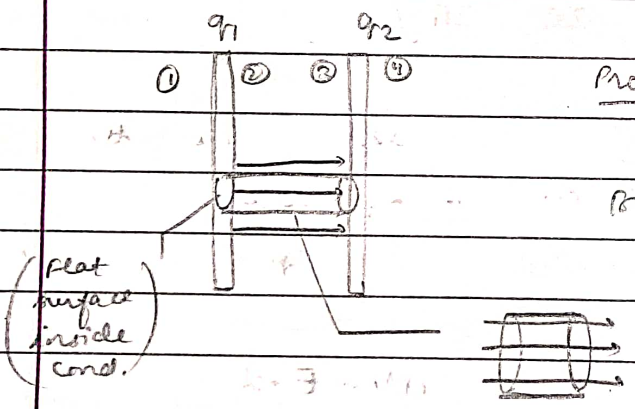
So)



$\Rightarrow \sigma = \frac{q}{2\epsilon_0} \Rightarrow E = \frac{\sigma}{\epsilon_0} = \frac{q}{2A\epsilon_0}$



$\Rightarrow q = \frac{\sigma}{\epsilon_0} \Rightarrow E = \frac{\sigma}{2\epsilon_0} = \frac{q}{2A\epsilon_0}$



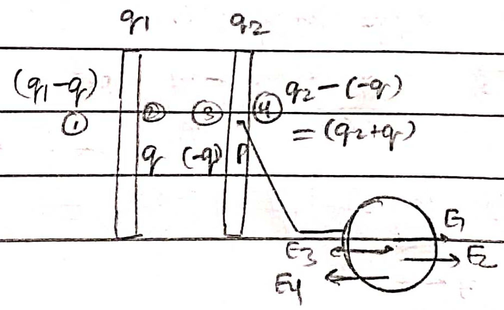
Proof: $q_② + q_③ = 0$

By G.L, $\phi = \frac{q_{en}}{\epsilon_0} = \frac{q_② + q_③}{\epsilon_0}$

But $E \perp dS$ for curved surface & flat surface inside cond.

$\Rightarrow \phi = 0$

$\Rightarrow q_② + q_③ = 0$



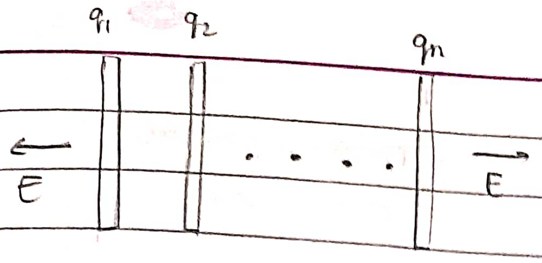
$E_1 + E_2 - E_3 - E_4 = 0$ (P inside cond.)

$\Rightarrow \left(\frac{q_1 - q}{2A\epsilon_0}\right) + \left(\frac{q}{2A\epsilon_0}\right) - \left(\frac{q}{2A\epsilon_0}\right) - \left(\frac{q_2 + q}{2A\epsilon_0}\right) = 0$

$\Rightarrow \boxed{q = (q_1 - q_2)/2}$

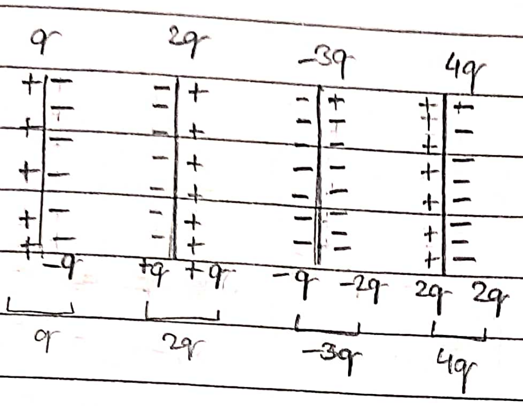
In general,

12/05/2023



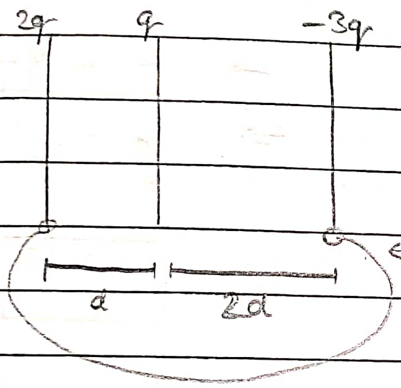
$$E = \left(\frac{\Sigma q_i}{2}\right)$$

eg —



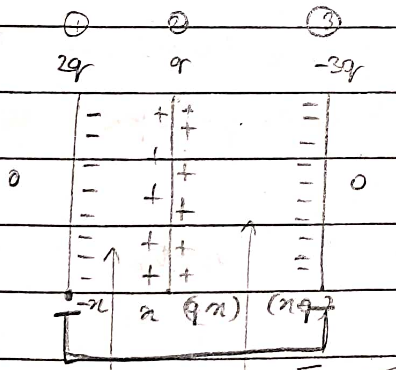
$\left(\frac{q+2q-3q+4q}{2}\right)\left(\frac{1}{\epsilon_0}\right)$ $\left(\frac{2q}{\epsilon_0}\right)$

Q.



connected by wire.

A. Still $(2q+q-3q)=0$ on outside surfaces (by proof argument)



(i) $V_1 = V_3$ (connected by wire)

(ii) $q_1 + q_3 = \text{const}$ (charge can flow)

$= 2q - 3q = (-q)$

$dV = E \cdot dr$

$\Rightarrow V_3 - V_2 = (E_{23})(2d) = \frac{2\sigma d}{\epsilon_0}$

$V_1 - V_2 = (E_{21})(d) = \frac{\sigma' d}{\epsilon_0}$

$\frac{\sigma'}{\epsilon_0}$ $\frac{\sigma}{\epsilon_0}$

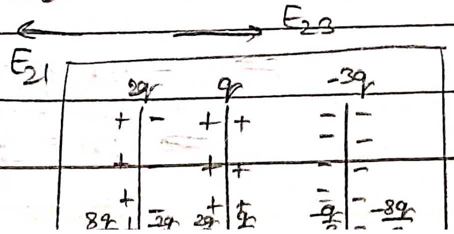
$\Rightarrow \frac{2\sigma d}{\epsilon_0} = \frac{\sigma' d}{\epsilon_0} \Rightarrow \sigma' = 2\sigma$

E_{21} E_{23}

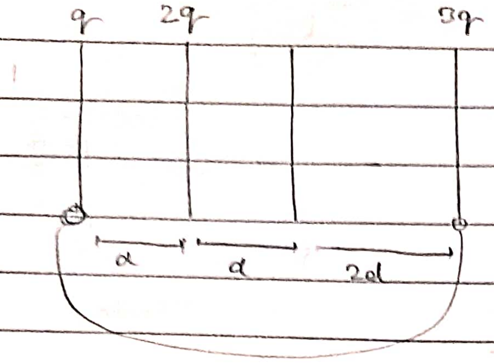
$\Rightarrow \frac{x}{A} = \frac{2(q-x)}{A}$

$\Rightarrow \boxed{x = \frac{2q}{3}}$

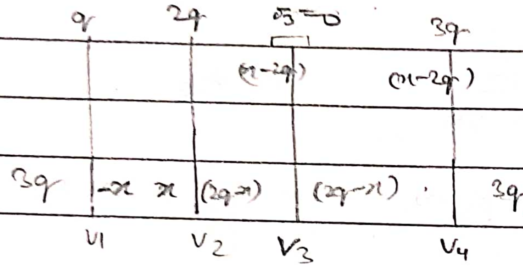
=



Q. Find charge distribution.

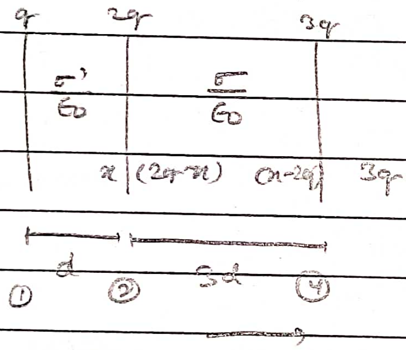


A.



⇓ Irrelevant (as does not contribute to field)

$\equiv$



$V_1 = V_4$.

$V_4 - V_2 = E_{24}(3d)$

$V_1 - V_2 = E_{21}(d)$

$\Rightarrow \left(\frac{\sigma}{\varepsilon_0}\right)(3d) = \left(\frac{\sigma'}{\varepsilon_0}\right)(d)$

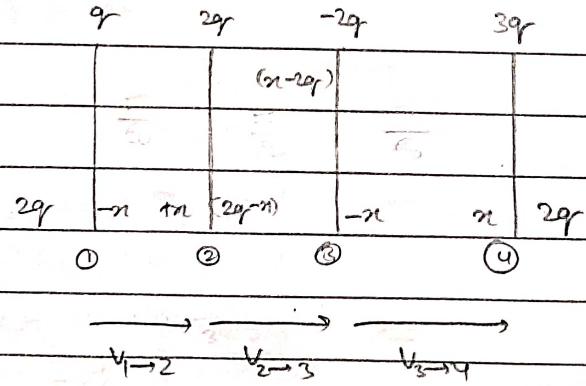
$\Rightarrow \sigma' = 3\sigma$

$\Rightarrow \frac{x}{A} = \frac{3(2q-x)}{A}$

$\Rightarrow \boxed{x = \frac{3q}{2}}$

★ **Q.** If in above Q, find charge distribution, if neutral plate was charged with $-2q$.

A.



$V_1 = V_4$

$V_{1\to2} + V_{2\to3} + V_{3\to4} = 0$

$\Rightarrow \frac{(-x)(d)}{A\varepsilon_0} + \frac{(2q-x)(d)}{A\varepsilon_0} + \frac{(-x)(2d)}{A\varepsilon_0} = 0$

$\Rightarrow -x + 2q - x - 2x = 0$

$\Rightarrow \boxed{x = \frac{q}{2}}$

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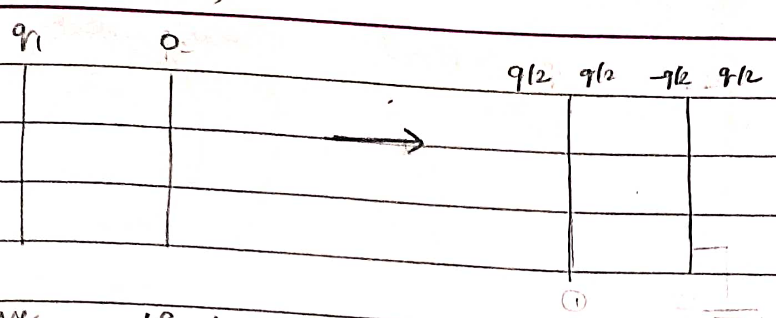
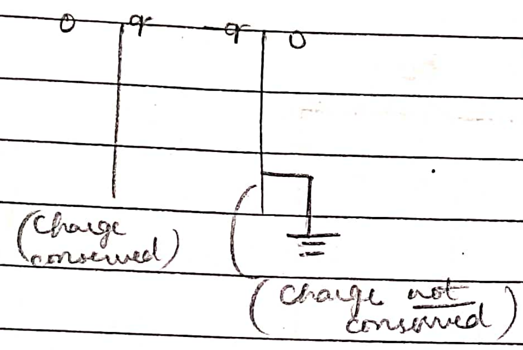
Let us consider,



Observe that outer plates repel & inner plates attract. If a source / sink of e^- is attached, available e^- would like to settle in inner plates.

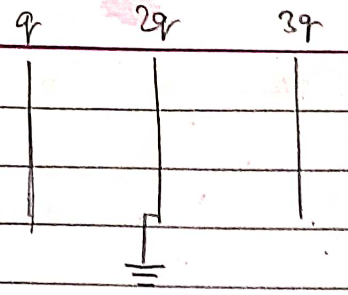
So,



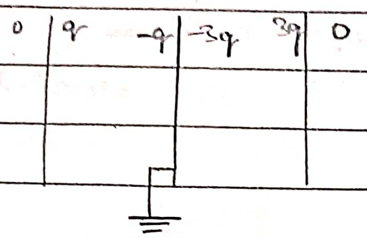
For multiple plate systems,

1. $q_{(\text{out most})} = 0$ (for min V)
2. Charge on ungrounded systems (plates or plates joined by wires) conserved
3. Charge on grounded plate <u>not</u> conserved.

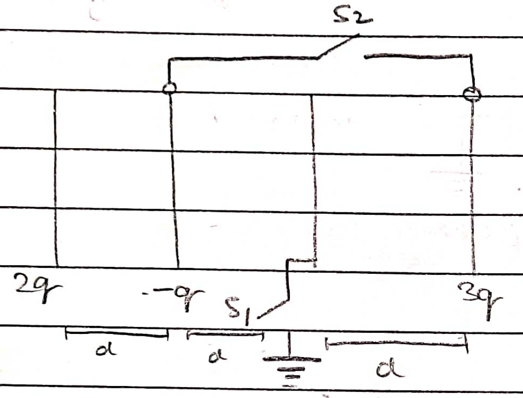
Q. Find charge distro



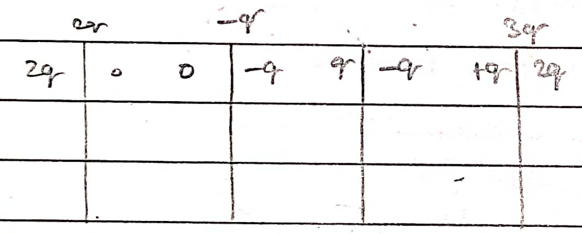
A.



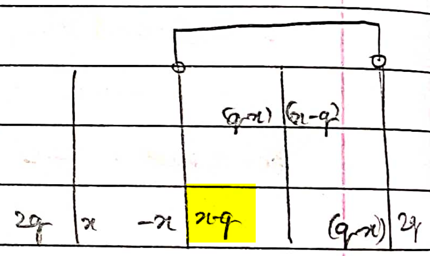
Q.



A.

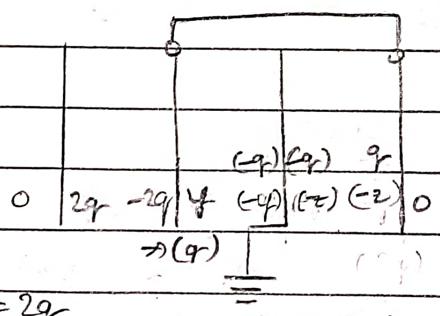


S_2 closed



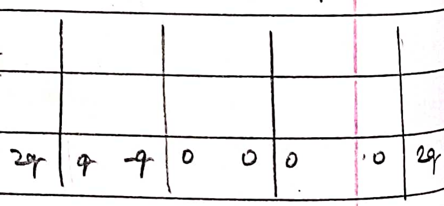
$$\frac{(x-q)d}{A\epsilon_0} + \frac{(x-q)d}{A\epsilon_0} = 0$$

$\Rightarrow$ $\underline{x = q}$



S_1 & S_2 closed

=



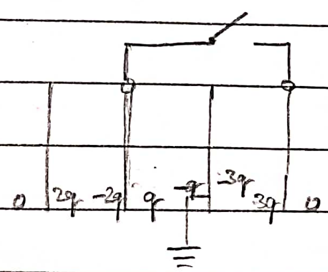
(i) $y - z = 2q$

(ii) $\frac{y}{\epsilon_0 A} d + \frac{z}{\epsilon_0 A} d = 0$

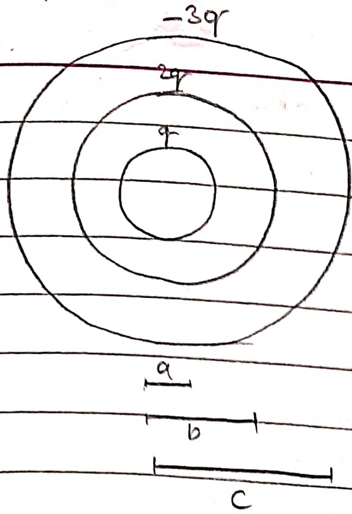
$\Rightarrow$ $\underline{y + z = 0}$

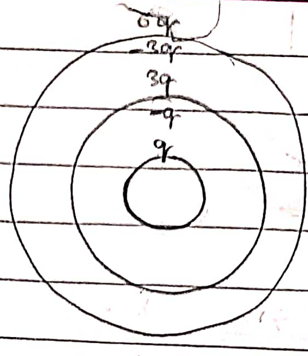
$y = q$
$z = -q$

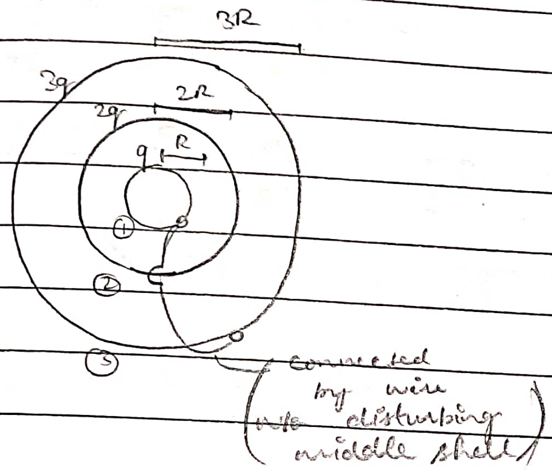
S_2 closed



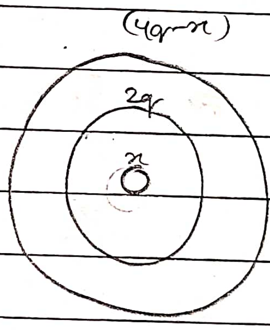
S_1 closed

Q. 

$\Rightarrow$ 

Q. 

Find charge flowing from innermost to outermost shell.



A. $V_1 = V_3$ & $q_1 + q_3 = 4q$

$\Rightarrow K\left(\frac{x}{R} + \frac{2q}{2R} + \frac{(4q-x)}{3R}\right)$

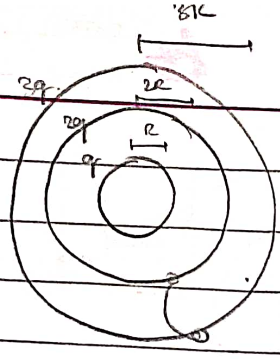
$= \cancel{K}\left(\frac{x}{3R} + \frac{2q}{3R} + \frac{(4q-x)}{3R}\right)$

$\Rightarrow \frac{2x}{3R} = \frac{-q}{3R} \Rightarrow \boxed{x = \frac{-q}{2}}$

$\Rightarrow q_{1\to3} = q_3' - q_3$

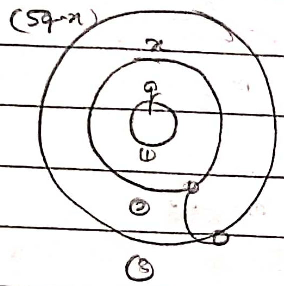
$= (4q-x) - 3q$

$= q - x = \frac{3q}{2}$



Q. Same Q as above.

A.



$\underline{V_3 = V_2}$

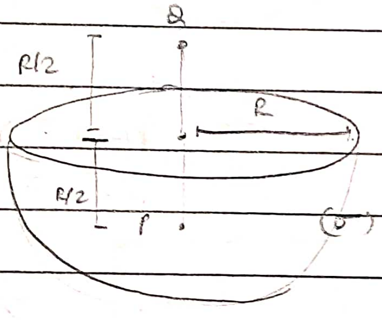
$$\Rightarrow k\left(\frac{q}{3R} + \frac{x}{3R} + \frac{5q-x}{3R}\right) = k\left(\frac{q}{2R} + \frac{x}{2R} + \frac{5q-x}{3R}\right)$$

$$\Rightarrow \frac{q+x}{3} = \frac{q+x}{2R} \Rightarrow \underline{x = -q}$$

$$q_{2\to3} = q_3' - q_3 = 5q - x - 3q$$
$$= 2q - x = 3q$$

☆ $V_2 = V_3 \Rightarrow \boxed{E_{2\to3} = 0}$

Q.

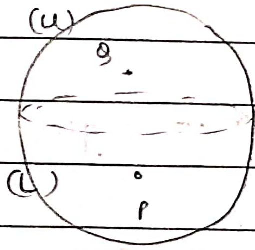


find E_Q if $E_P = E_0$

A. If whole shell,

$$\vec{E_Q} = 0$$

$$\vec{E}_{Q/L} + \vec{E}_{Q/U} = 0$$



By sym., $\vec{E}_{Q/L} = -\vec{E}_{P/U} = -\vec{E_0}$

$$\Rightarrow \boxed{\vec{E}_{Q/U} = +\vec{E_0}}$$

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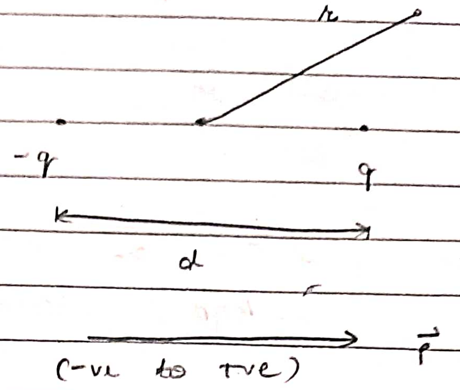
DIPOLE

• Dipole moment –



$r >> d$

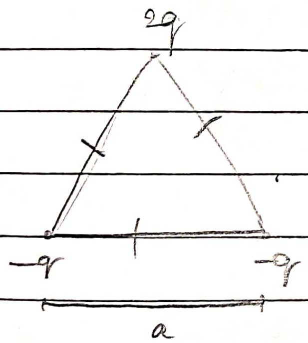
$|\vec{p}| = qd$

Type : Vector

Ways to find $\vec{p}$ of systems :–

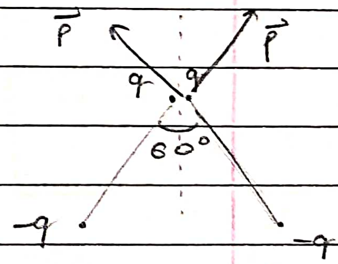
① Vector Addⁿ

② Centre of Charge – CoC(–) & CoC(+)

Q. 

Find $\vec{p}$.

