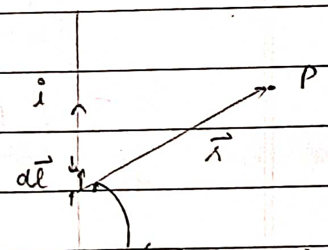
MAGNETIC EFFECT OF CURRENT

14/06/2023

BIOT-SAVART'S LAW

$$\boxed{d\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\frac{i\,(d\vec{l}\times\vec{r})}{r^3}}$$

(Permeability of free space)



* ($d\vec{l}$ in dirn of current)

$$\frac{\mu_0}{4\pi} = 10^{-7}$$

Unit (B): Tesla (T)

Dirn of $\vec{B}$: (Representation)

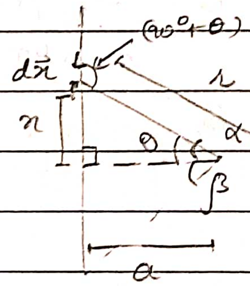
⊗ Into the plane

⊙ out of the plane

- Straight wire

$$dB = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{i\,dx\,r}{r^3}\right) s(90^\circ+\theta)$$

$$= \left(\frac{\mu_0}{4\pi}\right)\frac{i\,a\sec^2(\theta)\,d\theta\;c\theta}{a\sec^2(\theta)}$$



$$c\theta = \frac{a}{r}$$

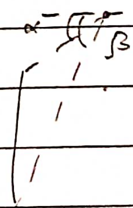
$$\Rightarrow r = a\sec(\theta)$$

$$\boxed{B = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{i}{a}\right)[s\alpha + s\beta]}$$

$$t\theta = \frac{x}{a}$$

$$\Rightarrow \sec^2(\theta)\,d\theta = \frac{dx}{a}$$

NOTE: 1.



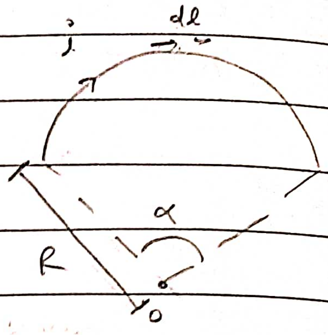
$\alpha \rightarrow (-\alpha)$

2. – Infinite wire –

$$\alpha = \beta = 90^\circ$$

– Semi-infinite wire – α or $\beta = 90^\circ$

- Circular arc

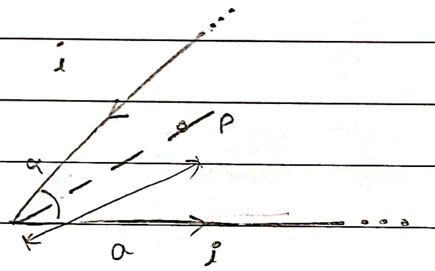


$$dB = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{i\, dl\ r \sin 90^\circ}{r^3}\right)$$

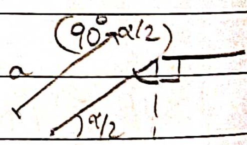
$$= \left(\frac{\mu_0}{4\pi}\right)\left(\frac{i}{R^2}\right) dl$$

$$B = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{i}{R^2}\right)\overset{R\alpha}{(l)} = \boxed{\left(\frac{\mu_0}{4\pi}\right)\left(\frac{i}{R}\right)(\alpha)}$$

Q. Find B_P



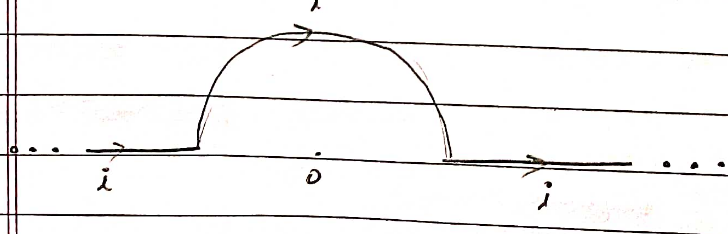
A. $B_{\swarrow} = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{i}{a \sin\left(\frac{\alpha}{2}\right)}\right)(1 + \cos\alpha/2)$ ⊙

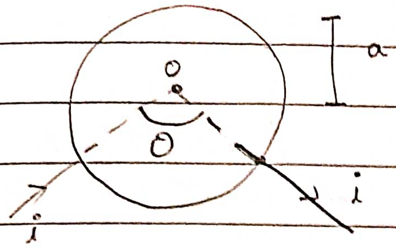


$B_{\rightarrow} = \left(\frac{\mu}{4\pi}\right)\left(\frac{i}{a \sin\alpha/2}\right)(1 + \cos\alpha/2)$ ⊙

$$B = B_{\swarrow} + B_{\rightarrow} = \left(\frac{\mu_0}{2\pi}\right)\left(\frac{i}{a \sin\alpha/2}\right)(1 + \cos\alpha/2)$$

Q Find B_O.