A①



$p_{net} = 2p\cos 30° = \sqrt{3}p$

$= \sqrt{3}qa$

$(p = qa)$

A②

$p = (2q)\left(\frac{\sqrt{3}a}{2}\right)$

$= \sqrt{3}qa$

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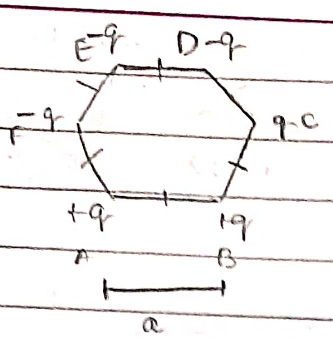
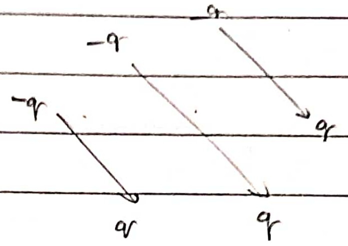
Q.



Find $\vec{p}$

A. (1) $p = qa + 2qa + qa$

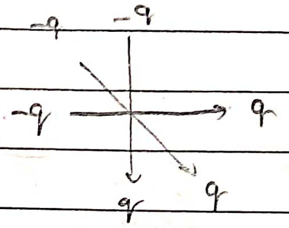
$= \underline{4qa}$ (along EB)



(2)

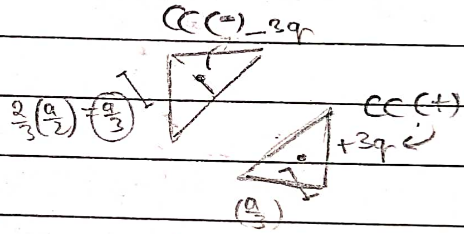
$p = 2qa + 2(2qa)\cos 60°$

$= \underline{4qa}$



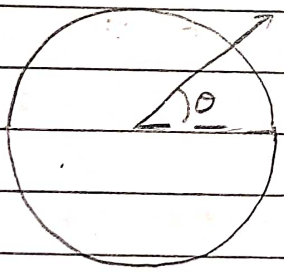
(3) $p = (3q)\left(2a - \frac{2a}{3}\right)$

$= \underline{4qa}$



Q.

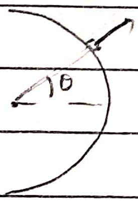
$\lambda = \lambda_0 \cos\theta$



Find $\vec{p}$

CC

A. (1)



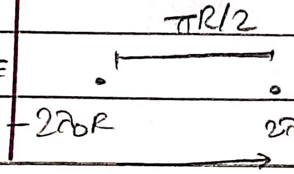
$dq = \lambda R\, d\theta$
$= \lambda_0 R \cos\theta\, d\theta$

By sym. $\underline{COC(+)_y = 0}$

$COC(+)_x = \dfrac{\int R\cos\theta\, dq}{\int dq} = \dfrac{\int \lambda_0 R^2 \cos^2\theta\, d\theta}{\int \lambda_0 R \cos\theta\, d\theta}$

$= \dfrac{R}{2} \dfrac{\left[\theta + \frac{\sin 2\theta}{2}\right]_{-\pi/2}^{\pi/2}}{[\sin\theta]_{-\pi/2}^{\pi/2}} = \left(\dfrac{\pi R}{4}\right)$

$\equiv$



$p = \left(\frac{\pi R}{2}\right)(2\lambda_0 R) = \pi \lambda_0 R^2$

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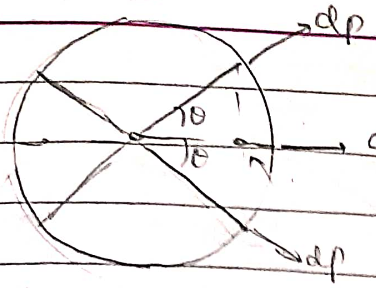
(2)



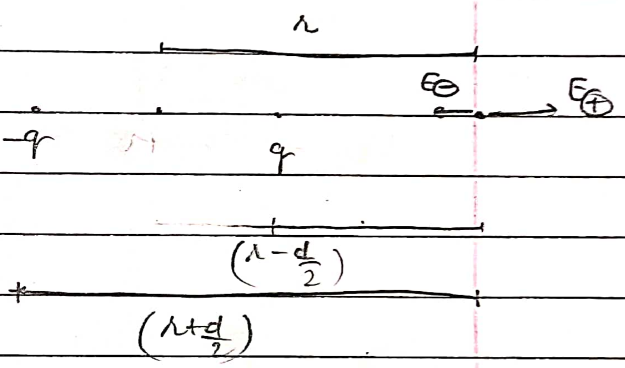
$$dp_{net} = 2\,dp\cos\theta = 2(\lambda_0 \cos\theta\, R\,d\theta)(\cos\theta)$$

$$\Rightarrow p_{net} = \lambda_0 R \int_0^{\pi/2} 2\cos^2\theta\, d\theta = \lambda_0 R\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi/2}$$

$$= \pi\lambda_0 R^2$$

→ Field & Potential due to dipole

Axial



$$E_{ax} = E_\oplus - E_\ominus$$

$$= \frac{kq}{(r-\frac{d}{2})^2} - \frac{kq}{(r+\frac{d}{2})^2}$$

$$= \left(\frac{1}{4\pi\epsilon_0}\right)(q)\frac{(2r d)}{(r^2-\frac{d^2}{4})^2}$$

$$= \left(\frac{2pr}{4\pi\epsilon_0}\right)\left(\frac{1}{r^4}\right) \quad [r>>d] \Rightarrow \boxed{E_{ax} = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{2p}{r^3}\right)}$$

along p

$$\Rightarrow \vec{E}_{ax} = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{2\vec{p}}{r^3}\right)$$

$$V_{ax} = V_\oplus + V_\ominus = kq\left[\frac{1}{r-\frac{d}{2}} - \frac{1}{r+\frac{d}{2}}\right]$$

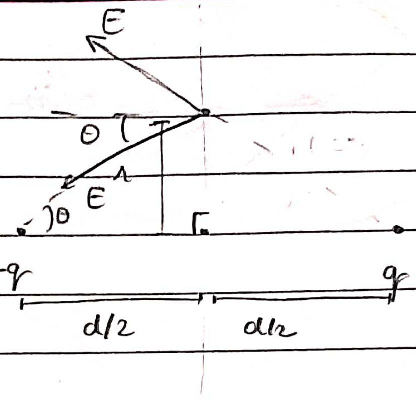
$$= \frac{kqd}{r^2-\frac{d^2}{4}}$$

$$= \boxed{\left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{p}{r^2}\right)} \quad [r>>d]$$

$V_{ax} > 0,\ r > 0$

$V_{ax} < 0,\ r < 0$

Equatorial



$$E_{eq.} = 2E\cos\theta$$

$$= \frac{2kq}{\left(\sqrt{r^2+\frac{d^2}{4}}\right)^2}\frac{d/2}{\sqrt{r^2+\frac{d^2}{4}}}$$

$$= \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{p}{r^3}\right) \quad [r >> d]$$

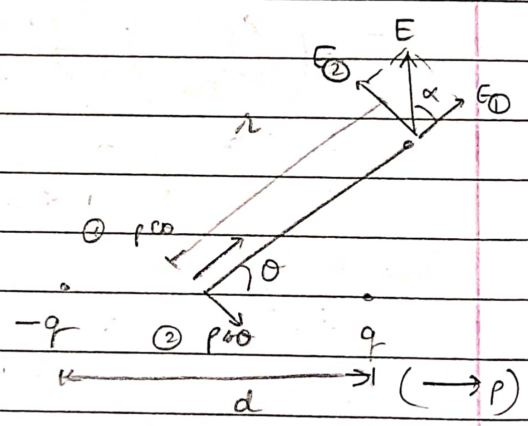
(against p)

$$\boxed{E_{eq} = \left(\frac{-1}{4\pi\epsilon_0}\right)\left(\frac{\vec{p}}{r^3}\right)}$$

$$V_{eq} = V_{\oplus} + V_{\ominus} = \frac{kq}{\sqrt{r^2+\frac{d^2}{4}}} - \frac{kq}{\sqrt{r^2+\frac{d^2}{4}}} = 0$$

General

(I) For ① → ax.
② → eq.



$$E_1 = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{2p\cos\theta}{r^3}\right)$$

$$E_2 = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{p\sin\theta}{r^3}\right)$$

$$E = \sqrt{E_1^2 + E_2^2} = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{p}{r^3}\right)\sqrt{4\cos^2\theta + \sin^2\theta}$$

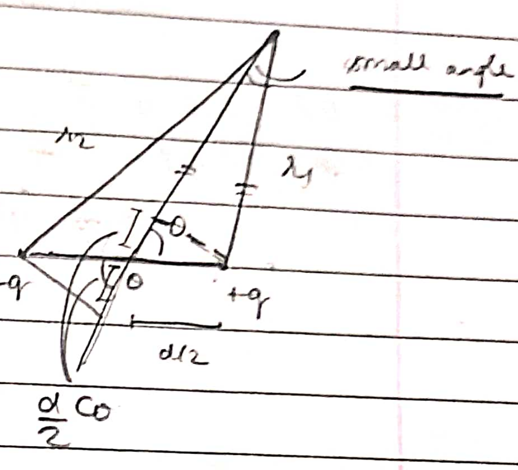
$$= \boxed{\left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{p}{r^3}\right)\sqrt{1+3\cos^2\theta}}$$

$$\tan\alpha = \frac{E_2}{E_1} = \frac{\tan\theta}{2} \Rightarrow \boxed{\tan\alpha = \frac{\tan\theta}{2}}$$

$$V = V_r + V_\theta = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{p c_\theta}{r^2} + 0\right)$$

$$= \boxed{\left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{p c_\theta}{r^2}\right)}$$

(II) $V = \left(\frac{1}{4\pi\epsilon_0}\right)\left[\frac{q}{r_1} - \frac{q}{r_2}\right]$

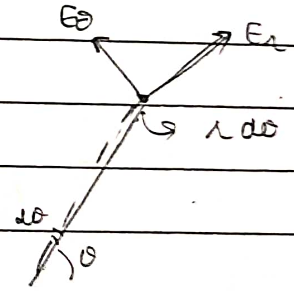


$$= \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q}{r + \frac{dc_\theta}{2}} - \frac{q}{r - \frac{dc_\theta}{2}}\right)$$

$$= \left(\frac{1}{4\pi\epsilon_0}\right)\frac{qdc_\theta}{\left(r^2 - \frac{d^2 c_\theta^2}{2}\right)}$$

[r << d]

$$= \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{p c_\theta}{r^2}\right)$$



$$E_r = -\frac{\partial V}{\partial r} = \left(\frac{-1}{4\pi\epsilon_0}\right)\left(\frac{2p c_\theta}{r^3}\right)$$

$$E_\theta = -\frac{\partial V}{r\,d\theta} = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{p s_\theta}{r^3}\right)$$

$$E = \sqrt{E_r^2 + E_\theta^2} = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{p}{r^3}\right)\sqrt{4c_\theta^2 + s_\theta^2} = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{p}{r^3}\right)\sqrt{1 + 3c_\theta^2}$$

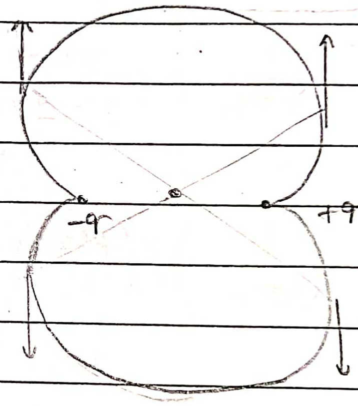
NOTE: (i) $E \neq 0 \quad \forall\ r, \theta \Rightarrow F$ b/w Q (pt. charge) (dipole) $\neq 0$

no matter how they are placed wrt each other.

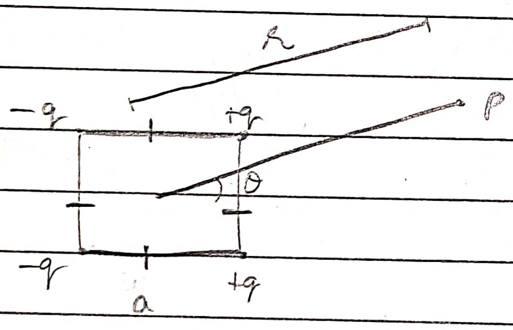
(ii) For $\vec{E} \perp \vec{p}$, $\underline{\alpha + \theta = \pi/2}$ $\Rightarrow \tan\left(\frac{\pi}{2} - \theta\right) = \tan\theta/2$

$\Rightarrow \underline{\tan\theta = \pm\sqrt{2}}$

$\Rightarrow \theta = \tan^{-1}(\pm\sqrt{2})$

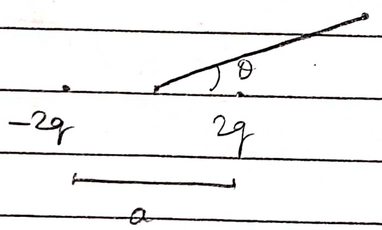


Q.



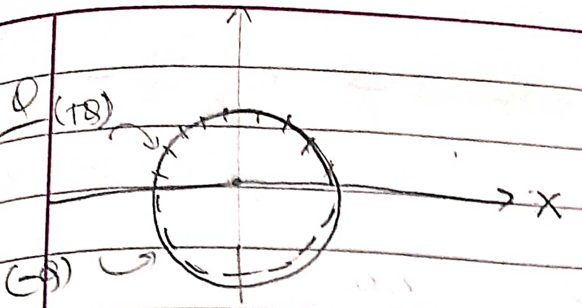
Find E & V at P $(r \gg a)$

A.



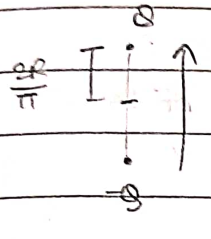
$$E = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{2qa}{r^3}\right)\sqrt{1 + 3\cos^2\theta}$$

$$V = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{2qa\cos\theta}{r^2}\right)$$



Find E & V at P $(0,0,z)$.

$(z >> a)$

A. 

$p = \left(\frac{4RQ}{\pi}\right)$

$E = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{4RQ}{\pi}\right)\left(\frac{1}{z^3}\right)\left(\sqrt{1+0}\right)$

$\Rightarrow \vec{E} = \left(\frac{RQ}{\pi^2\epsilon_0}\right)\left(\frac{1}{z^3}\right)(-\hat{j})$

$\theta = \pi/2$

$V = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{4RQ}{\pi}\right)\left(\frac{1}{z^2}\right)c(\pi/2)$

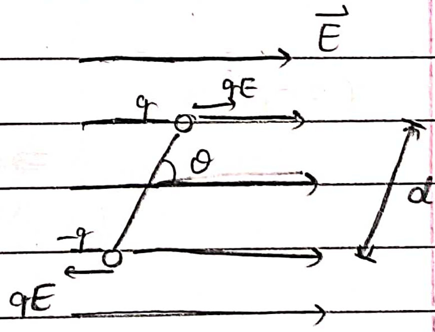
$= 0$

→ Behaviour of Dipole in $\vec{E}$

$\boxed{\text{Uniform Field}}$

$F_{net} = 0$ (Stable eq.)

$\tau_{net} = 0$, $\theta = 0, \pi$ (unstable eq.)



$\tau = (qEd)(s\theta)$

$= pE\, s\theta$

$\Rightarrow \vec{\tau} = \vec{p} \times \vec{E}$

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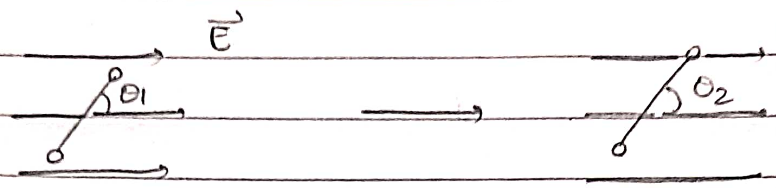
NOTE: τ has tendency to align $\vec{p}$ || to $\vec{E}$.

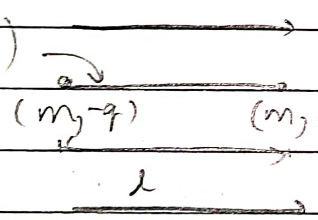


$\tau_{ext} = pE\sin\theta \Rightarrow W_{ext} = \int \tau_{ext}\, d\theta = \int_{\theta_1}^{\theta_2} pE\sin\theta\, d\theta$

$\Rightarrow U = pE(\cos\theta_1 - \cos\theta_2)$

$\theta_1 = \pi/2$
$\theta_2 = \theta$
$\Rightarrow \boxed{U = -pE\cos\theta}$

Q (light rod)



Slightly rotated from eq. posit. & released. Find T (time period)

A. $\tau = -pE\sin\theta \sim \tau = -(pE_0)(\theta)$ $(\theta << \Rightarrow \theta \sim \sin\theta)$

$\Rightarrow \alpha = -\left(\frac{pE_0}{I}\right)(\theta)$

Free body rotates about CM.

$\Rightarrow I = \left(\frac{ml^2}{2}\right)$

$T = 2\pi\sqrt{\frac{I}{pE_0}}$

$= 2\pi\sqrt{\frac{ml}{2qE_0}}$

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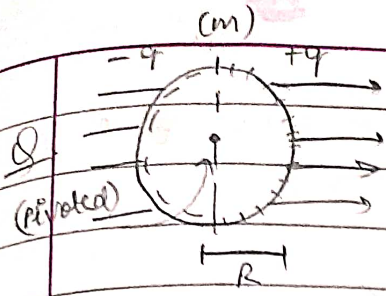
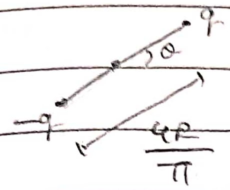
(Charge on periphery)

Q 

Disc rotated by small angle (in its plane) & released P.T it will execute SHM & find T.

A. 

since charge on periphery $\Rightarrow$ CC (ring)

$\tau = -pE\sin\theta \sim \tau = -pE_0\theta$

$\Rightarrow \alpha = -\left(\frac{pE_0}{I}\right)\theta \Rightarrow$ SHM

$T = 2\pi\sqrt{\frac{I}{pE_0}}$ $\quad I = \frac{mR^2}{2}$

$= 2\pi\sqrt{\frac{mR\pi}{8qE_0}}$ $\quad p = \frac{4Rq}{\pi}$

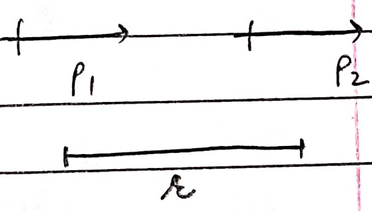
Non-uniform Field

$U = -\vec{p}\cdot\vec{E} \Rightarrow F = -\frac{\partial U}{\partial r}$

$= +\frac{\partial}{\partial r}(\vec{p}\cdot\vec{E})$

$= \boxed{(\vec{p})\left(\frac{\partial\vec{E}}{\partial r}\right)}$

eg. $U = \overbrace{\left(-\frac{1}{4\pi\epsilon_0}\frac{2p_1}{r^3}\right)}^{E_{p_2/p_1}\text{ (Axial)}}(p_2)\,(\cos 0)$



$F = -\frac{dU}{dr} = \left(\frac{2p_1p_2}{4\pi\epsilon_0}\right)\left(\frac{-3}{r^4}\right)$

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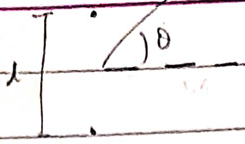
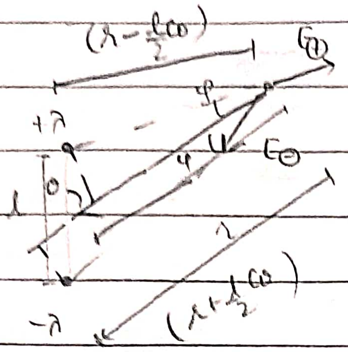
Q 3.12 Irodov



Find $\vec{E}$ & V.

A



$$E_{\oplus} = \frac{2k\lambda}{(r-\frac{l}{2}c_\theta)}$$

$$E_{\ominus} = \frac{2k\lambda}{(r+\frac{l}{2}c_\theta)}$$

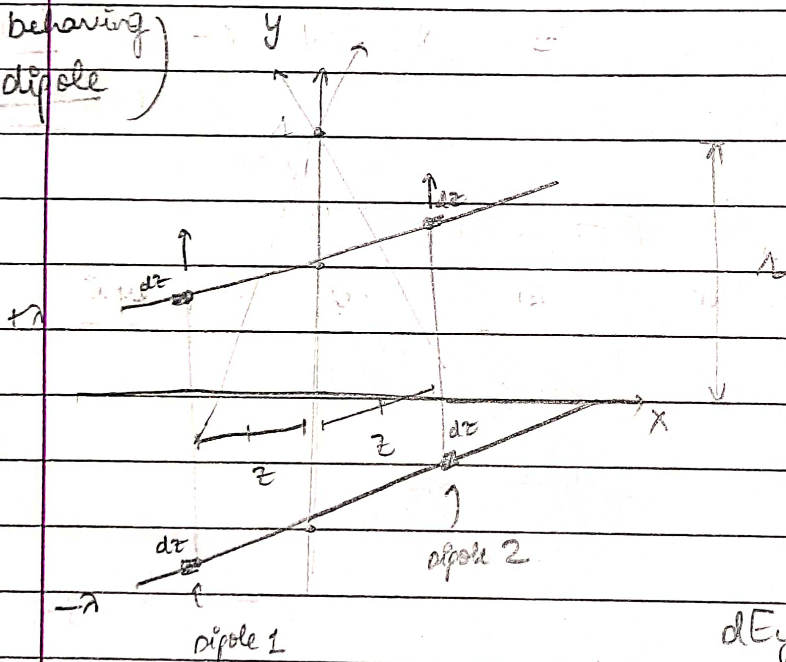
$\varphi << \theta \Rightarrow E = E_{\oplus} - E_{\ominus} = \frac{2k\lambda l c_\theta}{r^2 - \frac{l^2c_\theta^2}{4}} \sim \left(\frac{2k\lambda l}{r^2}\right)$

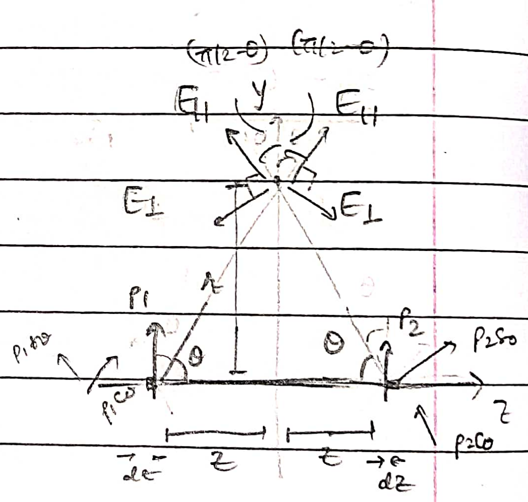
$\Rightarrow$ (Almost in same line)

observe, $E = \frac{2kl(\lambda r)}{r^3}$ (behave as p & q of dipole respectively)

Proof:

(for behaving as dipole)





$$dE_y = 2E_{\parallel}s_\theta - 2E_{\perp}c_\theta$$

$$= \left(\frac{4kp\,s_\theta}{(r^2+z^2)^{3/2}}\right)\cdot s_\theta - \left(\frac{2kp\,c_\theta}{(r^2+z^2)^{3/2}}\right)c_\theta$$

$$= \frac{2kp}{(r^2+z^2)^{3/2}}\underbrace{(2s_\theta^2 - c_\theta^2)}_{2-3c_\theta^2}$$

$$E_y = \int_0^\infty \frac{2kp\,(2-3c_\theta^2)}{(r^2+z^2)^{3/2}}$$

$p = (\lambda\, dz)(l)$

$z = r\cot(\theta)$

$dz = -r\csc^2(\theta)\,d\theta$

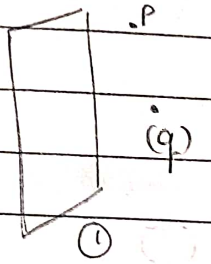
$z:(0,\infty) \longrightarrow \theta:(\pi/2, 0)$

$$= \int_0^\infty \frac{2k(\lambda l)(dz)(2-3c_\theta^2)}{(r^2+z^2)^{3/2}} = \int_{\pi}$$

Lord Kelvin Method (Image Method) for conductors

For an infinitely large / grounded conductor, we can recreate its influence by the following method.

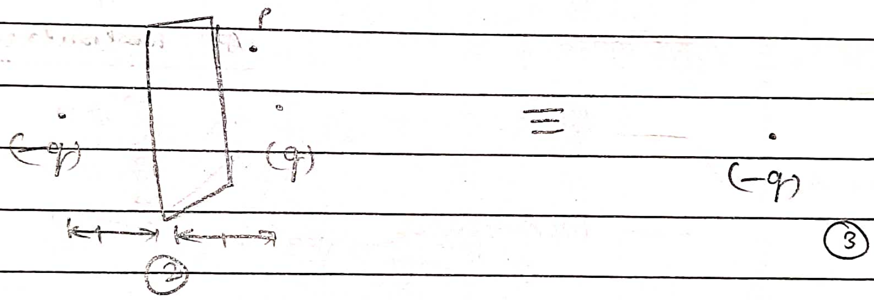
eg (i)



By sym. $E \perp$ plane & $V = 0$.

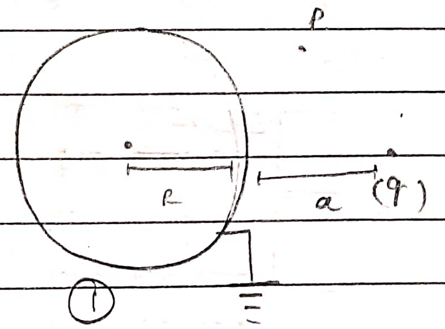
But to calc. E & V at any random pt. in space is difficult as $\sigma_{induced}$ on plane will be non-uniform

So, we come up with a charge distr. in space s.t $E \perp$ plane & $V = 0$.

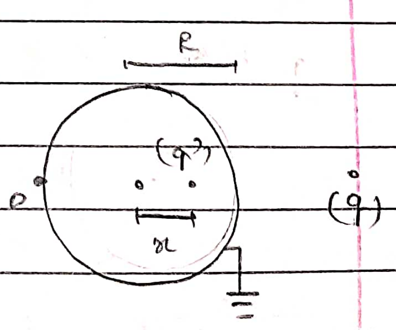


So, finding V & E at P in sys. 1 is equivalent to finding V_P & E_P in 3.

(ii)



So,



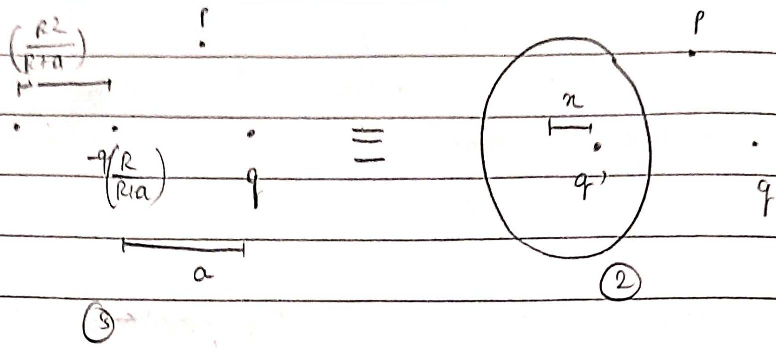
1. $V_{surface} = 0 \Rightarrow \frac{kq'}{(R-x)} + \frac{kq}{a} = 0$

2. $V_D = 0 \Rightarrow \frac{kq}{(2R+a)} + \frac{kq'}{R+x} = 0$

Using these eq^n's, we can find q' & x s.t $V = 0$ & $E \perp$ surface

In conductors → Induced charge (Mobile)
dielectrics → Polarised charge (Immobile)

$$x = \frac{R^2}{(R+a)} \qquad q' = -q\left(\frac{R}{R+a}\right)$$



So, finding V_P & E_P in ① is eq. to finding V_P & E_P in ③

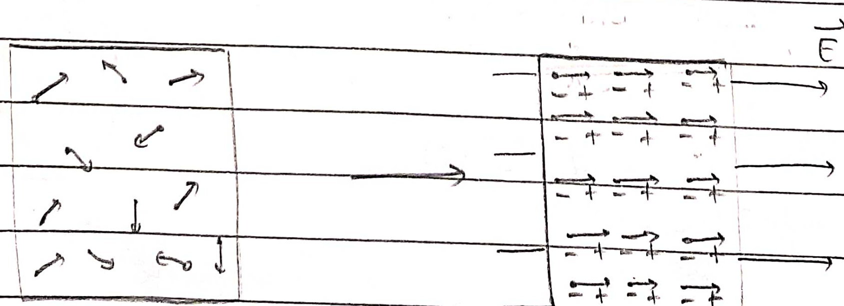
DIELECTRICS (diE)

At molecular level

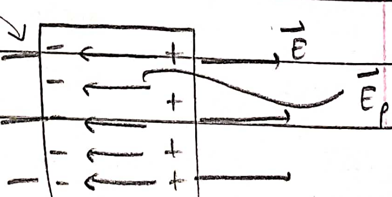
Insulators
- Polar: $\vec{p} \neq 0$
- Non-polar: $\vec{p} = 0$

Dielectrics are polar insulators.

If dielectrics placed in a field, mc. try to align $\vec{p}$ to $\vec{E}_{ext}$



$\boxed{\text{Polarised Charge}}$ ≡



So, $\vec{E}_{net} = \vec{E}_{applied} + \vec{E}_{polarisation}$

$\Rightarrow E_{net} = E_A - E_p$

$\Rightarrow \boxed{\dfrac{E_0}{k} = E_0 - E_p}$

$k \rightarrow$ (relative permittivity of medium)
(or ϵ_r)

$$E_m = \left(\frac{1}{4\pi\epsilon_m}\right)\left(\frac{q_1 q_2}{r^2}\right) \Leftarrow \quad \epsilon_m = k\epsilon_0$$
$$= \left(\frac{E_0}{k}\right)$$

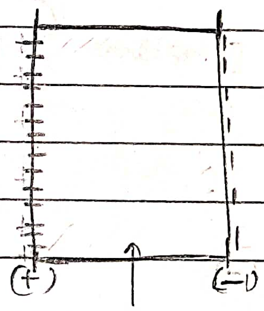
$\uparrow$ (permittivity of medium)

For charged plates with diE in b/w

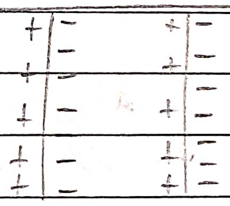
(polarisation charge density)

$$\frac{\sigma}{k\epsilon_0} = \frac{\sigma}{\epsilon_0} - E_p = \frac{\sigma}{\epsilon_0} - \frac{\sigma_p}{\epsilon_0}$$

$\Rightarrow \boxed{\sigma_p = \sigma\left(1 - \frac{1}{k}\right)}$



NOTE:

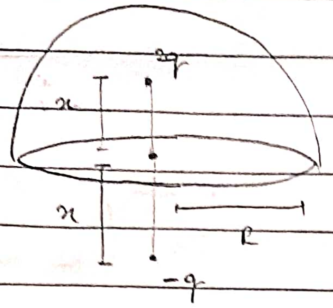


Do not cancel

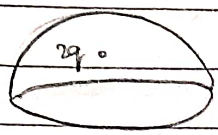
Do not cancel

since polarised charge is immobile

Q. Find x for which, net flux on curved surface & plane surface will be equal.



A. Curved



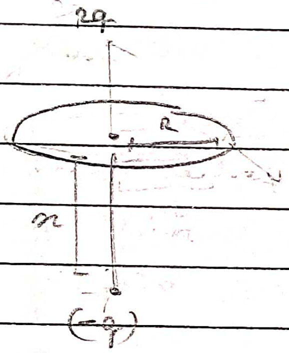
$$\left\{\begin{array}{l} \varphi_{c/2q} + \varphi_{F/2q} = \frac{2q}{\epsilon_0} \\ \varphi_{c/(-q)} + \varphi_{F/(-q)} = 0 \end{array}\right\}$$

$$\varphi_{curved} = \varphi_{c/2q} + \varphi_{c/(-q)}$$

$$= \left(\frac{2q}{\epsilon_0} - \varphi_{Flat/2q}\right) + \left(-\varphi_{Flat/(-q)}\right)$$

$$= \frac{2q}{\epsilon_0} - \frac{2q}{2\epsilon_0}(1-c_0) - \left(\frac{q}{2\epsilon_0}\right)(1-c_0)$$

Plane



$$\varphi_{Flat} = \varphi_{F/2q} + \varphi_{F/(-q)}$$

$$= \frac{2q}{2\epsilon_0}(1-c_0) + \frac{q}{2\epsilon_0}(1-c_0)$$

$$\varphi_{curved} = \varphi_{Flat}$$

$$\Rightarrow 4 - 2(1-c_0) - (1-c_0) = 2(1-c_0) + (1-c_0)$$

$$\Rightarrow 6(1-c_0) = 4 \Rightarrow \boxed{c_0 = 1/3}$$

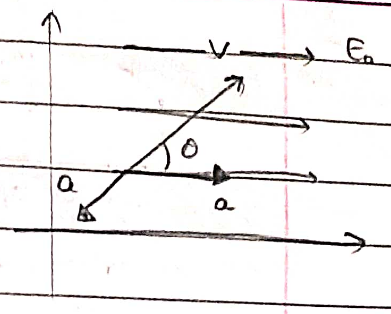
Q. In space $\vec{E} = \langle E_0 \;\; 0 \;\; 0 \rangle$. A particle (q, m) projected from origin $\vec{v} = \langle 0 \;\; v \;\; 0 \rangle$
Apart from F_E, a resistive force of mag. qE_0 acts on the particle opp. to the dirxn of velo of particle.
Find speed of particle when its vel. becomes $\parallel$ to x-axis.

A: **Along X :** $(v - 0) = \int a - a\cos\theta \, dt$

Along v : $(v - v_0) = \int a\cos\theta - a \, dt$

$= -v$

$\Rightarrow \boxed{v = \frac{v_0}{2}}$



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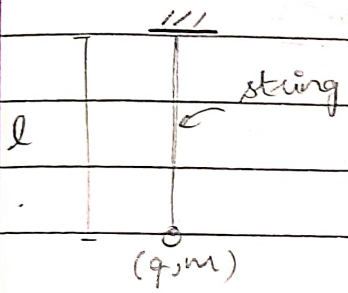
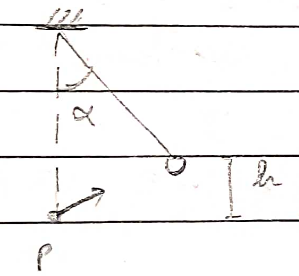
Q.

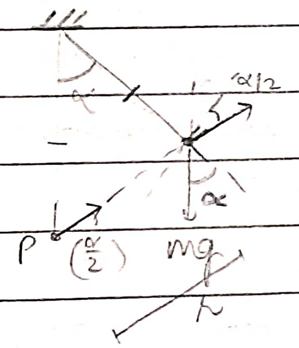


Dipole, very far initially is brought towards charge & is aligned as shown. String makes α in eq.



If particle rises vertically h above initial posⁿ, find W in bringing dipole.

A.



$mg\sin\alpha = qE_p\cos\alpha/2 \Rightarrow qE_p = 2mg\sin\alpha/2$

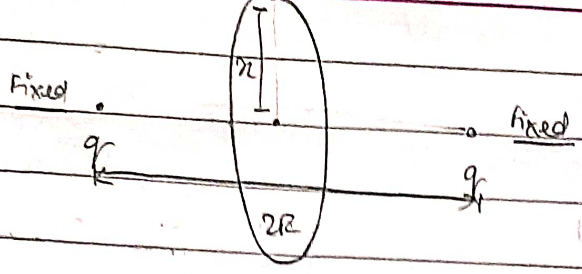
$\Rightarrow q\left(\frac{\cancel{2}kp}{r^2}\right) = \cancel{2}mg\sin\alpha/2$

$\Rightarrow \frac{qkp}{r} = mgr\sin\alpha/2$

$W_{on\,p} + W_{m/g} + W_{m/p} = 0 \Rightarrow W_{on\,p} - mgh + qV_p = 0$

$W_{on\,p} = mgh - \frac{q(kp)\cos\alpha/2}{r}$

Q.

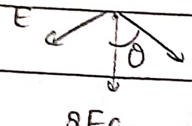


Find x for V_{max}.

(gravity free space)

(Executing circular motion in ~~Director~~ plane)

A



$E = \frac{kqQ}{d^2}$

$$\frac{mv^2}{x} = 2E\cos = \frac{2kqQ}{d^2}\cos$$
$$= \frac{2kqQx}{d^3}$$

$$\Rightarrow \quad v = \left(\frac{2kqQ}{m}\right)^{1/2}\left(\frac{x^2}{(x^2+R^2)^{3/2}}\right)^{1/2}$$

$$\ell(v) = \ell(x) - \frac{3}{4}\ell(x^2+R^2) \Rightarrow \frac{1}{v}(v') = \frac{1}{x} - \frac{3}{2}\frac{x}{(x^2+R^2)} = 0$$

$$\Rightarrow 2x^2+2R^2 = 3x^2$$

$$\Rightarrow \boxed{x = \sqrt{2}R}$$

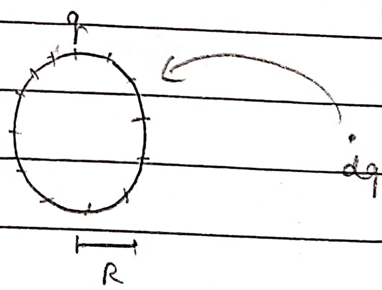
SELF ENERGY

Work done in assembling the charges into a specific config. (or released when dismantled)

→ **Shell**

$V_f = \frac{kq}{R}$

$V_i = 0$



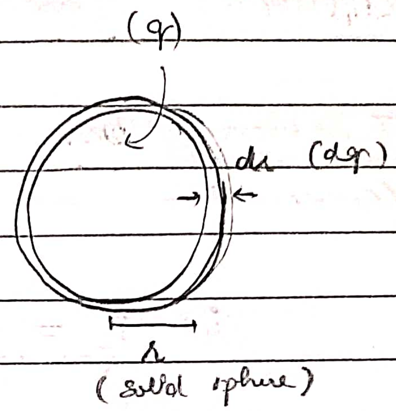
$$dW = (dq)(V_f - V_i) = \frac{kq}{R}dq.$$

$$W = \int_0^Q \frac{kq}{R}dq = \frac{kQ^2}{2R}$$

→ Solid sphere

$V_f = \frac{kq}{r} = \frac{k\rho}{r}\left(\frac{4\pi r^3}{3}\right) = \left(\frac{4}{3}\pi k\rho\right) r^2$

$V_i = 0$



$dW = dq\,(V_f - V_i)$

$= \rho\,(4\pi r^2)(dr)\left(\frac{4\pi k\rho}{3}\right) r^2$

$\Rightarrow W = \int_0^R \frac{16\pi^2\rho^2 k}{3}\, r^4\, dr \rightarrow W = \frac{16\pi^2}{3}\rho^2 k\,\frac{R^5}{5}$ $\qquad \rho = \frac{Q}{\frac{4\pi R^3}{3}}$

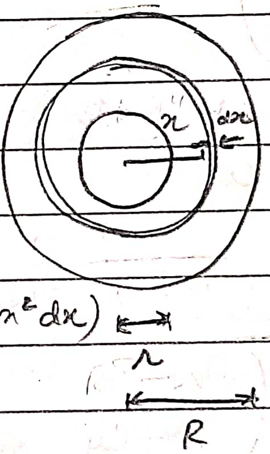
$= \frac{\cancel{16\pi^2}}{3}\left(\frac{9k}{\cancel{16\pi^2}}\right)(Q^2)\left(\frac{R^5}{5R^6}\right)$

$\Rightarrow \boxed{W = \frac{3kQ^2}{5R}}$

→ Thick spherical shell

$V_f = \frac{kq}{x} = \frac{k\rho}{x}\left(\frac{4\pi}{3}\right)(x^3 - r^3)$

$V_i = 0$



$dW = dq\,(V_f - V_i) = \left(\frac{4\pi k}{3}\right)\left(\frac{x^3 - r^3}{x}\right)\rho\,(\rho\, 4\pi x^2 dx)$

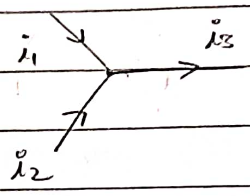
$= \left(\frac{16\pi^2\rho^2 k}{3}\right)(x^4 - r^3 x)\,dx$

$W = \frac{16\pi^2\rho^2 k}{3}\int_r^R x^4 - r^3 x\; dx = \left(\frac{16\pi^2\rho^2 k}{3}\right)\left[\frac{R^5}{5} - \frac{r^5}{5} - \frac{r^3 R^2}{2} + \frac{r^5}{2}\right]$

$= \left(\frac{16\pi^2\rho^2 k}{3}\right)\left[\frac{R^5}{5} - \frac{r^3 R^2}{2} + \frac{3r^5}{10}\right]$

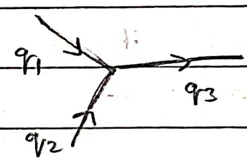
KIRCHOFF'S LAW

→ Junction law
(Conservation of charge)



$i_1 + i_2 = i_3$

Similarly, $q_1 + q_2 = q_3$



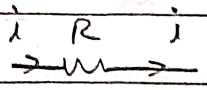
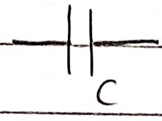
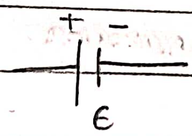
S'ment – Algebraic sum of current at any junction is zero

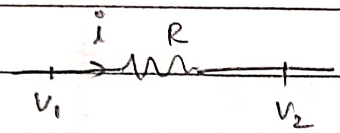
→ Loop / Mesh law
(Conservative nature of $\vec{E}$)

S'ment – Sum of potential drop in any closed mesh will be equal to sum of EMF in the mesh.

Used as – Sum of potential drop in any closed mesh will be zero.

Here, we will take EMF as V drop

	Resistance	Capacitor	Cell
Component :			
ΔV :	$-iR$ (if going in dirn of current)		

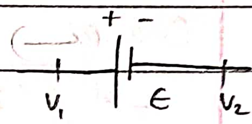
i.e 

$(\rightarrow)$ $V_1 - iR = V_2$

$\Rightarrow$ $V_2 - V_1 = -iR$

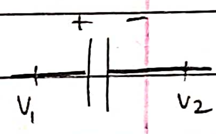
$(\leftarrow)$ $V_2 + iR = V_1$

$\Rightarrow$ $V_2 - V_1 = -iR$

i.e $(\longrightarrow)$ 

$(\rightarrow)$ $V_1 - \epsilon = V_2$

$(\leftarrow)$ $V_2 + \epsilon = V_1$

i.e 

$(\rightarrow)$ $V_1 - \frac{q}{C} = V_2$

$(\leftarrow)$ $V_2 + \frac{q}{C} = V_1$

CAPACITANCE & CAPACITOR

Def. Property of cond. by virtue of which it can store charge (on its surface) is called Capacitance.

It depends on shape on cond., but not on the material

Experimentally, $q \propto V$

$\Rightarrow$ $\boxed{q = CV}$

Capacitance

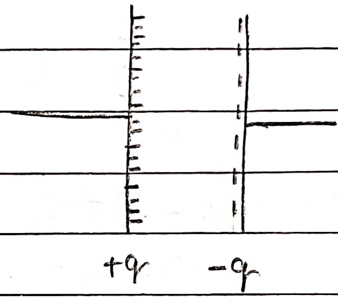
unit: C/V or F (Farad)

→ **Capacitor** (symbol $-||-$ or $-|(-$)

Device for storing charge.

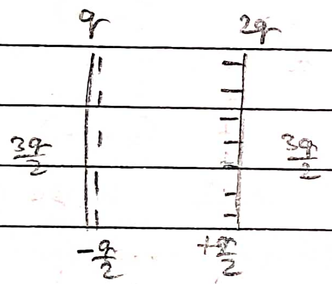
Charge on capacitor

= (Charge on one of the inner surface of plates)



Total charge on inner surfaces is always zero.

eg –



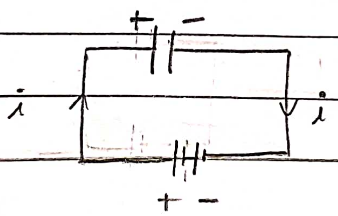
→ Charge on capacitor

= $q/2$

NOTE: (1) We can consider an isolated conductor as a capacitor by considering the second conductor at ∞.