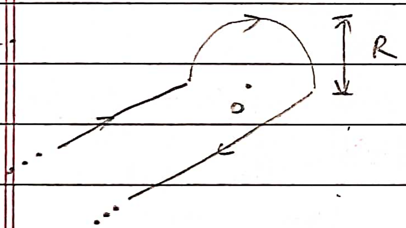


Q  Find B_O

A Current divided in inverse ratio of resistance,

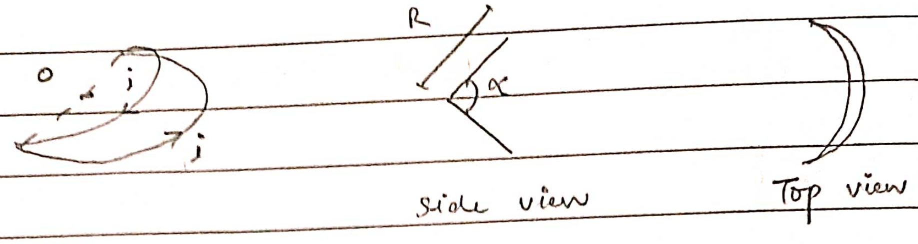
$$R \propto \frac{1}{l} \Rightarrow i_{branch} \propto l = a\theta$$

$$\Rightarrow i_{branch} \propto \theta$$

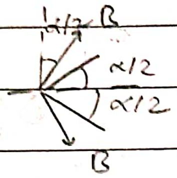
Q  Find B_O.

A $B_{W_1} = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{i}{R}\right) \downarrow$ $B_{W_2} = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{i}{R}\right) \downarrow$

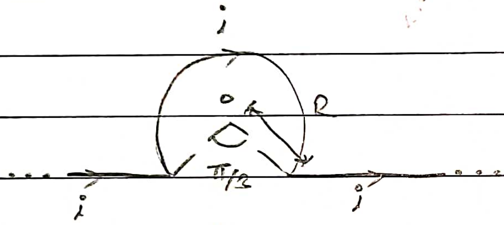
$B_C = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{i}{R}\right)(\pi) \nearrow$

Q. 

Find B_0

A. 

$$B = 2B\cos\left(\frac{\pi}{2}-\frac{\alpha}{2}\right) = \left(2\sin\frac{\alpha}{2}\right)\left(\frac{\mu_0}{4\pi}\right)\left(\frac{i}{R}\right)(\pi)$$

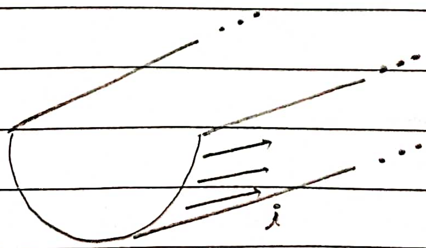
Q. 

Find B_0

A. $$B_W = B_{\cdots\text{—}\cdots} - B_{\text{—}} = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{2i}{\sqrt{3}R}\right)\left[2-\left(\frac{1}{2}+\frac{1}{2}\right)\right] \odot$$

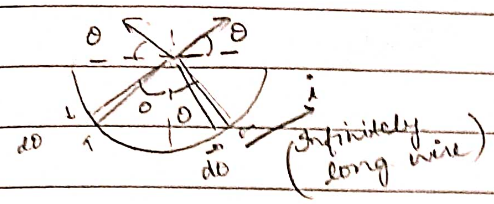
$$B_C = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{i}{R}\right)\left(\frac{5\pi}{3}\right) \otimes$$

$$B = B_W + B_C = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{i}{R}\right)\left(\frac{2}{\sqrt{3}}-\frac{5\pi}{3}\right) \odot$$

Q. 

Thin semi-cylinderical shell carries i along length.
Find B_{axis}

A.