(2) Polarity of capacitor depends on polarity of battery providing the charge.

eg.



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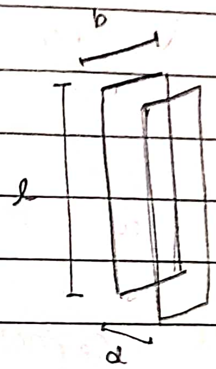
For capacitor, $C = \frac{q}{V}$ ← potential diff.

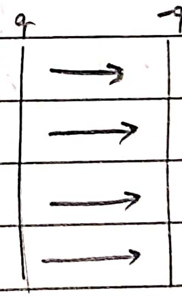
conductor $C = \frac{q}{V}$ ← Potential



Types
- Parallel plate
- Spherical
- Cylinderical

$d << b$
$d << l$

- Parallel Plate



$dV = -\vec{E}\cdot d\vec{r}$

$\Rightarrow V = \int -E\, dr = -Ed$

$E = \frac{\sigma}{\epsilon_0}$

Potential drop $= Ed$

$= \left(\frac{\sigma}{\epsilon_0}\right)(d) = \frac{qd}{A\epsilon_0}$

$\boxed{C = \frac{q}{V} = \left(\frac{A\epsilon_0}{d}\right)}$ ← (Air capacitor)

If diE introduced, (completely) $C = (\epsilon_r)\left(\frac{A\epsilon_0}{d}\right)$ $\left[\because E = \frac{\sigma}{\epsilon_r \epsilon_0}\right]$

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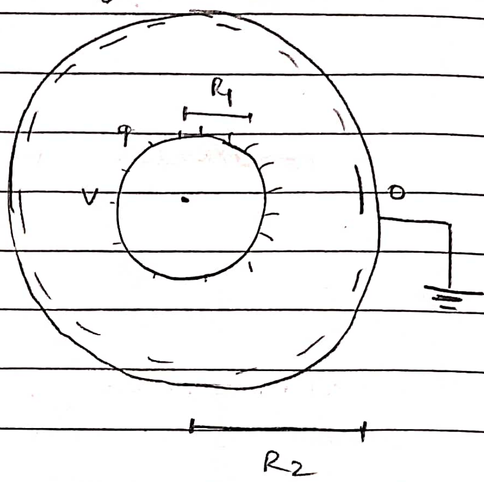
- **Spherical Capacitor**

Concentric shells

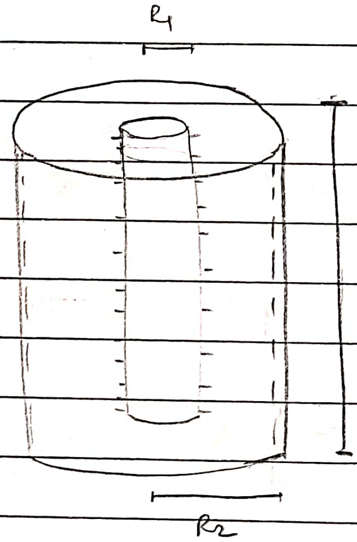
$$V = k\left(\frac{q}{R_1} - \frac{q}{R_2}\right) - 0$$



$$C = \frac{q}{V} = \frac{R_1 R_2}{k(R_2 - R_1)}$$

$$= \boxed{4\pi\epsilon_0\left(\frac{R_1 R_2}{R_2 - R_1}\right)}$$

- **Cylinderical Capacitor**



(R<<l)

$$\text{Potential drop} = \int_{R_1}^{R_2} E \cdot dr$$

$$= \int_{R_1}^{R_2} \frac{2k\lambda}{r}\, dr$$

$$\left(\lambda = \frac{q}{l}\right) \quad = 2k\lambda \ln\left(\frac{R_2}{R_1}\right)$$

$$C = \frac{q}{V} = \boxed{\frac{2\pi\epsilon_0 l}{\ln(R_2/R_1)}}$$

Battery is voltage source → maintains fixed V across circuit.

(↑) is current source → Gives fixed current in circuit

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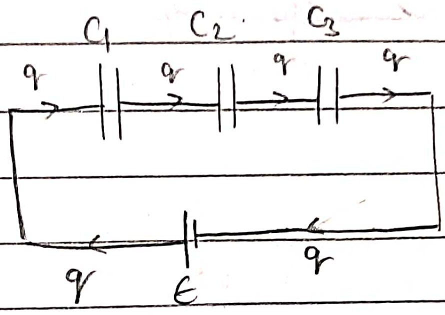
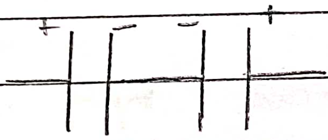
→ Equivalent Capacitance

• Series



Cond^n: same charge in all capacitors with same polarity.

i.e



cannot be taken in series.

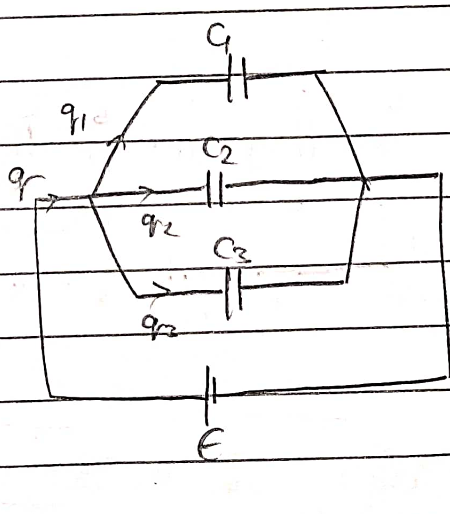
By def, $C_{eq} = \frac{q}{E}$

Since $q_1 = q_2 = q_3 = q$ & $V_1 + V_2 + V_3 = E$

$\Rightarrow \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3} = \frac{q}{C_{eq}}$.

$\Rightarrow \boxed{\frac{1}{C_{eq}} = \sum \frac{1}{C_i}}$

• Parallel



$q_1 + q_2 + q_3 = q$

$\Rightarrow C_1E + C_2E + C_3E = C_{eq}E$

$\Rightarrow \boxed{C_{eq} = \sum C_i}$

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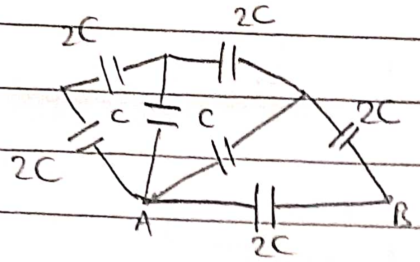
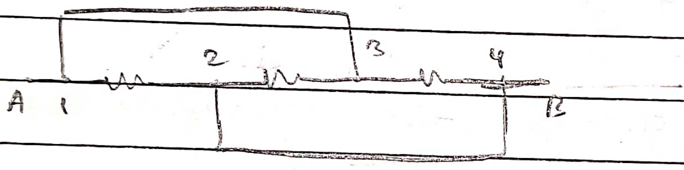
Q. 

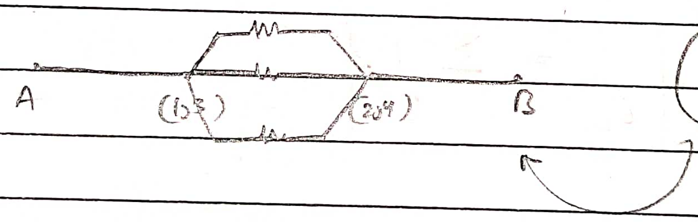
Find C_{eq} b/w A & B

A.

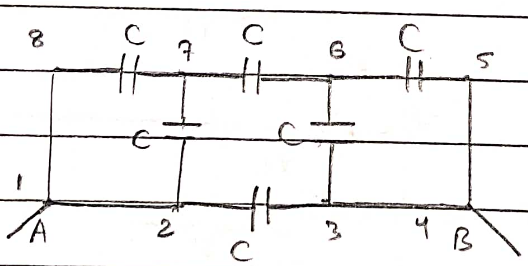
NOTE: When 2 pts. connected by wire,
$\Rightarrow$ Potential to those pts is same.

Q. 

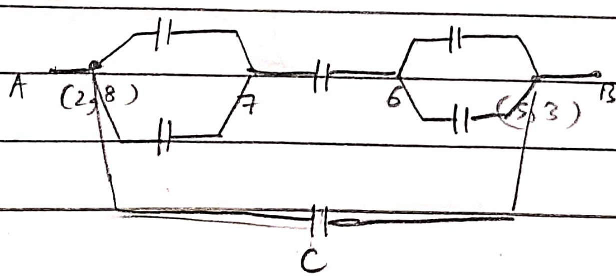
Find R_{eq}(AB)

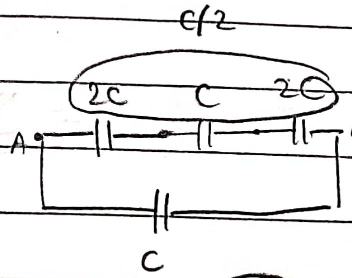
A. 

(Redrawn figure)

Q. 

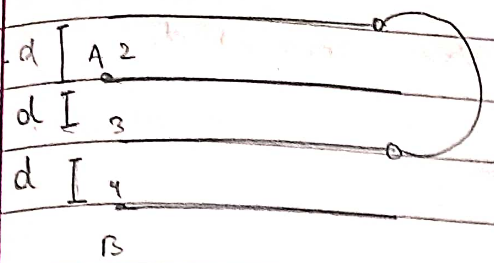
Find C_{eq} (AB)

A. 

= 

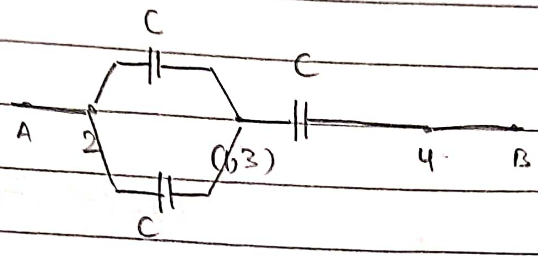
$C_{eq} = \frac{C}{2} + C = \frac{3C}{2}$

Q.



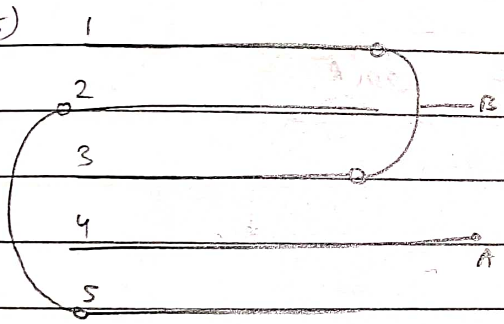
Find $C_{eq}(AB)$

A.



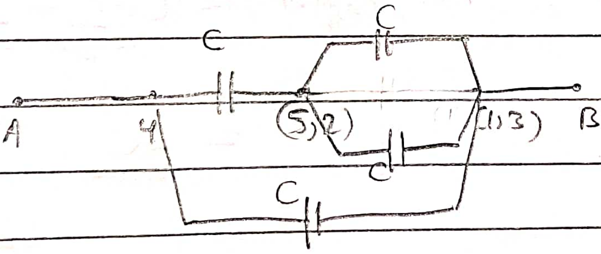
$$C_{eq}(AB) = \frac{2}{3}C = \left(\frac{2}{3}\right)\left(\frac{A\varepsilon_0}{d}\right)$$

Q.



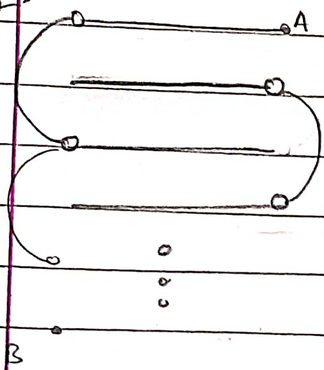
Find $C_{eq}(AB)$

A.



$$C_{eq}(AB) = C + \frac{2}{3}C = \frac{5}{3}C$$

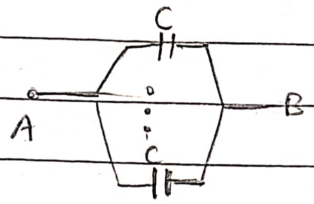
(d, A)

Q. 

n plates

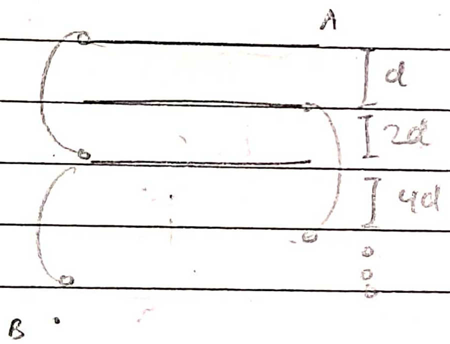
Alternate plates connected

Find $C_{eq}(AB)$

A. 

$(n-1)$ Capacitors

$\underline{C_{eq}(AB) = (n-1)C}$

Q. 

Find $C_{eq}(AB)$

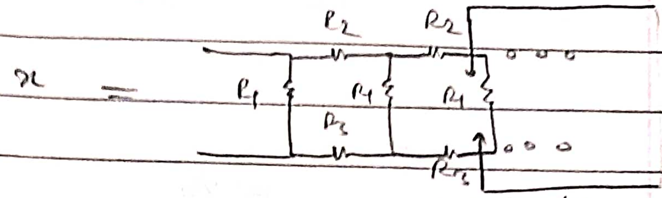
A All in ||.

$C_{eq}(AB) = C + \frac{C}{2} + \frac{C}{4} + \dots$

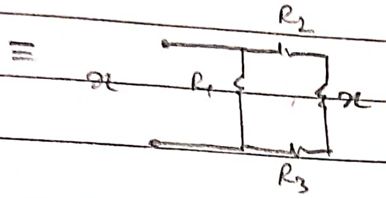
$= \underline{2C}$

(Symmetry Based Qs)

1. Infinite ladder

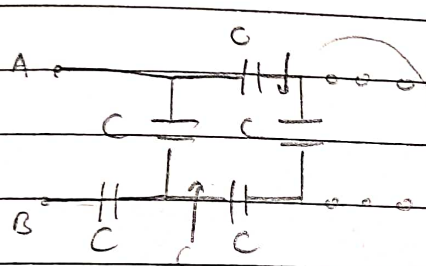


(Replaced)



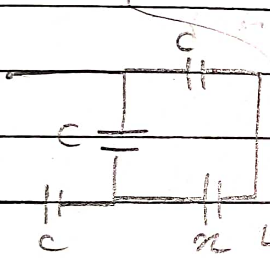
* (We replace from the pt. where the part & the whole look alike)

Q.



find C_{eq}

A.



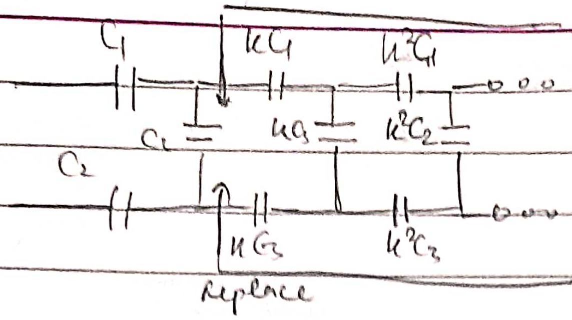
(Replaced)

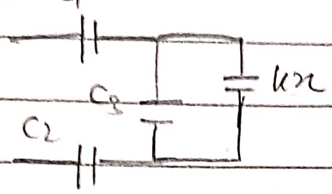
$$\Rightarrow x = \frac{C\left(C + \frac{xC}{C+x}\right)}{C + C + \frac{xC}{C+x}} = \frac{C(C^2+2Cx)}{(2C^2+3Cx)} = \frac{C(2x+C)}{(3x+2C)}$$

$$\Rightarrow 3x^2 + 2Cx = 2xC + C^2$$

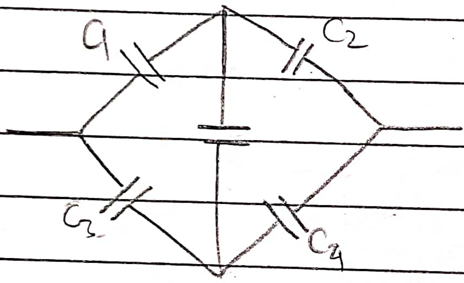
$$\Rightarrow \boxed{x = \frac{C}{\sqrt{3}}}, \quad \left(x \neq \frac{-C}{\sqrt{3}}\right)$$

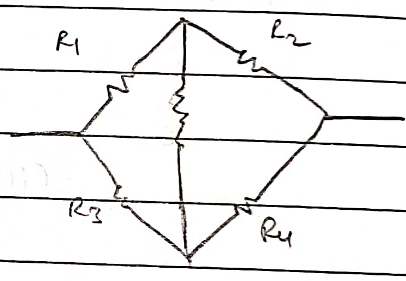
since $x > 0$

$x =$ 

$=$ 

② Wheatstone Bridge

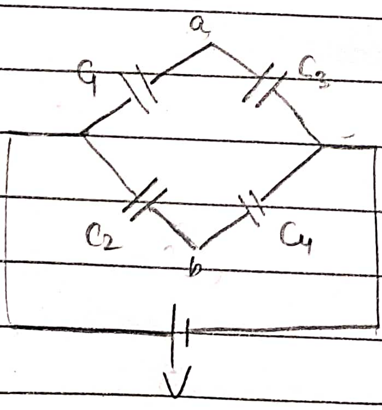




if $\dfrac{C_1}{C_2} = \dfrac{C_3}{C_4}$ $\qquad \dfrac{R_1}{R_2} = \dfrac{R_3}{R_4}$

$\Rightarrow$ Balanced bridge $\Rightarrow$ Bridge capacitor/resistor doesn't work

Proof



if $V_a = V_b$

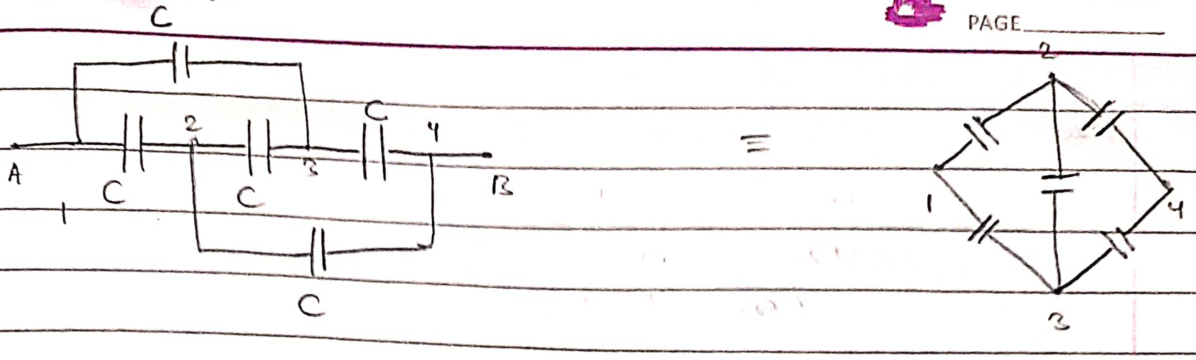
$\Rightarrow \left(\dfrac{\cancel{C_1}C_2}{C_1+C_2}\right)\left(\dfrac{V}{\cancel{C_1}}\right) = \dfrac{\cancel{C_3}C_4}{(C_3+C_4)}\left(\dfrac{V}{\cancel{C_3}}\right)$

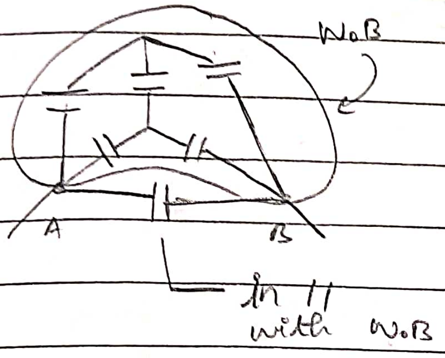
$\Rightarrow \dfrac{C_1}{C_2} + 1 = \dfrac{C_3}{C_4} + 1$

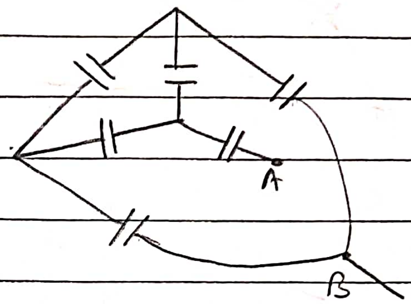
$\Rightarrow \boxed{\dfrac{C_1}{C_2} = \dfrac{C_3}{C_4}}$

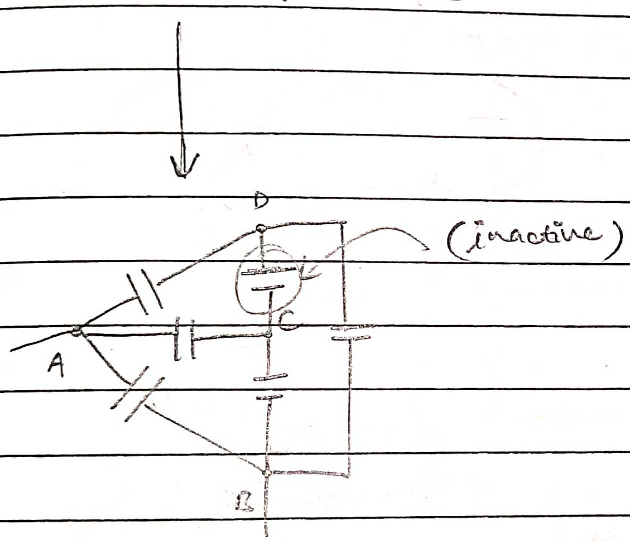
$\Rightarrow$ If this condn satisfied, no current b/w a & b (if connected)

common configs.

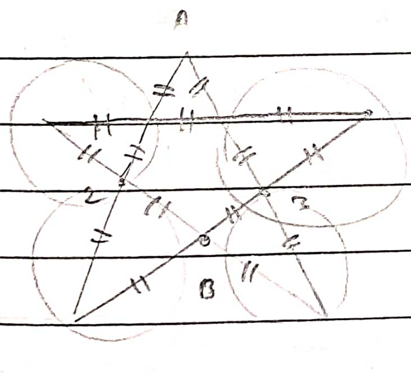






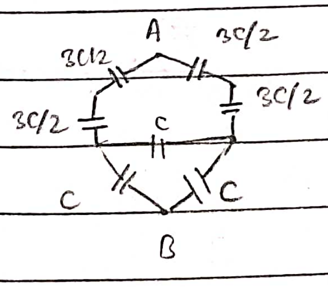


Q.



Find Ceq.(AB)

A

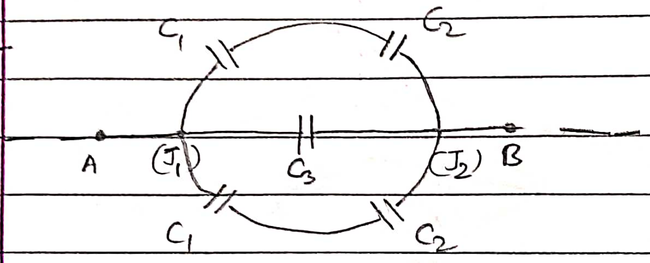


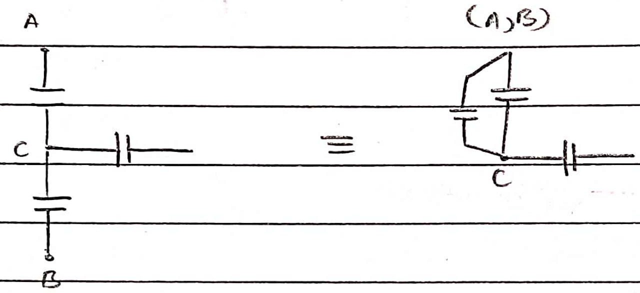
24/03/2023

(3) Mirror Image Symmetry
& Reverse Polarity Symmetry
(Perpendicular)

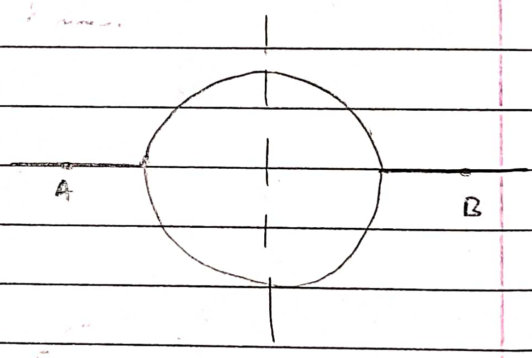
MI : Junctions at same potential

eg -

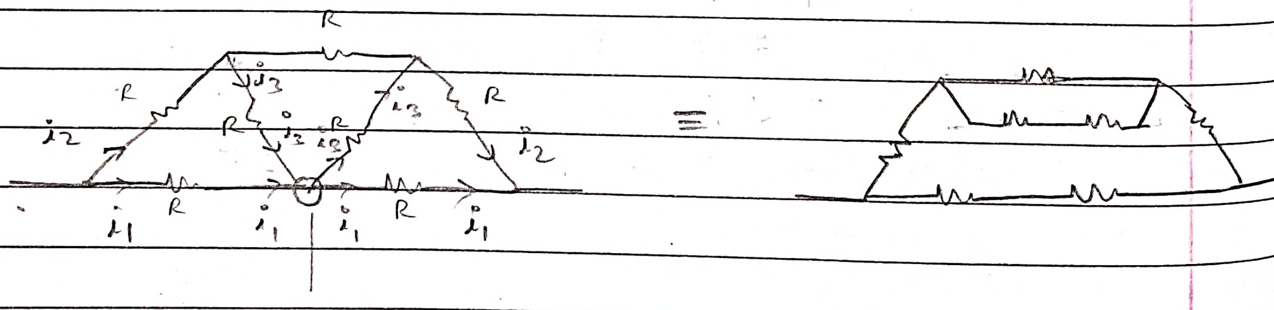


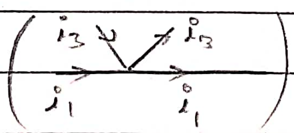


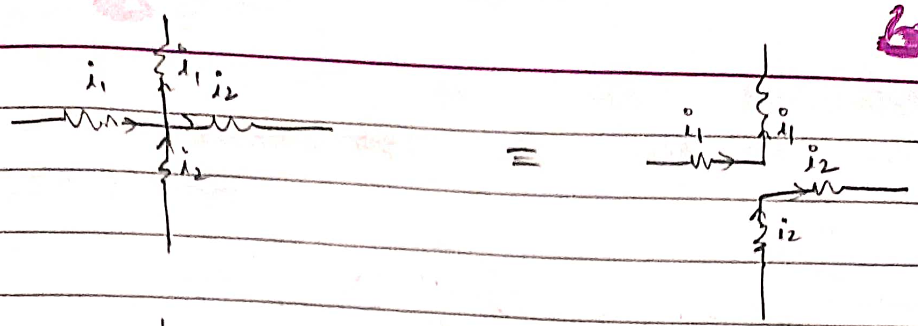
RP: Same current through elems. sym abt ⊥ bisector

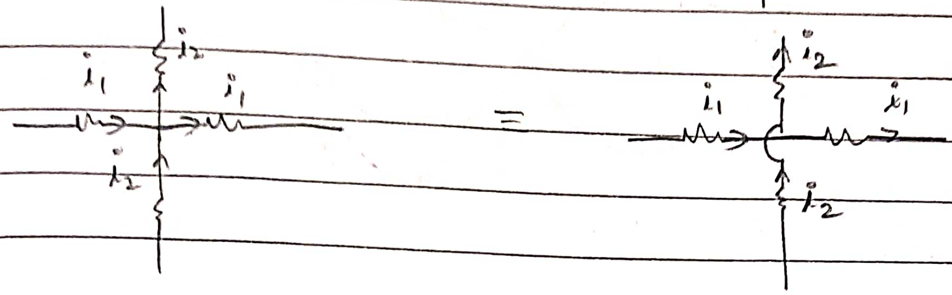


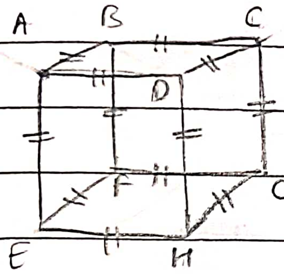
eg





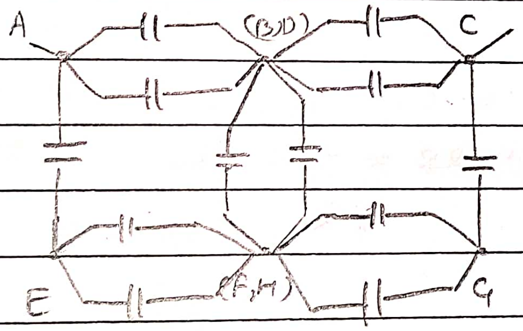




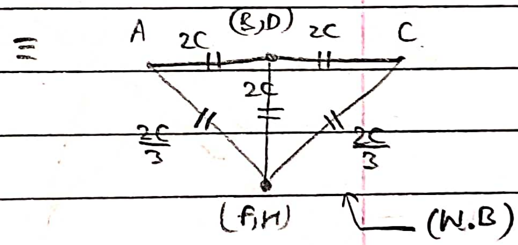
Q. 

Find Ceq.
(i) About face diagonal
(ii) About side
(iii) About body diagonal.

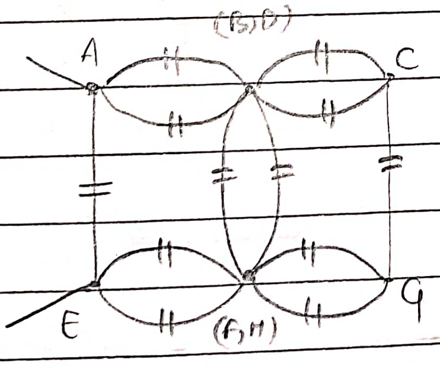
A
(i)
(AC)



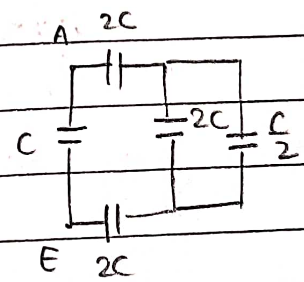
(Mirror plane through ABFH)



(ii)
(AE)



(Mirror plane through AEGC)



* In general, if all resistors are replaced by capacitors, $\text{coeff}(C_{eq}) = \frac{1}{\text{coeff}(R_{eq})}$ i.e here $C_{eq} = \frac{5C}{6}$

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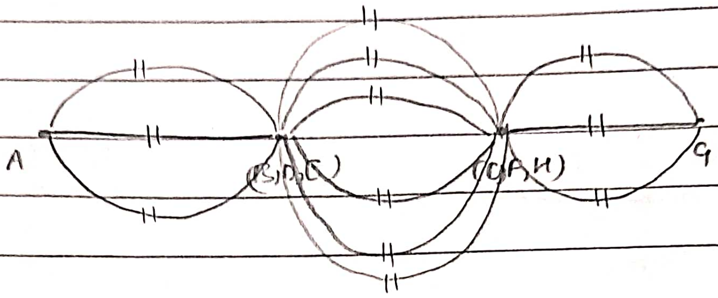
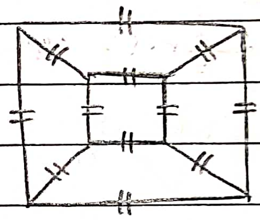
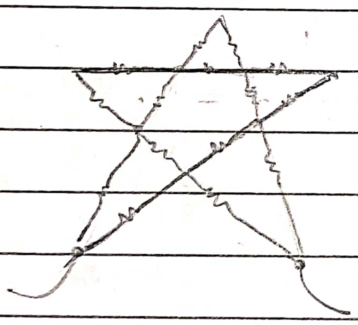
(iii) (AG) Rotational sym of (B, D, E) & (C, F, H) about AG line. ⟹ Same potentials of (B, D, E) & (C, F, H) respectively



Alternate form –

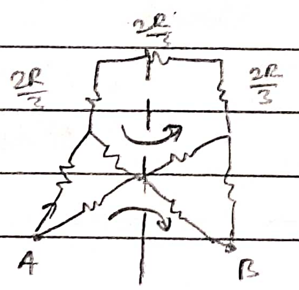


Q

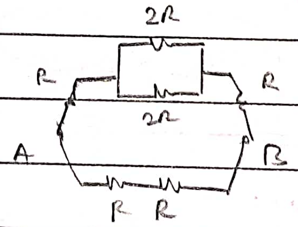


Find Req.

A



≡

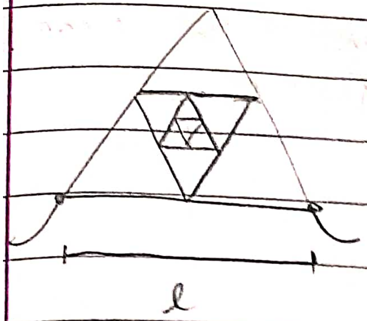


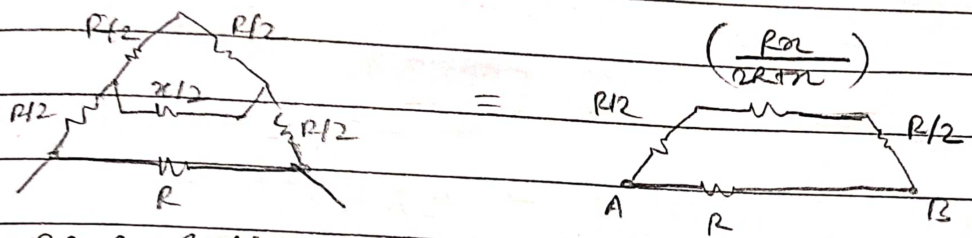
⟹ * $\boxed{R_{eq} = \frac{6R}{5}}$

(7 Ω/m)

Infinite Δs.

Find Req.





[By RP & Infinite replacement]

=R)

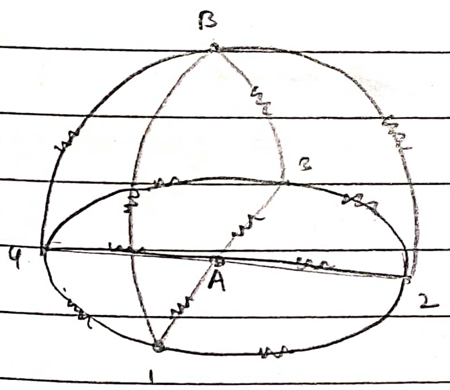
$$\frac{1}{x} = \frac{1}{R} + \frac{1}{R+\frac{Rx}{2R+x}}$$

$$\Rightarrow \frac{1}{x} = \frac{1}{R} + \frac{2R+x}{2R^2+2Rx}$$

$$\Rightarrow (R-x)(2\cancel{R})(R+x) = x\cancel{R}(2R+x)$$

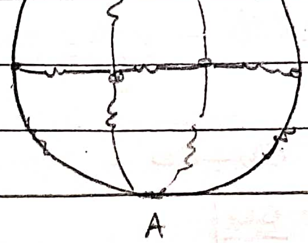
$$\Rightarrow 2R^2-2x^2 = 2Rx+x^2 \Rightarrow 3x^2+2Rx-2R^2=0$$

$$\Rightarrow x = \left(\frac{\sqrt{7}-1}{3}\right)R$$



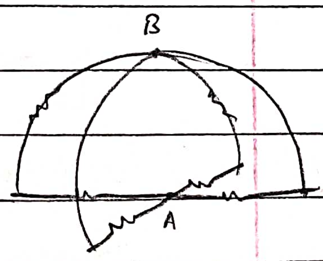
Find Req.

=



(RP)

≡



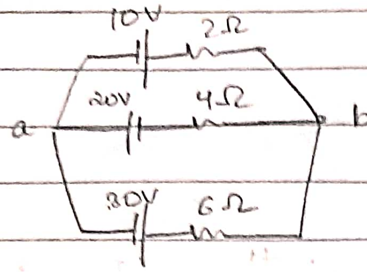
(MI)

1, 2, 3, 4 at same potential

25/05/2022

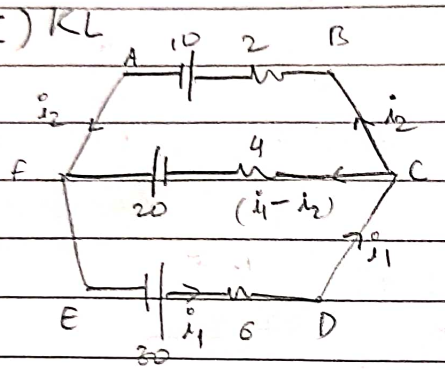
Node Eq^n

Q.



Find $V_b - V_a$

A. (I) KL



① EDCFE

$30 - 6i_1 - 4(i_1 - i_2) + 20 = 0$

$\Rightarrow 10i_1 - 4i_2 = 80$

$\Rightarrow 5i_1 - 2i_2 = 25$

② FCBAF

$-20 + 4(i_1 - i_2) - 2i_2 - 10 = 0$

$\Rightarrow 4i_1 - 6i_2 = 30$

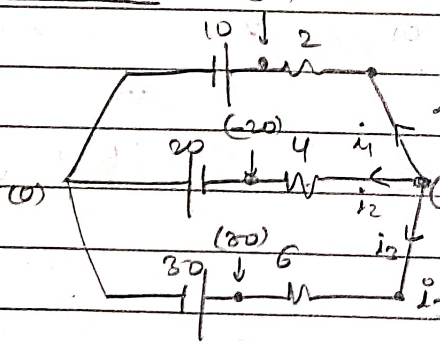
$\Rightarrow 2i_1 - 3i_2 = 15$

$i_1 = \frac{45}{11}$

$i_2 = \frac{-25}{11}$

(II) Node eq^n
(Preferred)

Cond^n: # Variables req. $= (\#\text{Nodes}) - 1$



$i_1 = \left(\frac{V-10}{2}\right)$

$i_2 = \left(\frac{V+20}{4}\right)$

$i_3 = \left(\frac{V-30}{6}\right)$

$i_1 + i_2 + i_3 = 0 \Rightarrow \frac{V-10}{2} + \frac{V+20}{4} + \frac{V-30}{6} = 0$

$\Rightarrow 6V - 60 + 3V + 60 + 2V - 60 = 0$

$\Rightarrow \boxed{V = \frac{60}{11}}$

$i_1 = \frac{\frac{60}{11} - 10}{2} = \frac{-25}{11}$

$i_3 = \frac{\frac{60}{11} - 30}{6} = -\frac{45}{11}$

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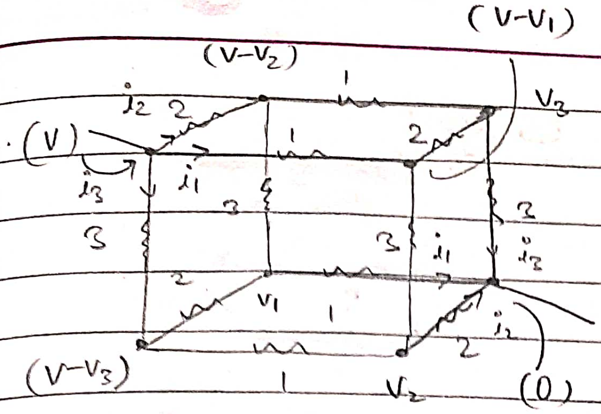
Q. 

Find V

A. By symmetry, i & ΔV across 1, 2, 3 resistors is as shown above

On pt. at V_1, $\frac{V_1 - 0}{1} + \frac{V_1 - (V - V_2)}{3} + \frac{V_1 - (V - V_3)}{2} = 0$

$\Rightarrow \underline{11V_1 + 2V_2 + 3V_3 = 5V}$ — ①

at V_2, $\frac{V_2 - 0}{2} + \frac{V_2 - (V - V_1)}{3} + \frac{V_2 - (V - V_3)}{1} = 0$

$\Rightarrow 3V_2 + 2V_2 - 2V + 2V_1 + 6V_2 - 6V + 6V_3 = 0$

$\Rightarrow \underline{2V_1 + 11V_2 + 6V_3 = 8V}$ — ②

at V_3, $\frac{V_3 - 0}{3} + \frac{V_3 - (V - V_1)}{2} + \frac{V_3 - (V - V_2)}{1} = 0$

$\Rightarrow \underline{3V_1 + 6V_2 + 11V_3 = 9V}$ — ③

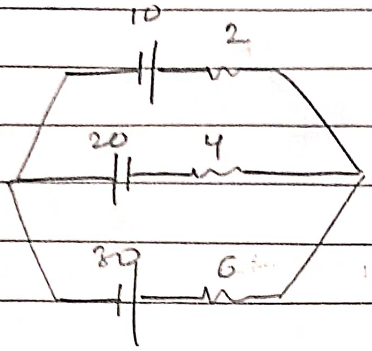
$2① - ②$ $\quad 20V_1 - 7V_2 = 2V$

$11② - 6③$ $\quad 4V_1 + 85V_2 = 34V$

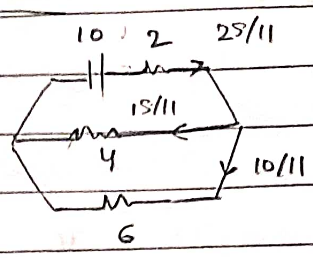
$\Rightarrow$ $V_1 = (17/72)V$, $V_2 = (7/18)V$, $V_3 = (13/24)V$

$i = i_1 + i_2 + i_3 = \frac{V_1}{1} + \frac{V_2}{2} + \frac{V_3}{3} = \frac{17}{72} + \frac{7}{36} + \frac{13}{72} = \frac{11}{18}$

- ## Superposition Law of Current

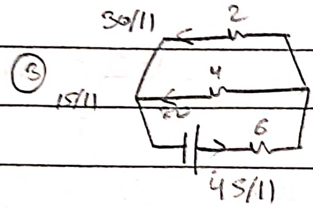


①



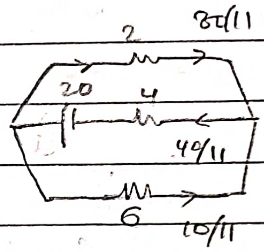
$$i = \frac{10}{22/5} = \frac{25}{11}$$

③



$$i = \frac{30}{22/3} = \frac{45}{11}$$

②

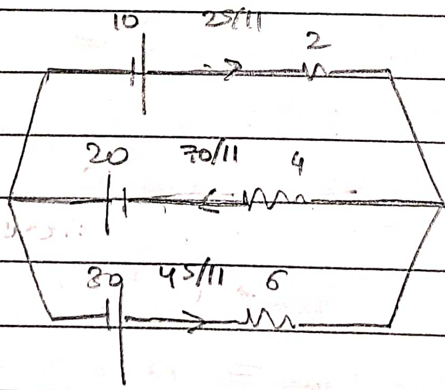


$$i = \frac{20}{11/2} = \frac{40}{11}$$

We assume only one source active at a time.

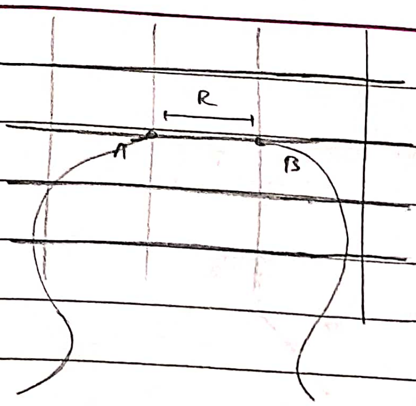
Net current = Σ (Current) with only one source active

$\equiv$



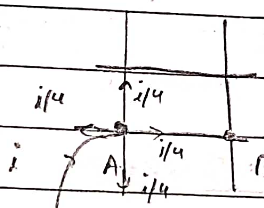
Q. Infinite grid.
Resistance of each side R.

Find R_{eq}.



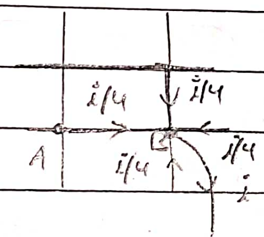
A. We assume 2 sources at infinity:
S_1: sends i from $A \to \infty$
& S_2: sends i from $\infty \to B$

Due to S_1,



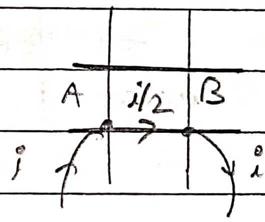
current won't return as infinite grid.