By sym, $B_x = 0$

$dB_y = 2\,dB\sin\theta$ $\quad \left(\frac{i}{\pi R}\right)(R\,d\theta)$

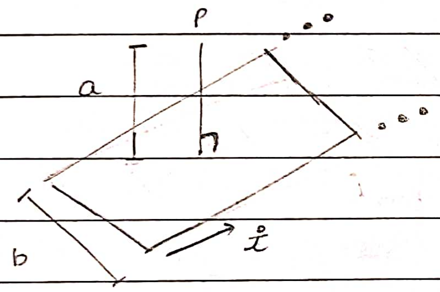
$di = \left(\frac{i}{\pi R}\right)(R\,d\theta)$

$$B_y = \int_0^{\pi/2} (2\sin\theta)\left(\frac{\mu_0}{4\pi}\right)\left(\frac{di}{R}\right)(2)$$

$$= \int_0^{\pi/2} \left(\frac{\mu_0 i}{\pi^2 R}\right)\sin\theta\, d\theta$$

$$= \frac{\mu_0 i}{\pi^2 R}$$

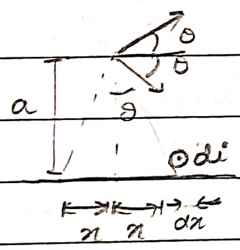
Q.



Find B_p

A.

By sym, $B_x = 0$



$dB = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{di\sin\theta}{x}\right)(2)$

$x = a\tan\theta$

$\Rightarrow dx = a\sec^2(\theta)\,d\theta$

$di = \left(\frac{i}{b}\right)dx$

$dB_y = 2\,dB\cos\theta$

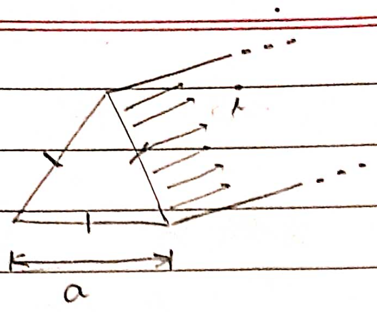
$= (\cancel{2}\cos\theta)\left(\frac{\mu_0}{\cancel{4}\pi}\right)\left(\frac{\cancel{2}\sin\theta}{\cancel{a}\tan\theta}\right)\left(\frac{i}{b}\cancel{a}\sec^2(\theta)\,d\theta\right)$

$= \frac{\mu_0 i}{b\pi}\,d\theta$

$$B = \frac{\mu_0 i}{b\pi}\tan^{-1}\left(\frac{b}{2a}\right)$$

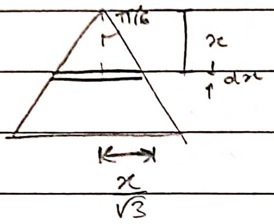
Page

Q.



Find B_{edge}.

A.



$$dB = \left(\frac{\mu_0}{\frac{2x}{\sqrt{3}}\pi}\right) t^{-1}\left(\frac{x/\sqrt{3}}{x}\right) di$$

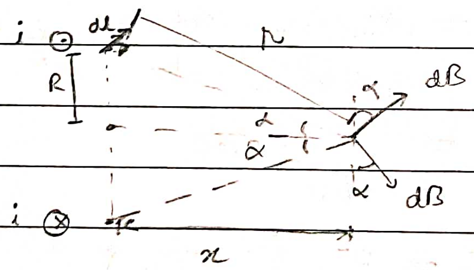
$$= \left(\frac{\sqrt{3}\mu_0}{2\cancel{x}\pi}\right)\left(\frac{\pi}{6}\right)\left(\frac{8i}{3a^2}\right)(\cancel{x}\, dx)$$

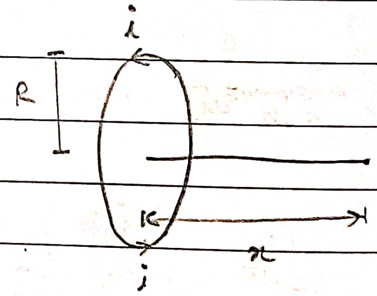
$$di = \frac{4i}{\sqrt{3}a^2}\left(\frac{2x\, dx}{\sqrt{3}}\right)$$

$$B = \left(\frac{\cancel{2}\sqrt{3}\mu_0\cancel{\pi}i}{9\cancel{\pi}a^2}\right)\left(\frac{\sqrt{3}a}{\cancel{2}}\right)$$

$$= \frac{\mu_0 i}{3a}$$

- Axis of Ring





$$dB_{net} = 2\,dB\sin\alpha = (2\sin\alpha)\left(\frac{\mu_0}{4\pi}\right)\left(\frac{i\,dl\sin 90^\circ}{r^2}\right)$$

$$B_{net} = \frac{\mu_0 \sin\alpha\, i}{2\pi r^2}\int_0^{\pi R} dl = \left(\frac{\mu_0 i}{2}\right)\left(\frac{R^2}{r^3}\right)$$