Due to S_2,



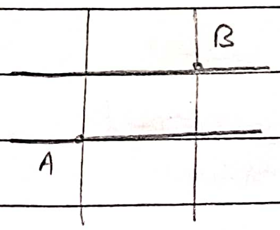
(due to sym)

So, by superpost. principle,



$$\Rightarrow R_{eq} = \frac{\overbrace{(R)(i/2)}^{V_{AB}}}{(i)} = R/2$$

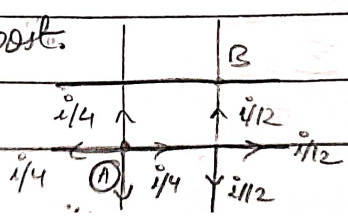
Q. Each edge is of res. R.
Find R_{eq}.



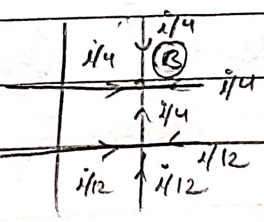
$$i R_{eq} = (i/3)R + (i/3)R$$

$$\Rightarrow \boxed{R_{eq} = \frac{2R}{3}}$$

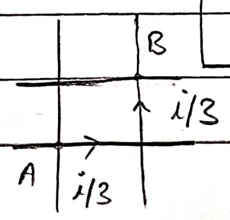
A. By superpost. principle,



$+$



$=$



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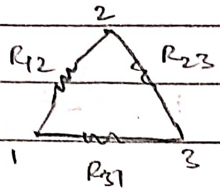
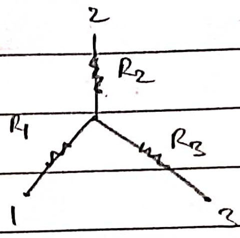
- Star – Delta Connection

(I) D to S



$\longrightarrow$



By Superpost. law.

For source b/w 1 & 3, $\dfrac{R_{31}(R_{12}+R_{23})}{R_{12}+R_{23}+R_{31}} = R_1+R_3$

1 & 2, $\dfrac{R_{12}(R_{23}+R_{31})}{R_{12}+R_{23}+R_{31}} = R_1+R_2$

2 & 3, $\dfrac{R_{23}(R_{31}+R_{12})}{R_{12}+R_{23}+R_{31}} = R_2+R_3$

On adding,

$$+\left\{\begin{aligned} \Sigma R_1 &= \frac{\Sigma R_{12}R_{23}}{\Sigma R_{12}} \\ -(R_2+R_3) &= -\frac{R_{23}(R_{31}+R_{12})}{\Sigma R_{12}} \end{aligned}\right.$$

$$\Rightarrow \boxed{R_1 = \frac{R_{12}R_{31}}{\Sigma R_{12}}}$$

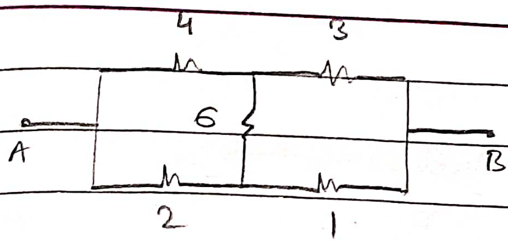
(II) S to D

By above formula, $R_{31} = \left(\dfrac{R_3}{R_2}\right)(R_{12})$

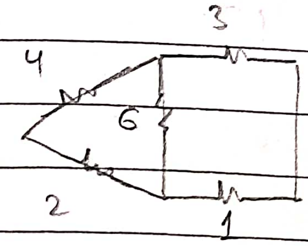
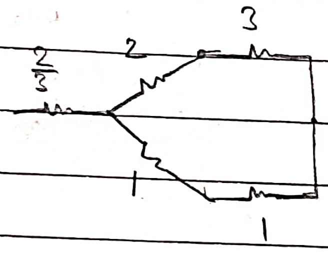
& $R_{23} = \left(\dfrac{R_3}{R_1}\right)(R_{12})$

Since $\cancel{R_3} = \dfrac{R_{31}\cdot R_{23}}{\Sigma R_{12}} = \dfrac{\left(\dfrac{\cancel{R_3}}{R_2}\right)(\cancel{R_{12}})\left(\dfrac{\cancel{R_3}}{R_1}\right)(R_{12})}{(\cancel{R_{12}})(\cancel{R_3})\left(\Sigma \dfrac{1}{R_1}\right)}$

$$\Rightarrow \boxed{R_{12} = (R_1R_2)\left(\Sigma \frac{1}{R_1}\right)}$$

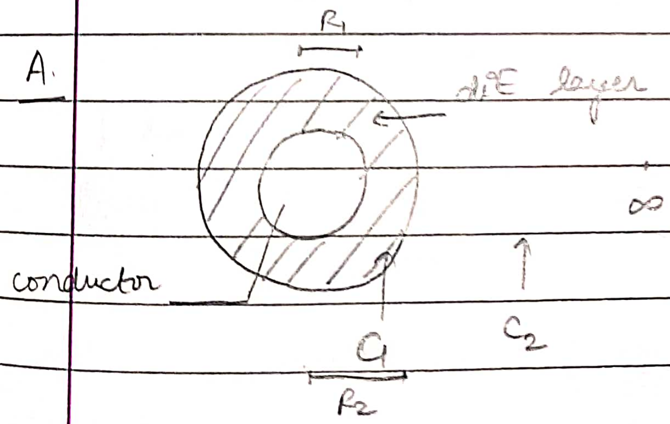
Q. 

Find Req.

A.  $\equiv$ 

$$R_{eq} = \frac{2}{3} + \frac{5\cdot 2}{7} = \left(\frac{44}{21}\right)$$

Q. Spherical conductor (R_1) surrounded by diE ($\epsilon_r = \epsilon$) of thickness ($R_2 - R_1$). Find capacitance.

A. 

Capacitance of this arrangement is eq. to 2 capacitors connected in series

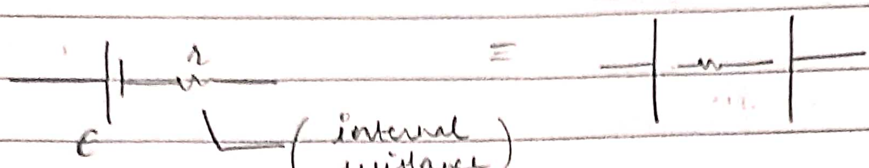
$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

$$C_1 = 4\pi\epsilon_0\epsilon\left(\frac{R_1 R_2}{R_2 - R_1}\right)$$

$$C_2 = 4\pi\epsilon_0\left(\frac{R_2 R_\infty}{R_\infty - R_2}\right) = 4\pi\epsilon_0\left(\frac{R_2}{1 - \frac{R_2}{R_\infty}}\right)$$

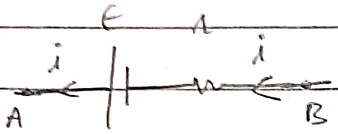
$$= 4\pi\epsilon_0 R_2$$

BATTERY

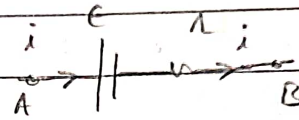


(internal resistance)

— Resistance b/w terminals of battery.



(Discharging battery)



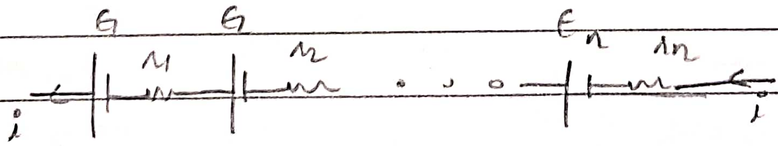
(Charging battery)

$V_B - V_A$: $-E + ir$ $\quad\quad$ $-E - ir$

If $i = 0$, $\underline{\Delta V = E}$

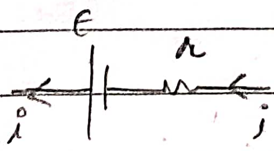
→ Combination of Battery

• Series



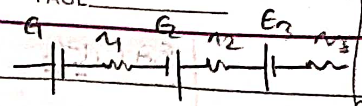
(Properly connected i.e identical polarity)

$\equiv$



$\boxed{\epsilon = \sum \epsilon_i}$

NOTE:



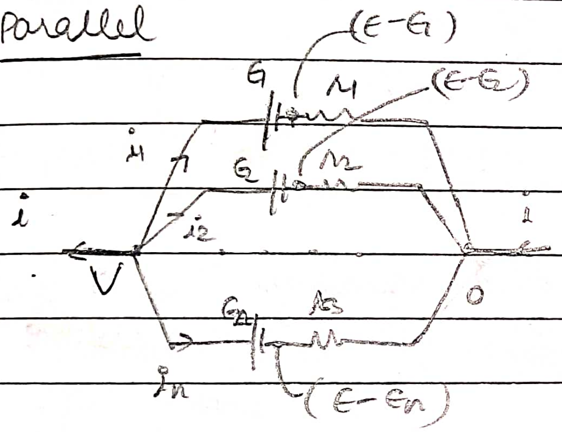
$\epsilon = \epsilon_1 - \epsilon_2 + \epsilon_2$

If i flowing,

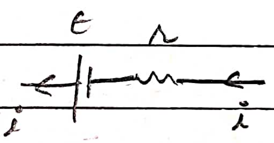
$$\cancel{\epsilon} - ir = (\epsilon_1 - ir_1) + (\epsilon_2 - ir_2) + \dots + (\epsilon_n - ir_n)$$
$$= (\cancel{\textstyle\sum \epsilon_i}) - i(\textstyle\sum r_i)$$

$$\Rightarrow \boxed{r = \sum r_i}$$

• Parallel



$\equiv$



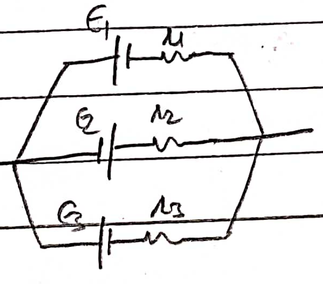
When $i = 0$ in ext circuit, $V = \epsilon$

by node eqn $\sum i_i = 0 \Rightarrow \sum \frac{\epsilon - \epsilon_i}{r_i} = 0$

$$\Rightarrow \epsilon\left(\sum \frac{1}{r_i}\right) = \sum\left(\frac{\epsilon_i}{r_i}\right)$$

$$\Rightarrow \boxed{\epsilon = \left(\frac{\sum \epsilon_i / r_i}{\sum 1/r_i}\right)}$$

NOTE:



$$\epsilon = \left(\frac{\epsilon_1/r_1 - \epsilon_2/r_2 + \epsilon_3/r_3}{1/r_1 + 1/r_2 + 1/r_3}\right)$$

when i being supplied to ext. circuit,

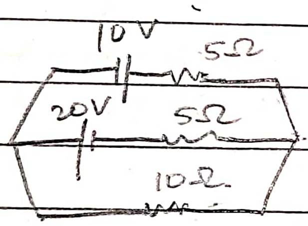
by Node eqn, $\sum\left(\frac{V-\epsilon_i}{r_i}\right) + i = 0$

$\Rightarrow V\left(\sum 1/r_i\right) = \sum(\epsilon_i/r_i) - i$

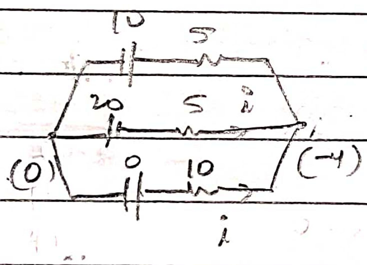
$\Rightarrow V = \underbrace{\frac{\sum(\epsilon_i/r_i)}{\sum(1/r_i)}}_{\epsilon} - \frac{i}{(\sum 1/r_i)}$

$\Rightarrow r = 1/\sum(1/r_i)$

OR $\boxed{\frac{1}{r} = \sum\left(\frac{1}{r_i}\right)}$

Q. 

Find current through 10Ω resistor

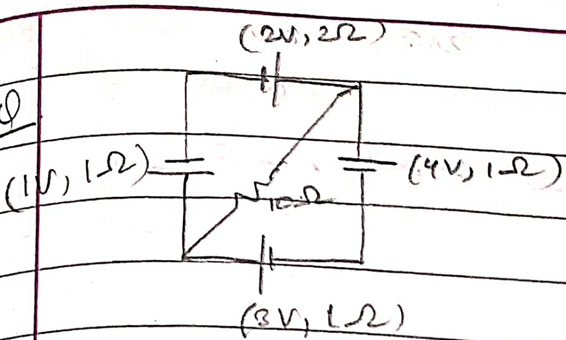
A. $\equiv$ 

$\epsilon = \frac{\frac{10}{5} - \frac{20}{5} + \frac{0}{10}}{\frac{1}{5} + \frac{1}{5} + \frac{1}{10}}$

$= -4$

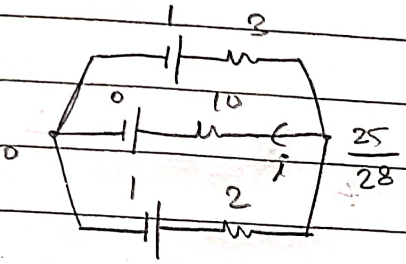
$0 - 5i = -4 \Rightarrow i = 4/5$

Q



Find current in 10 Ω resistor

A. $\equiv$

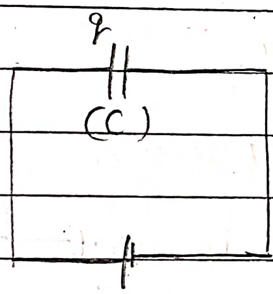


$$\mathcal{E} = \frac{\frac{1}{3}+\frac{0}{10}+\frac{1}{2}}{\frac{1}{3}+\frac{1}{2}+\frac{1}{10}} = \frac{50}{56} = \frac{25}{28}$$

$$\frac{25}{28} - i(10) = 0 \Rightarrow \underline{i = 5/56}$$

ENERGY OF CAPACITOR

Let q charge be stored at a moment of time



To store dq more charge on capacitor, work needs to be done.

$$dW = V\,dq = \frac{q}{C}\,dq \Rightarrow W = \int_0^Q \frac{q}{C}\,dq.$$

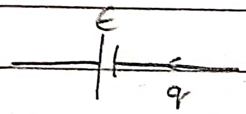
$$\Rightarrow \boxed{U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV}$$

• Energy density of $\vec{E}$ — Energy per unit vol. in E

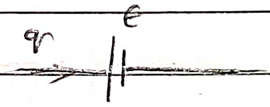
$$\frac{E}{\text{Vol.}} = \frac{\frac{1}{2}Q^2}{C(Ad)} = \frac{\frac{1}{2}Q^2}{\left(\frac{A\epsilon_0}{d}\right)(Ad)} = \frac{\sigma^2}{2} = \left(\frac{\sigma}{\epsilon_0}\right)^2\left(\frac{\epsilon_0}{2}\right)$$

$$= \boxed{\frac{1}{2}\epsilon_0 E^2}$$

• EMF — Work done by battery in supplying unit charge in ext. circuit.



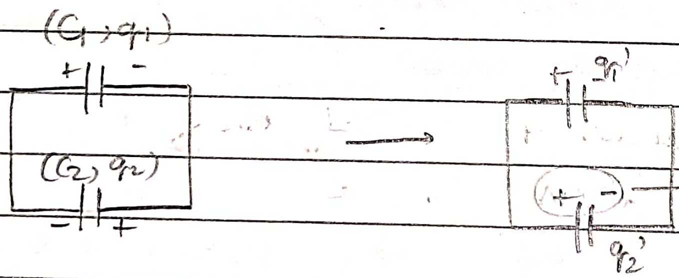
$W = \epsilon q$



$W = -\epsilon q$

→ Redistribution of Charge

Q.



*(same polarity finally)

Find q_1' & q_2'.

A q flows till V across both capacitors is same.

*(opp polarity)

CoC $\quad q_1 - q_2 = q_1' + q_2' = C_1V + C_2V \Rightarrow V = \left(\frac{q_1+q_2}{C_1+C_2}\right)$

$q_1' = C_1V = \left(\frac{C_1}{C_1+C_2}\right)(q_1+q_2) \qquad q_2' = C_2V = \left(\frac{C_2}{C_1+C_2}\right)(q_1+q_2)$

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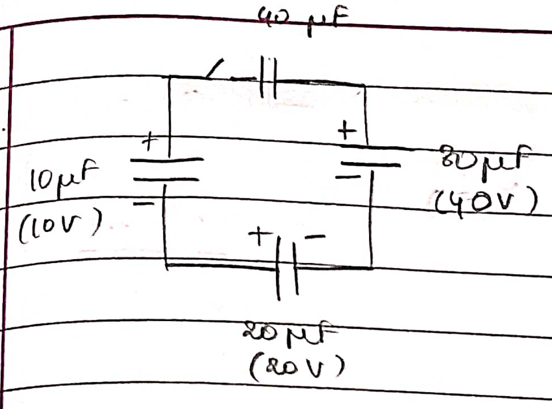
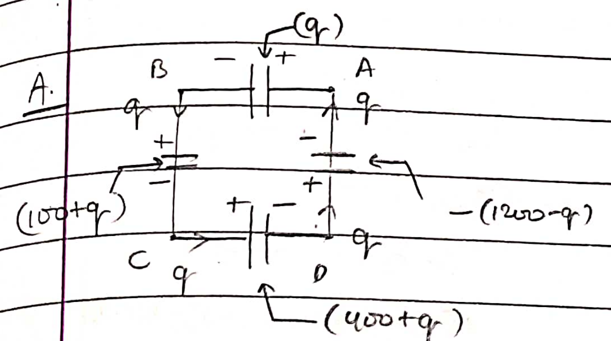
Q.



Switch closed, find final charge on initially uncharged capacitor.

A.

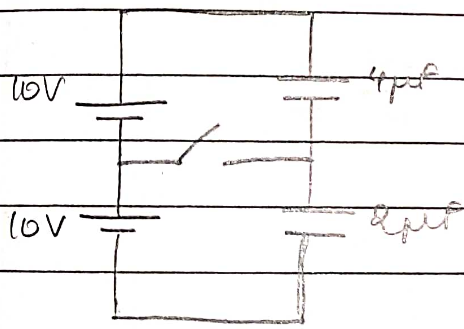


By mesh law (ADCBA)

$$\frac{-(1200-q)}{30} + \frac{(400+q)}{20} + \frac{(100+q)}{10} + \frac{q}{40} = 0$$

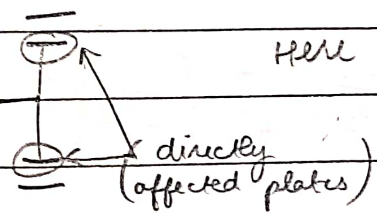
$\Rightarrow$

Q.



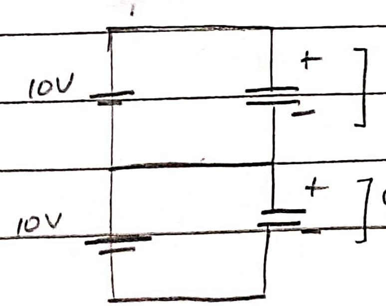
Find charge flowing through switch when it is closed.

A. ① Identify plates directly affected by battery & note their total charge q_i



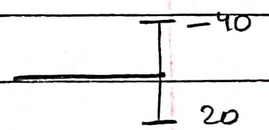
Here, $q_i = 0$ since caps. in series.

② Find q_f



$q_1 = (10)(4) = -40$

$q_2 = (10)(2) = (20)$



$q_f = -40 + 20$
$= -20\ \mu C$

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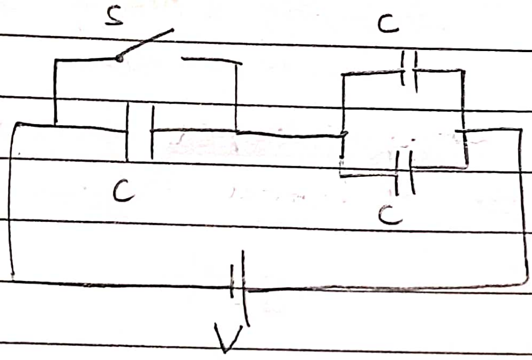
Charge flown through switch (in the dirn of directly affected plates) $= q_f - q_i$

$= \underline{-20\ \mu C}$

Q. 

Find addn charge supplied by battery & heat produced after closing of switch

A. (1) $C_{eq} = \frac{2C}{3}$ $\quad q = \left(\frac{2CV}{3}\right)$

$C'_{eq} = 2C \rightarrow q' = 2CV$

$\Delta q = q' - q = \left(\frac{4CV}{3}\right)$

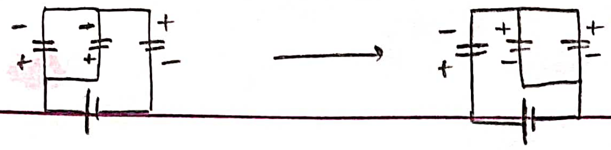
(2) CoE $\quad$ Heat $= (U_i + W_{battery}) - U_f$

$= \frac{1}{2}\left(\frac{2C}{3}\right)(V^2) + V(\Delta q) - \frac{1}{2}(2C)(V^2)$

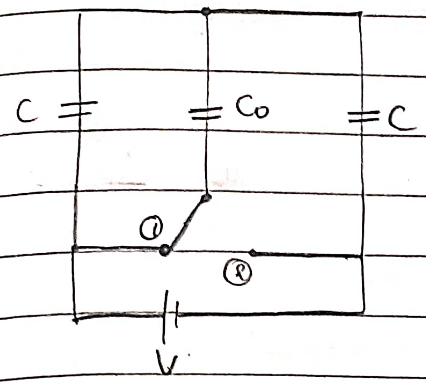
$= (V)\left(\frac{4CV}{3}\right) - \left(\frac{V^2}{2}\right)\left(\frac{4C}{3}\right)$

$= \underline{2CV^2/3}$

Here, battery supplies charge as polarity of caps. change



Q.



Find extra charge supplied by battery & heat produced when switch is shifted from posi. 1 to 2.

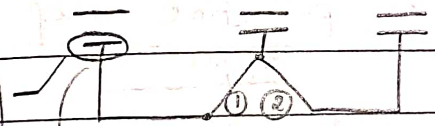
A.

$$C_{eq} = \frac{C(C+C_o)}{(C_o+2C)} \qquad q = \frac{(C+C_o)CV}{(C_o+2C)}$$

$$C_{eq}' = \frac{C(C+C_o)}{(C_o+2C)} \qquad q' = \frac{(C+C_o)CV}{(C_o+2C)}$$

①

directly affected plate



$$q_1 = \underbrace{\frac{q}{(C+C_o)}}_{\text{V across the cap.}} C = \frac{C^2V}{(C_o+2C)}$$

$$q_2 = \underbrace{\frac{q}{C}}_{\text{V across the cap.}} C = \frac{(C+C_o)CV}{(C_o+2C)}$$

$$\Delta q = q_2 - q_1 = \frac{CC_oV}{(C_o+2C)}$$

② **CoE**

$$\text{Heat} = (U_i + W_{battery}) - U_f$$

$$= W_{battery}$$

$$= V\Delta q$$

$$= \frac{CC_oV^2}{(C_o+2C)}$$

$(\because U_i = U_f)$

$$\left[\frac{1}{2}\frac{q^2}{C_{eq}} = \frac{1}{2}\frac{q'^2}{C_{eq}'}\right]$$

{ Battery supplies CV charge if all cap. in circuit were uncharged initially. }

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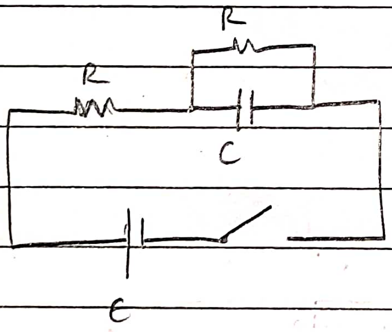
R–C CIRCUIT

Types of Qs :–

1. Just after closing switch
2. Steady state
3. Transient charge / current

- When switch closed, if $q=0$, but $\frac{dq}{dt} \neq 0$, hence current flows through circuit but no potential diff. across cap. i.e it behaves as a wire.
- If $q \neq 0$, current flows & potential drop CV across cap.
- In steady state, no current in branch of circuit containing capacitor. i.e it behaves as open circuit

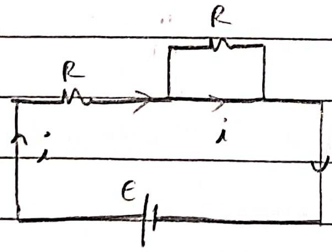
Q.



a) Find i after just closing switch

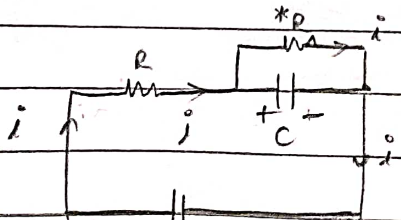
b) Find steady state current & q on cap

A. a)



$i = \left(\frac{E}{R}\right)$

b)



$i = (E/2R)$

Since V across *R & C same

$\Rightarrow iR = \frac{q}{C} \Rightarrow \boxed{q = \frac{EC}{2}}$

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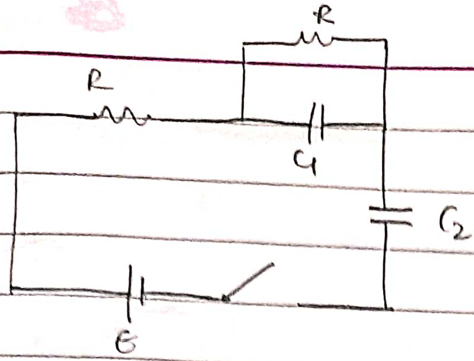
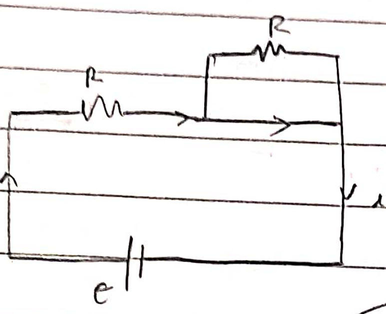
Q.

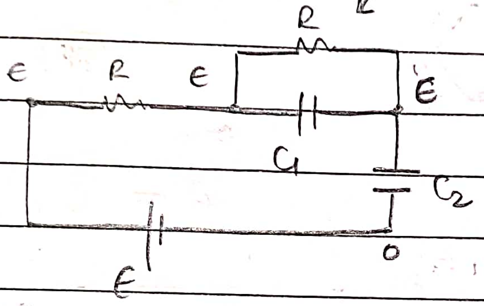


A. a)



$i = E/R$

b)

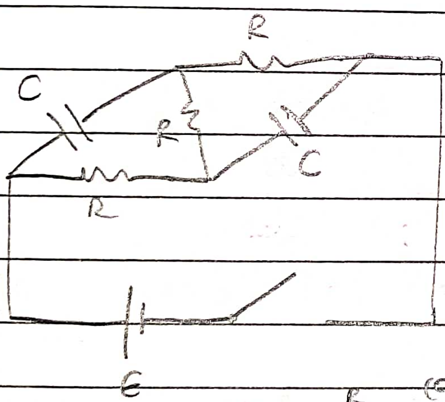


$\underline{i = 0} \Rightarrow \underline{V_R = 0}$

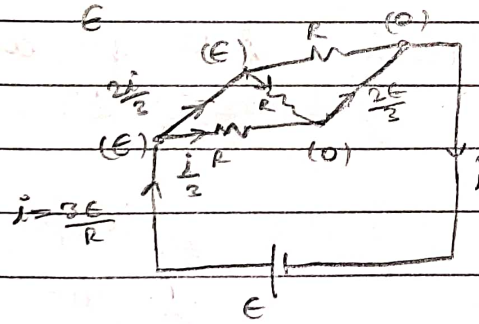
$V_{C_1} = 0 \Rightarrow q_{C_1} = 0$

$V_{C_2} = E \Rightarrow q_{C_2} = EC_2$

★ Q.



A. a)



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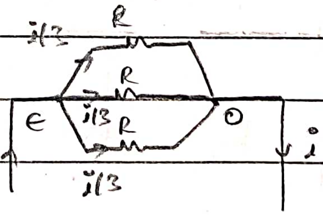
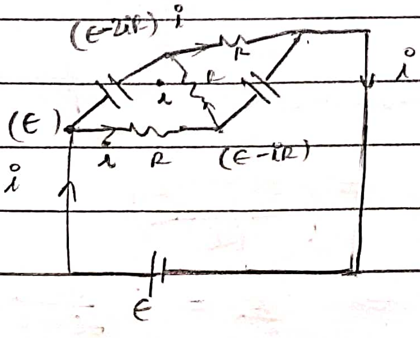


$i = 3E/R$

$i_C = 2E/R$

b)

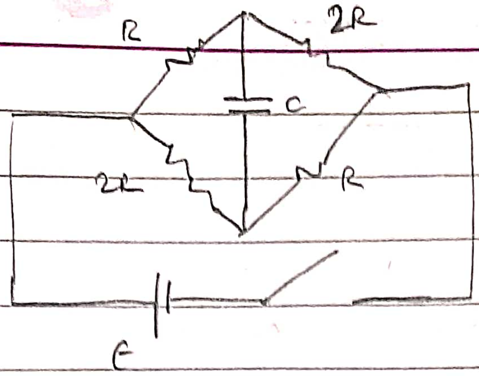


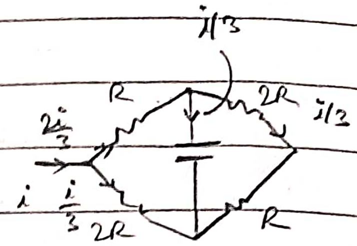
$i = E/3R$

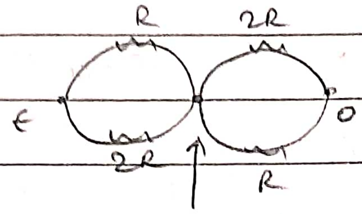
// Complete

find q on cap

$q = C(E-(E-2iR)) = 2iRC = \frac{2EC}{3}$

Q. 

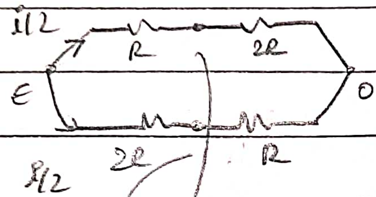


A a) 

(Cap acts as wire)

$$i = \frac{E}{2\left(\frac{2R}{3}\right)} = \frac{3E}{4R}$$

$$i_{cap} = i/3 = E/4R$$

b) 

$$\left(E - \frac{iR}{2}\right) = \left(\frac{2E}{3}\right)$$

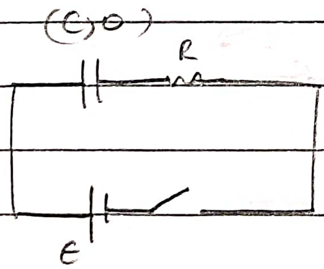
$$\left(E - \frac{2iR}{2}\right) = \left(\frac{E}{3}\right)$$

$$i = \frac{E}{3R/2} = \frac{2E}{3R}$$

$$q = C\left(\frac{2E}{3} - \frac{E}{3}\right)$$

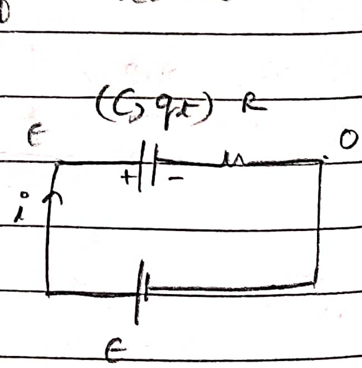
$$= CE/3$$

→ Charging of Cap.

Q 

Find i & q as fn of time.

A. Let q_t at $t = t$.



By Mesh law, $E - \frac{q_t}{C} - iR = 0$

Since $i = \frac{dq}{dt}$ ⇒ $\frac{dq}{dt} = \left(\frac{EC - q_t}{RC}\right)$

$$\Rightarrow \int_0^t \frac{1}{RC}\, dt = \int_0^q \frac{dq}{EC - q_t} \Rightarrow \frac{t}{RC} = l\left(\frac{EC}{EC - q}\right)$$

(Standard result to be remembered!)

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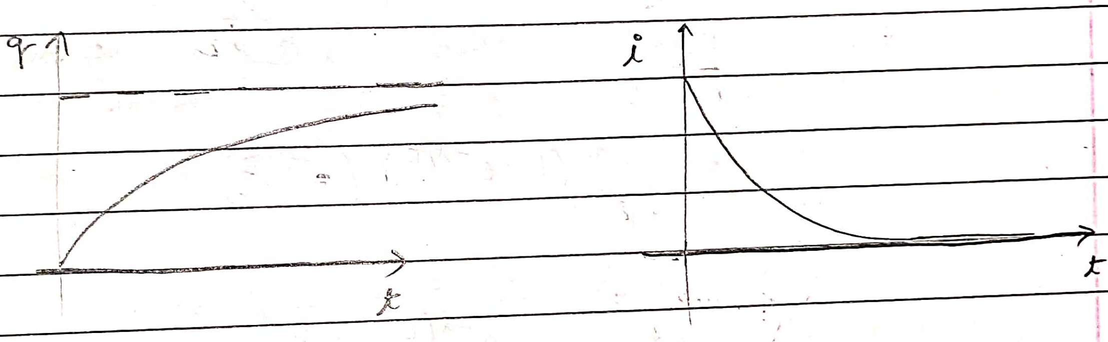
$$\Rightarrow \boxed{q = \epsilon C\left(1 - e^{-t/RC}\right)}$$

NOTE: ① {Sol^n of $R\dfrac{dq}{dt} + \dfrac{q}{C} = \epsilon$ for $q: 0 \rightarrow q$ & $t: 0 \rightarrow t$}

i.e $a\dfrac{dx}{dt} + \dfrac{x}{b} = c \Rightarrow x = bc\left(1 - e^{\frac{-t}{ab}}\right)$

$x: 0 \rightarrow x \quad t: 0 \rightarrow t$

② 

$$i = \frac{dq}{dt} = \frac{\epsilon}{R} e^{\frac{-t}{RC}}$$

since $[RC] = [t] \Rightarrow$ RC is called the time const of R–C circuit.

$$\boxed{\tau = RC}$$

Time const. – Time period after which charging [illegible] becomes $1/e$ of its initial value

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i.e $\boxed{i_{(t+\tau)} = i_t/e}$

(3) Work done by battery $= \epsilon q$

$$\Rightarrow P = \frac{dW}{dt} = \epsilon\frac{dq}{dt} = \epsilon i = \frac{\epsilon^2}{R}e^{-t/\tau}$$

Energy stored in cap. $= q^2/2C$

$$\Rightarrow \frac{dU}{dt} = \frac{q}{C}\left(\frac{dq}{dt}\right) = \frac{qi}{2C}$$

(Rate of Energy storage in cap.)

$$= \frac{\epsilon^2}{R}(1-e^{-t/\tau})(e^{-t/\tau})$$

$$\frac{dU}{dt} = \left(\frac{dU}{dt}\right)_{max} \text{ at } e^{-t/\tau} = \frac{1}{2}$$

$$\Rightarrow \boxed{t = \tau\, l(2)}$$

$$= 0.693\tau$$

Q After how many τ, will q on charging cap. will be half of q in steady state.

A $\underbrace{q}_{\frac{\epsilon C}{2}} = \epsilon C(1-e^{-\frac{t}{\tau}}) \Rightarrow e^{-t/\tau} = 1/2$

$$\Rightarrow t = \tau\, l(2)$$

Q. After how many τ, will charging cap. acquire 99.999% of max charge.

A. $(1-10^{-5})\cancel{\epsilon C} = \cancel{\epsilon C}(1-e^{-t/\tau}) \Rightarrow e^{-t/\tau} = 10^{-5}$

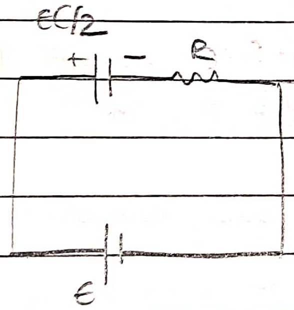
$\Rightarrow t = 5\,\ell(10)\,\tau$

$= \underline{11.515\,\tau}$

Q. The charging i becomes 90% of its initial value after 1 ms. What % of initial i will remain after 3 ms.

A. $i_3 = \frac{9}{10} i_2 = \left(\frac{9}{10}\right)^2 i_1 = \left(\frac{9}{10}\right)^3 i_0 \Rightarrow \underline{72.9\ \%}$

Q.



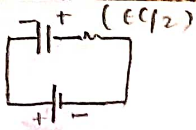
Initial charge $= \epsilon C/2$. Find q as fn of time.

A. Let extra charge acquired at time t be q_t.

$$\Rightarrow \epsilon - \frac{\left(\frac{\epsilon C}{2}+q_t\right)}{C} - iR = 0$$

$$\Rightarrow R\frac{dq}{dt} + \frac{q_t}{C} = \frac{\epsilon}{2} \Rightarrow q_t = \frac{\epsilon C}{2}\left(1-e^{-t/RC}\right)$$

Charge on cap. $q = \frac{\epsilon C}{2} + q_t = \frac{\epsilon C}{2} + \frac{\epsilon C}{2}\left(1-e^{-t/RC}\right)$

{ if  $(\epsilon C/2)$ Here, $q_0 = -\epsilon C/2$, not $\epsilon C/2$.

$\Rightarrow q = -\frac{\epsilon C}{2} + \frac{3\epsilon C}{2}\left(1 - e^{-t/RC}\right)$ }

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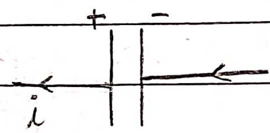
So, if q_0 initially,

$$\boxed{q = q_0 + (\epsilon C - q_0)\left(1 - e^{-t/\tau}\right)}$$

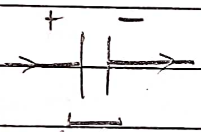
$\underline{t}$: $0 \rightarrow t$

NOTE: (1) For discharging cap.

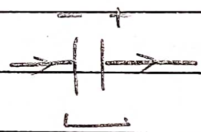
$$\boxed{i = -\frac{dq}{dt}}$$



eg (in above Q)



$\left(q - \frac{\epsilon C}{2}\right)$



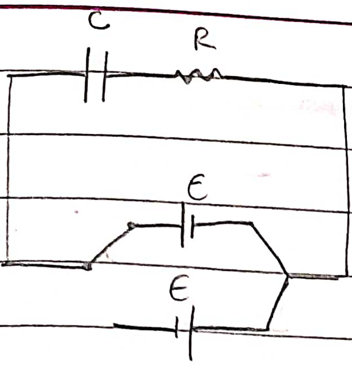
$\left(\frac{\epsilon C}{2} - q\right)$

$$i = \frac{d}{dt}\left(q - \frac{\epsilon C}{2}\right) = \frac{dq}{dt} \qquad i = -\frac{d}{dt}\left(\frac{\epsilon C}{2} - q\right) = +\frac{dq}{dt}$$

So, it makes no diff how we assume polarity of cap.
We generally take it acc. to final charge on cap. (as decided by the battery).

(2) Apply mesh law to obtain eqn of the form,

$$R\frac{dq}{dt} + \frac{q}{C} = \left(\epsilon - \frac{q_0}{C}\right)$$

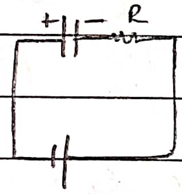
Q. 

Initially, switch was at posn. 1 for long time. At $t=0$, switch was shifted to posn. 2.

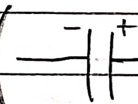
Find q on cap. as a fn^n of time. Also find the time at which the cap was momentarily fully discharged.

A. Steady state initially $\Rightarrow$ $\underline{q_0 = EC}$

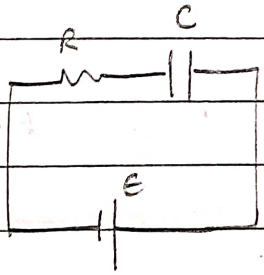
But polarity of battery changes.

i.e. 

So, $\underline{q_0 = -EC}$

Hence, $q = -EC + 2EC\left(1-e^{-t/\tau}\right)$ * ( Final polarity)

For $q=0$ $\quad e^{-t/\tau} = \frac{1}{2} \Rightarrow \underline{t = \tau \ell(2)}$

Q. 

At $t=0$, the capacitance was made ηC.

Find q on cap. as a fn^n of time

A. $q_0 = EC$ (steady state earlier)

$q = EC + (\eta EC - EC)\left(1 - e^{-\frac{t}{\eta RC}}\right)$

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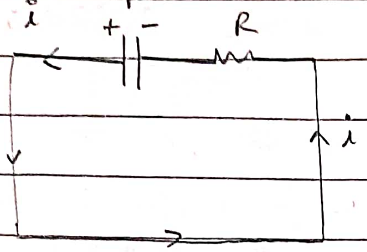
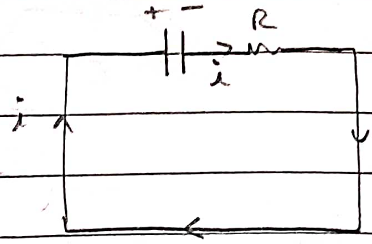
→ Discharging of Cap.



OR



By Mesh Law,

$$\frac{q}{C} - iR = 0 \qquad\qquad -\frac{q}{C} - iR = 0$$

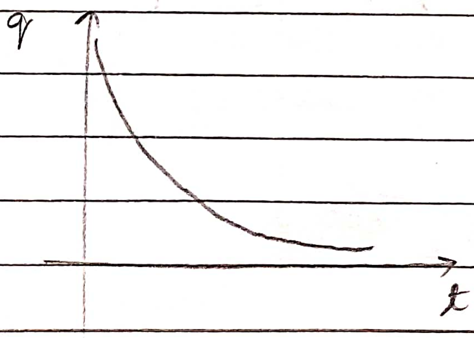
But, $i = -\frac{dq}{dt}$ (discharging current) $\qquad i = \frac{dq}{dt}$ (charging current)

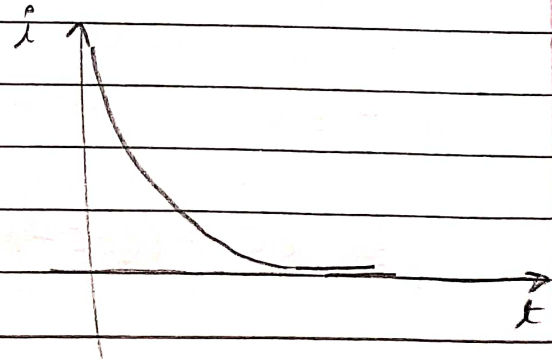
$$\Rightarrow R\left(\frac{dq}{dt}\right) + \frac{q}{C} = 0 \qquad\qquad \Rightarrow R\left(\frac{dq}{dt}\right) + \frac{q}{C} = 0$$

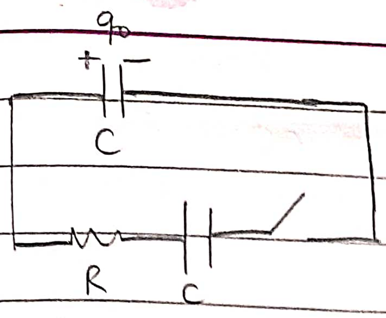
$$\Rightarrow \int_{q_0}^{q} \frac{dq}{q} = \int_0^t -\frac{dt}{RC}$$

$$\Rightarrow \boxed{q = q_0 e^{-t/RC}}$$

$$i = -\frac{dq}{dt} = \boxed{\frac{q_0}{RC} e^{-t/RC}}$$



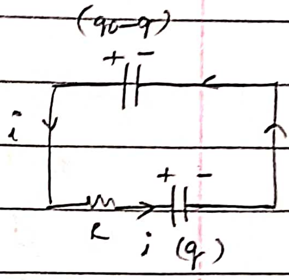


Q. 

Find charge on initially uncharged cap. as a fn of time.

A. ML:

$$\left(\frac{q_0-q}{C}\right) - iR - \frac{q}{C} = 0$$



$$\Rightarrow R\frac{dq}{dt} = \left(\frac{q_0-2q}{C}\right)$$

$i = \frac{dq}{dt}$

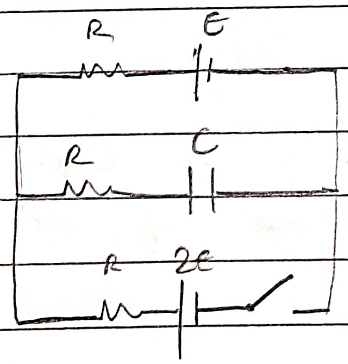
(Charging the uncharged cap)

$$\Rightarrow \int_0^q \frac{dq}{(q_0-2q)} = \int_0^t \frac{dt}{RC}$$

$$\Rightarrow \frac{1}{2}\ln\left(\frac{q_0}{q_0-2q}\right) = \frac{t}{RC}$$

$$\Rightarrow 1-\frac{2q}{q_0} = e^{-\frac{2t}{RC}} \Rightarrow q = \frac{q_0}{2}\left(1-e^{-2t/RC}\right)$$

$$\left(R\frac{dq}{dt} + \frac{2q}{C} = \frac{q_0}{C} \longrightarrow q = \left(\frac{q_0}{C}\right)\left(\frac{C}{2}\right)\left(1-e^{-\frac{t}{R(C/2)}}\right)\right)$$

Q. 

a) Find qcap in steady state

b) Find qcap as a fn of time after switch closed.