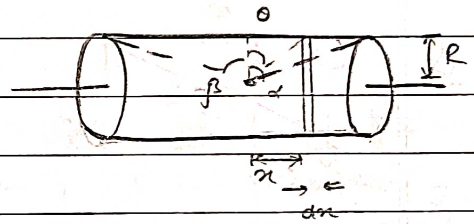
$$= \left(\frac{\mu_0}{4\pi}\right)\left(\frac{2\pi R^2 i}{(x^2+R^2)^{3/2}}\right)$$

$\pi R^2 i \longrightarrow$ (Magnetic dipole moment of ring along area vector)

$$\boxed{\vec{B}_{axis} = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{2\vec{M}}{r^3}\right)}$$

- Solenoid

$n \longrightarrow$ (# turns / length)



$$dB = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{2\pi R^2}{r^3}\right)(di)$$

$$= \left(\frac{\mu_0}{4\pi}\right)\frac{(2\pi R^2)\,(ni\,dx)}{(x^2+R^2)^{3/2}}$$

$di = i\,\underbrace{n\,dx}_{\#turns}$

$$= \left(\frac{\mu_0}{4\pi}\right)\frac{(2\pi \cancel{R^2})\,(ni)\,(\cancel{R}\sec^2(\theta)\,d\theta)}{\cancel{R^3}\sec^3(\theta)}$$

$x = R\tan\theta$

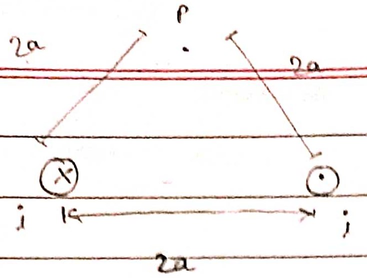
$dx = R\sec^2(\theta)\,d\theta$

$$= \frac{\mu_0 ni}{2}\cos d\theta$$

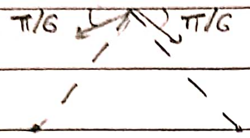
$$\boxed{B = \frac{\mu_0 ni}{2}\left[\sin\alpha + \sin\beta\right]}$$

For long solenoid, $\alpha = \beta = 90^\circ \Rightarrow \boxed{B = \mu_0 ni}$ (uniform magnetic field)

at the ends, $B = \left(\frac{\mu_0 ni}{2}\right)$

Q: 

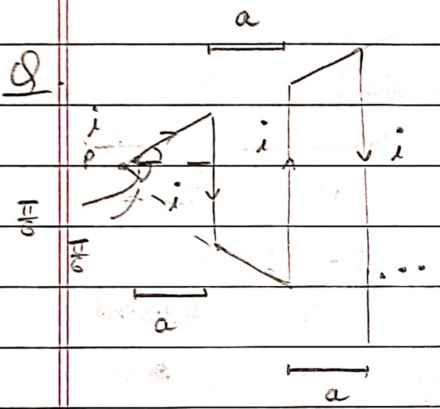
Find B_P

A. 

$B_P = 2B\sin\frac{\pi}{6} = B$

$= \left(\frac{\mu_0}{4\pi}\right)\left(\frac{i}{2a}\right)(2)$

$= \underline{\frac{\mu_0 i}{4\pi a}}$

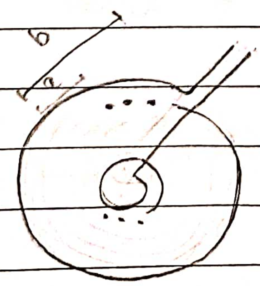
Q 

Find B_P.

A Field only due to vertical wires.

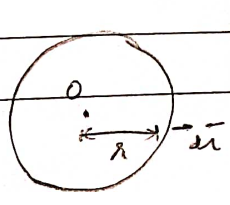
$$B = \left(\frac{\mu_0 i}{4\pi a}\right)(\sin 30^\circ + \sin 30^\circ)\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \dots\right]$$

$$= \underline{\frac{\mu_0 i \, l(2)}{4\pi a}}$$

Q 

N turns, very close

Find B_0

A. 

$dB = \left(\frac{\mu_0}{2}\right)\left(\frac{di}{r}\right) = \frac{\mu_0 N}{2(b-a)}\frac{dr}{r}$

$di = \frac{N}{(b-a)} dr$

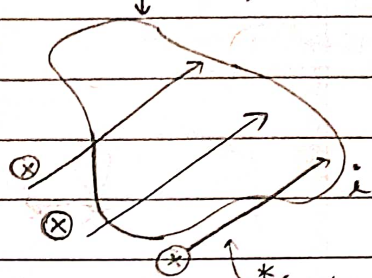
$B = \underline{\frac{\mu_0 N}{2(b-a)} l\left(\frac{b}{a}\right)}$

15/06/2023

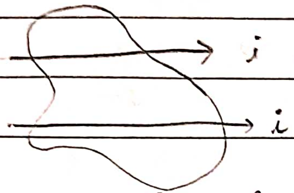
AMPERE'S CIRCUITAL LAW

$$\oint_L \vec{B}\cdot d\vec{l} = \mu_0 \, i_{enclosed}$$

(planar closed path)



*(Not necessarily $\perp$)

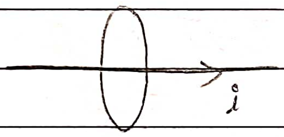


$B \longrightarrow \odot$

$\Rightarrow \underline{\vec{B}\cdot d\vec{l} = 0}$

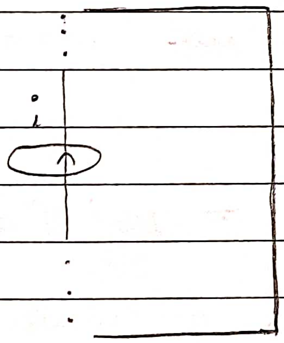
(current in plane of a planar closed path)

NOTE:



(X)

$\mu_0 i$ is not included here, since current not flowing in closed path. For applying ampere's law, we need current to travel in a closed path.

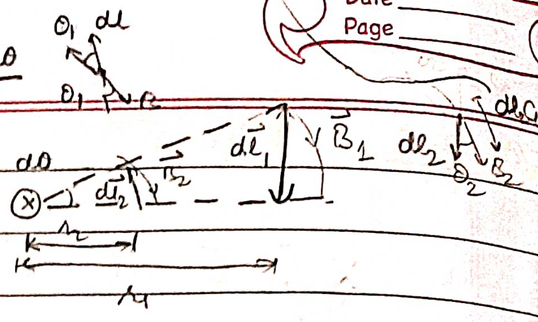


If we consider an ∞ long wire, then we can apply this law, assuming it to be closed by wires far enough that they do not contribute to $\oint_L \vec{B}\cdot d\vec{l}$

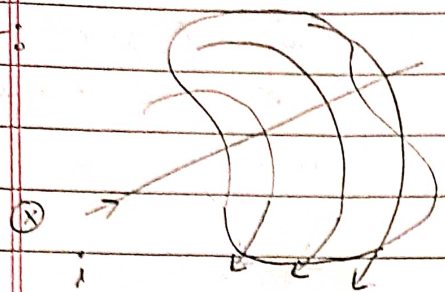
(✓)

$dl_2 \cos\theta_2 = r_2\, d\theta$

$dl_1 \cos\theta_1 = r_1\, d\theta$



Proof:



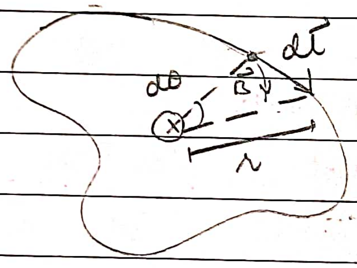
$\vec{B_1} \cdot \vec{dl_1} + \vec{B_2} \cdot \vec{dl_2}$

$\Rightarrow -B_1\, dl_1 \cos\theta_1 + B_2\, dl_2 \cos\theta_2$

$\Rightarrow -B_1 r_1\, d\theta + B_2 r_2\, d\theta$

$\Rightarrow -\dfrac{\mu_0 i}{2\pi r_1}(r_1\, d\theta) + \dfrac{\mu_0 i}{2\pi r_2} r_2\, d\theta$