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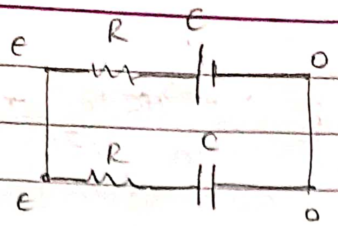
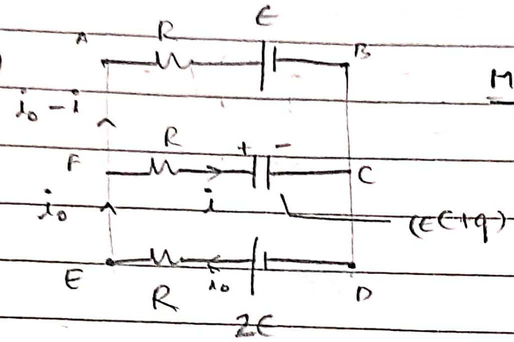
a) 

$q = \epsilon C$ (no current in circuit)

b) 

ML: EFCDE – (i)

$$-iR - \frac{\epsilon C + q}{C} + 2\epsilon - i_0 R = 0$$

ABDEA – (ii)

$$-(i_0 - i)R - \epsilon + 2\epsilon - i_0 R = 0$$

2(i) – (ii)

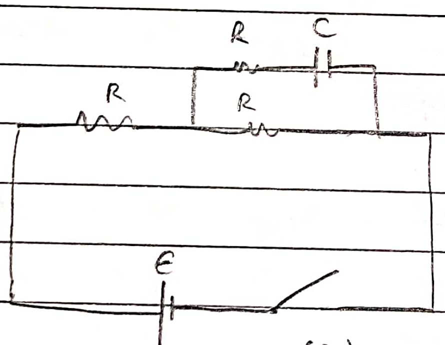
$$3R\left(\frac{dq}{dt}\right) + \frac{q}{(C/2)} = \epsilon$$

$$i = \frac{d}{dt}(\epsilon C + q)$$

$$= \frac{dq}{dt}$$

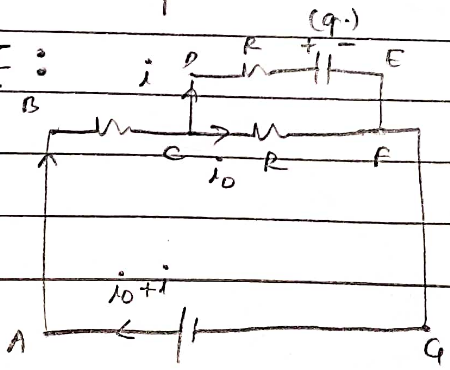
$$\Rightarrow \Delta q = \frac{\epsilon C}{2}\left(1 - e^{-\frac{2t}{3RC}}\right)$$

$$q = \epsilon C + \frac{\epsilon C}{2}\left(1 - e^{-\frac{2t}{3RC}}\right)$$

Q 

Find q on cap. as a fn of time.

A.

Method – I : 

C & R in ||

$$\Rightarrow iR + \frac{q}{C} = i_0 R$$

$$i = \frac{dq}{dt} \Rightarrow i_0 = \frac{dq}{dt} + \frac{q}{RC}$$

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ML: ABCFG

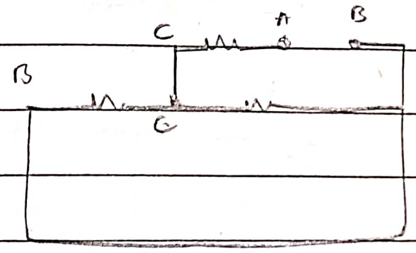
$$\epsilon - (i+i_0)R - i_0 R = 0$$

$$\Rightarrow \epsilon - R\frac{dq}{dt} = 2i_0 R = 2R\frac{dq}{dt} + \frac{2q}{C}$$

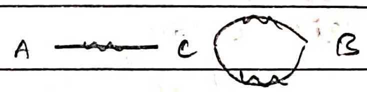
$$\Rightarrow 3R\frac{dq}{dt} + \frac{q}{(C/2)} = \epsilon$$

$$\Rightarrow q = \epsilon\left(\frac{C}{2}\right)\left(1 - e^{-\frac{2t}{3RC}}\right)$$

Method II : ① Find R_{eff} across cap.



$\equiv$



$$R_{AB} = \frac{3R}{2}$$

② Find q_{cap} in steady state (q_s)

$$\boxed{q = q_s\left(1 - e^{-\frac{t}{R_{eff} C}}\right)}$$

NOTE: This method only works when only one cap. given in circuit.

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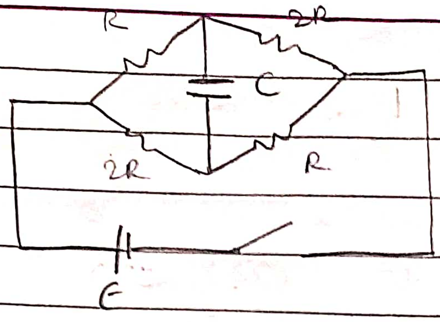
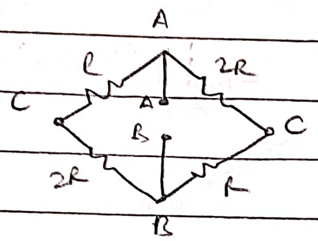
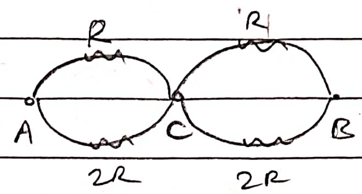
Q. 

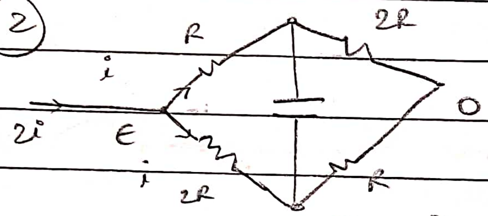
Find q_{cap} as a f^n of time.

A. ① 

$\equiv$ 

$R_{eff} = 4R/3$

$E - iR = 2iR$

② 

$E - 2iR = iR$

$E - 3iR = 0$

$q_s = (2iR - iR)\,C$

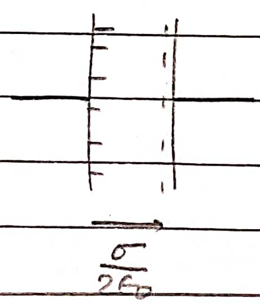
$= iRC = \dfrac{EC}{3}$

$2i = \dfrac{E}{3R/2} = \dfrac{2E}{3R} \Rightarrow i = \dfrac{E}{3R}$

$q = \dfrac{EC}{3}\left(1 - e^{-\frac{3t}{4R}}\right)$

PARALLEL PLATE CAPACITOR

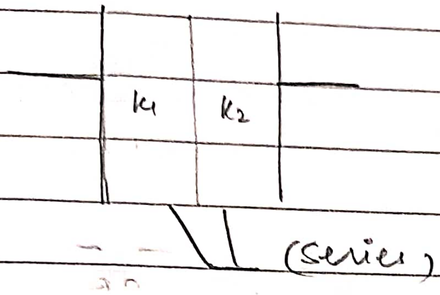
→ Force b/w plates of || cap.

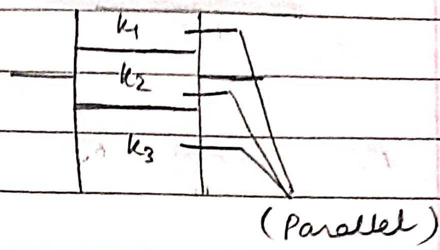


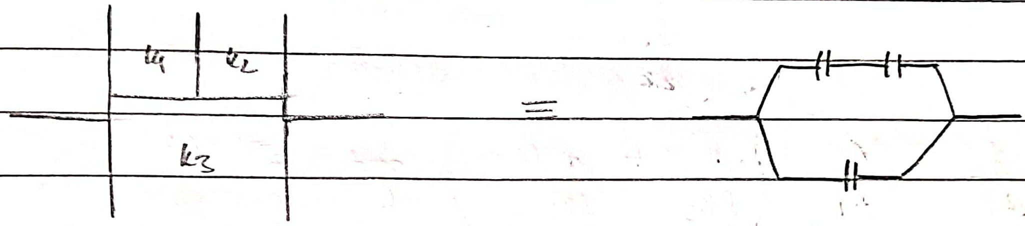
$F = qE = (\sigma A)\left(\dfrac{\sigma}{2\epsilon_0}\right) = \dfrac{\sigma^2 A}{2\epsilon_0}$

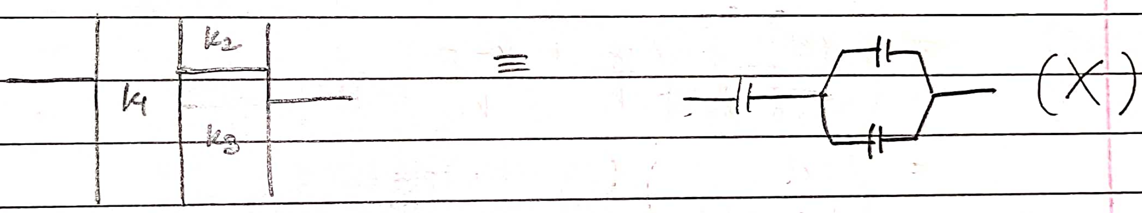
$= \dfrac{q^2}{2A\epsilon_0}$

→ Equivalent capacitance in $\|$ cap.

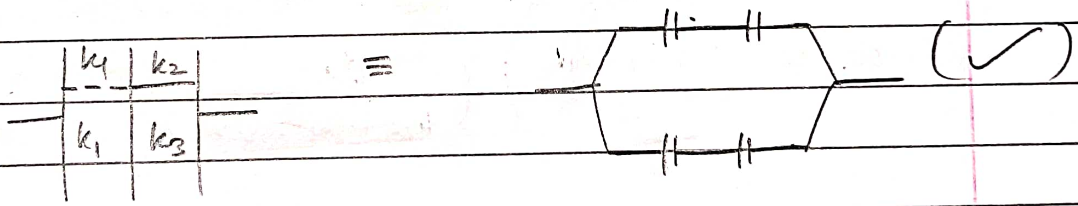






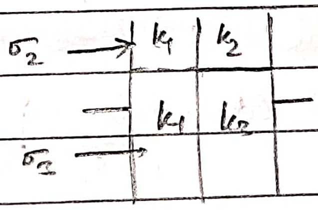


Actually,

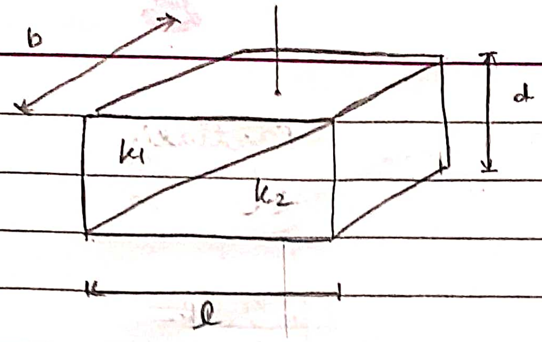


This is because

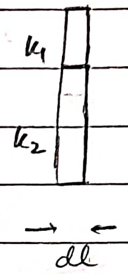
$\sigma_2 \neq \sigma_3$



☆ Q.

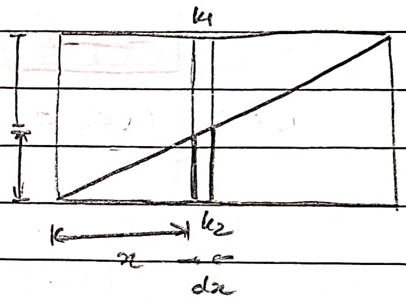


We take an elem. cap. of dl length.



All elems. in parallel

$$C = \sum C_i = \int dC$$



$$\frac{1}{dC} = \left(\frac{1}{A\varepsilon_0}\right)\left(\frac{d\,x}{l k_2} + \frac{d\,(l-x)}{l k_1}\right)$$

$$= \left(\frac{d}{b\varepsilon_0 l}\right)\left(\frac{x}{k_2} + \frac{l-x}{k_1}\right)\frac{1}{dx} \qquad A = b\,dx$$

$$= \left(\frac{d}{b\varepsilon_0 l k_1 k_2}\right)\frac{\left((k_1-k_2)x + k_2 l\right)}{dx}$$

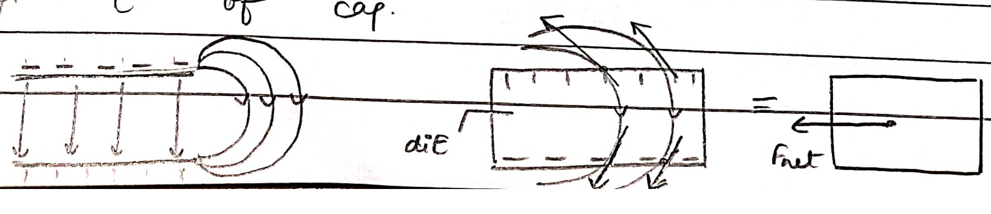
$$dC = \left(\frac{b\varepsilon_0 l k_1 k_2}{d}\right)\left(\frac{dx}{k_2 l + (k_1-k_2)x}\right)$$

$$C = \frac{b\varepsilon_0 l k_1 k_2}{d}\left(\frac{1}{k_1-k_2}\right)\Big[\ln\left(k_2 l + (k_1-k_2)x\right)\Big]_0^l$$

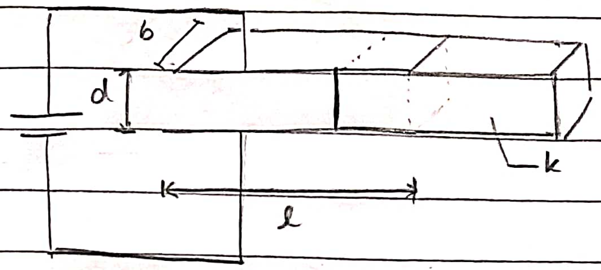
$$= \frac{b l \varepsilon_0}{d}\left(\frac{k_1 k_2}{k_1-k_2}\right)\ln\left(\frac{k_1}{k_2}\right)$$

In the Q given below, diE moves into the cap because of the force exerted on it by the fringing $\vec{E}$ of cap.

cap.

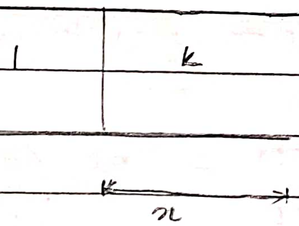


★ Q.



Partially inserted diE in ||cap. Find force with which cap. pulls diE

A.



$$U = \frac{1}{2}CV^2$$

$$\Rightarrow \frac{dU}{dx} = \frac{V^2}{2}\left(\frac{dC}{dx}\right)$$

$$C = \frac{b(l-x)\epsilon_0}{d} + \frac{kbx\epsilon_0}{d}$$

$$= \frac{V^2}{2}\left(\frac{b\epsilon_0}{d}\right)(k-1)$$

$$= \frac{b\epsilon_0}{d}(l+(k-1)x)$$

Since, ext agent (battery) doing work,

$$\frac{dC}{dx} = \frac{b\epsilon_0}{d}(k-1)$$

$$dW_{battery} = dU + dW_{on\ diE}$$

$$dW_{battery} = V\cdot\overbrace{(dC\cdot V)}^{dq} = V^2 dC$$

$$\Rightarrow F\cdot dx = V^2 dC - V^2 dC/2 = \frac{V^2 dC}{2}$$

$$dW_{on\ diE} = F_{diE}\cdot dx$$

$$\Rightarrow F = \frac{V^2}{2}\left(\frac{b\epsilon_0}{d}\right)(k-1)$$

$$dU = \frac{1}{2}V^2 dC$$

(const. force)

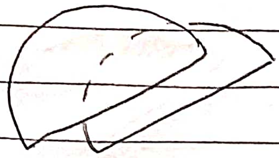
If no battery, work done by cap.

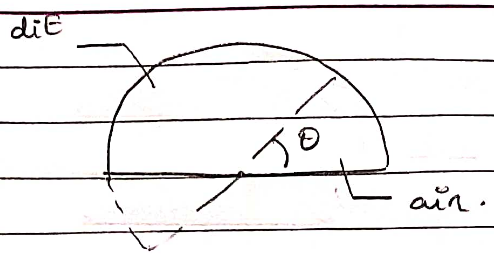
$$U = \frac{1}{2}\frac{q^2}{C} \Rightarrow \frac{dU}{dx} = \frac{q^2}{2}\left(\frac{-1}{C^2}\right)\left(\frac{dC}{dx}\right)$$

$$\Rightarrow F = -\frac{dU}{dx} = \frac{q^2}{2C^2}\left(\frac{dC}{dx}\right) = \left(\frac{q^2}{2C^2}\right)\left(\frac{b\epsilon_0(k-1)}{d}\right)$$

(C dependant on x. Hence variable force)

Q





Parallel plate cap. with semicircular plates.

Find torque on diE.

A.

$$C = C_{diE} + C_{air}$$

$$= \frac{\theta R^2 \epsilon_0}{d} + \frac{k(\pi-\theta)R^2\epsilon_0}{d}$$

$$= \frac{k\pi R^2\epsilon_0}{d} + \frac{\epsilon_0 R^2(1-k)\theta}{d}$$

$$U = \frac{1}{2}CV^2$$

$$\frac{dU}{d\theta} = \frac{V^2}{2}\left(\frac{dC}{d\theta}\right)$$

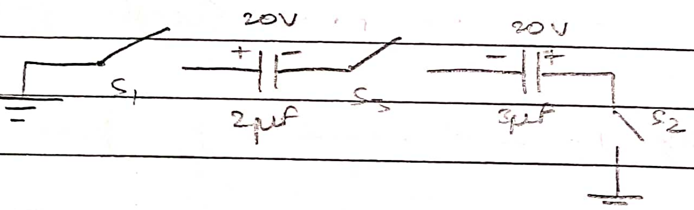
$$= \left(\frac{V^2}{2}\right)\left(\frac{\epsilon_0 R^2}{d}\right)(1-k)$$

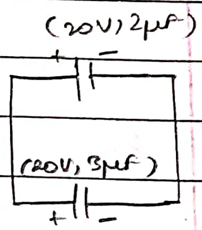
$$\frac{dC}{d\theta} = \frac{\epsilon_0 R^2(1-k)}{d}$$

$$\tau = \frac{dU}{d\theta} = \left(\frac{V^2}{2}\right)\left(\frac{\epsilon_0 R^2}{d}\right)(1-k)$$

WAT Doubts

Q.





When S_1, S_2, S_3 all are closed, then only charge flows.
($\because$ Charge flows only in closed path)

Otherwise no change in config.
(i.e not all switch closed)

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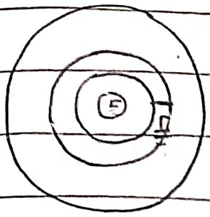
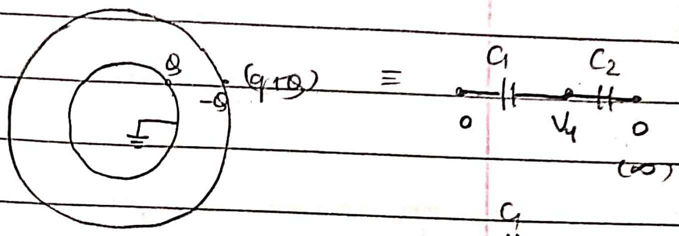
$r = a, 2a, 3a, 4a$

Q.



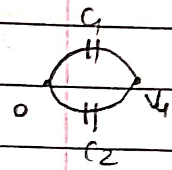
Find C_{eq} of sys.

A. $V_3 = 0 \Rightarrow V_2 = 0$, $V_1 = 0$ $\Big\}$ $\equiv$



$V_3 = 0 \Rightarrow \quad \frac{kQ}{3a} + \frac{kq}{4a} = 0$

$\Rightarrow \quad \underline{Q = -\frac{3q}{4}}$

$\equiv$ 

$C_{eq} = C_1 + C_2$

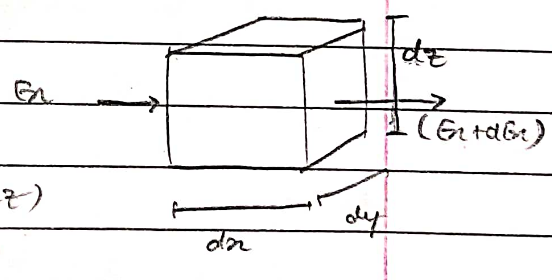
$= \frac{q_3}{V_4 - V_3} + \frac{q_4}{V_4 - 0} = \frac{7q}{V_4}$

$= \frac{16a}{k}$

$V_4 = \frac{kq}{4a} - \frac{k}{4a}\left(\frac{-3q}{4}\right) = \frac{7kq}{16a}$

→ Gauss Law (in differential form)

$\vec{E} = \langle E_x \quad E_y \quad E_z \rangle$



$d\varphi_x = (E_x + dE_x)\,A_x - (E_x)(A_x)$

$= dE_x\,dy\,dz \qquad \llcorner (dy\,dz)$

Similarly $\quad d\varphi_y = dE_y\,dx\,dz$

$d\varphi_z = dE_z\,dx\,dy$

$d\varphi = \frac{\sigma}{\epsilon_0} = \frac{\rho\,(dx\,dy\,dz)}{\epsilon_0} \Rightarrow \frac{dE_x}{dx} + \frac{dE_y}{dy} + \frac{dE_z}{dz} = \frac{\rho}{\epsilon_0}$

$$\boxed{\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \frac{\rho}{\epsilon_0}} \Rightarrow \nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

(Divergence of Field)