$= \underline{0}$



$\vec{B} \cdot \vec{dl} = B\, dl \cos\theta$

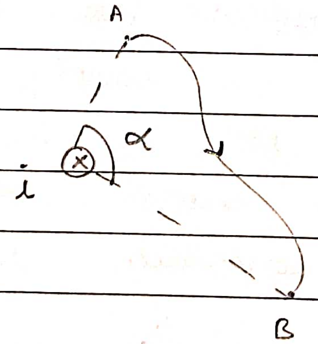
$= B\, r\, d\theta$

$= \dfrac{\mu_0 i}{2\pi \cancel{r}} \cancel{r}\, d\theta$

$\oint \vec{B} \cdot \vec{dl} = \dfrac{\mu_0 i}{2\pi} \int d\theta = \mu_0 i$

($\because$ for closed path $\int d\theta = 2\pi$)

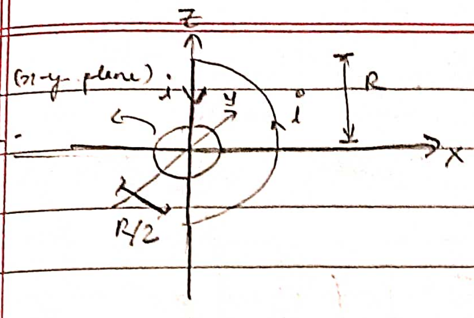
eg



$\oint_{AB} \vec{B} \cdot \vec{dl} = \dfrac{\mu_0 i}{2\pi} \int d\theta$

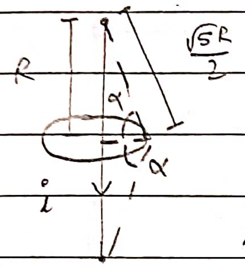
(open path)

$= \dfrac{\mu_0 i \alpha}{2\pi}$

Q. 

Find $\int \vec{B}\cdot d\vec{l}$ along the circle due to semi-circular part of loop.

A (AL) $\quad \left(\int B\,dl\right)_{wire} + \left(\int B\,dl\right)_{semi-circle} = \mu_0 i$



$$\left(\int B\,dl\right)_{wire} = \int \left(\frac{\mu_0}{4\pi}\right)\left(\frac{i}{R/2}\right)\overbrace{\left[\sin\alpha + \sin\alpha\right]}^{4/\sqrt{5}} dl \cos 0^\circ$$

$\sin\alpha = \frac{2}{\sqrt{5}}$

$$= \frac{2\mu_0 i}{R\pi\sqrt{5}} \underbrace{\int dl}_{2\pi\left(\frac{R}{2}\right)}$$

$B \parallel dl$

$\theta = 0^\circ$

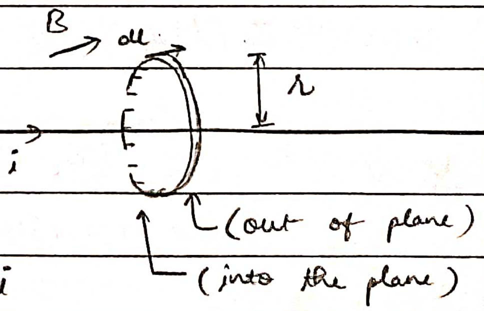
$$= \frac{2}{\sqrt{5}}\mu_0 i$$

$$\Rightarrow \left(\int B\,dl\right)_{semi-circle} = \left(1 - \frac{2}{\sqrt{5}}\right)\mu_0 i$$

→ **Calculating B using symmetrical current**

- Cylinderical sym.

$\boxed{\text{Thin wire}}$



$$\oint \vec{B}\cdot d\vec{l} = \oint B\,dl$$

$$= B\oint dl = \mu_0 i$$

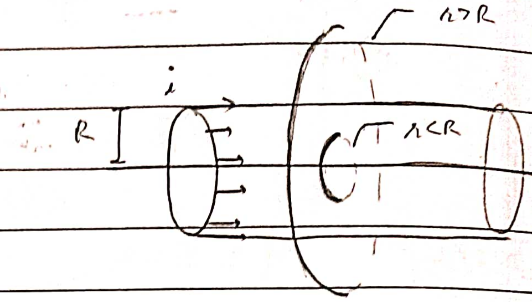
$$\Rightarrow B(2\pi r) = \mu_0 i$$

$$\Rightarrow B = \frac{\mu_0 i}{2\pi r}$$

Hollow Cylinder

1. $r < R \Rightarrow \underline{B = 0}$

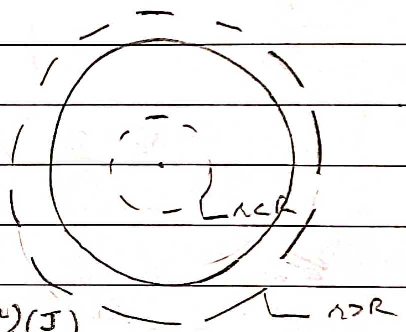
$(\because \oint B\,dl = 0)$



2. $r > R \Rightarrow B = \dfrac{\mu_0 i}{2\pi r}$

Solid Cylinder

- uniform current density 'J'



1. $r < R \Rightarrow (B)(2\pi r) = \mu_0 (\pi r^2)(J)$

$$\Rightarrow B = \left(\frac{\mu_0 J r}{2}\right) \Rightarrow \boxed{\vec{B} = \left(\frac{\mu_0}{2}\right)(\vec{J} \times \vec{r})}$$

2. $r > R \Rightarrow (B)(2\pi r) = \mu_0 (\pi R^2)(J)$

$$\Rightarrow B = \frac{\mu_0 i}{2\pi r}$$

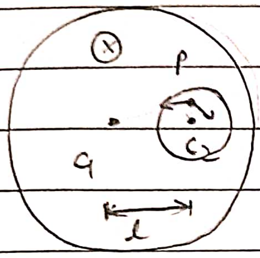
Q. $J(r) = J_0\left(1 + \dfrac{r}{R}\right)$

1. $r < R \Rightarrow (B)(2\pi r) = \mu_0 \int (2\pi r)(J_0)\left(1 + \dfrac{r}{R}\right) dr$

$$\Rightarrow (B)(2\pi r) = 2\pi \mu_0 J_0 \int_0^r r + \frac{r^2}{R}\, dr$$

$$\Rightarrow B = \mu_0 J_0 \left(\frac{r}{2} + \frac{r^2}{3R}\right)$$

Q. Long solid cylider has a cylinderical cavity. The axes of cyl. & cavity are parallel. Dist. b/w their axes is l. If current density of solid cyl. is J, find magnetic field inside cavity.

A 

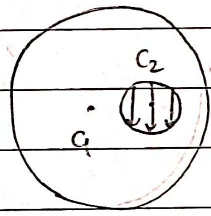
$$\vec{B}_P = \vec{B}_{C_1} - \vec{B}_{C_2} = \mu_0\left(\frac{\vec{J}\times\vec{C_1P}}{2}\right) - \mu_0\left(\frac{\vec{J}\times\vec{C_2P}}{2}\right)$$

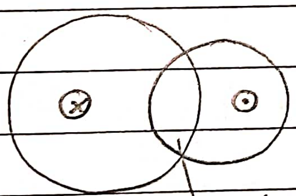
$$= \frac{\mu_0}{2}\vec{J}\times(\vec{C_1P} - \vec{C_2P})$$

$$= \frac{\mu_0}{2}\vec{J}\times(\vec{C_1P} + \vec{PC_2})$$

$$= \frac{\mu_0}{2}\vec{J}\times\vec{C_1C_2} = \underline{\frac{\mu_0(\vec{J}\times\vec{l})}{2}}$$

Hence, uniform field, $\perp$ to line joining centres



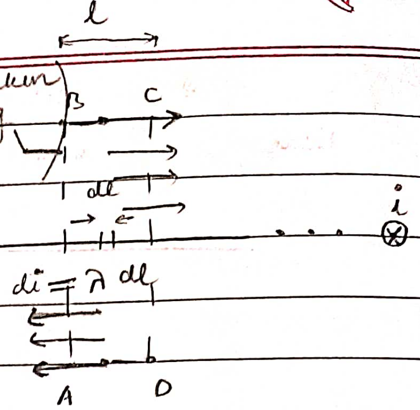
NOTE: 

Uniform field in intersection

(behaves as a cavity)

- Plane sym.

(Path taken for applying AL)



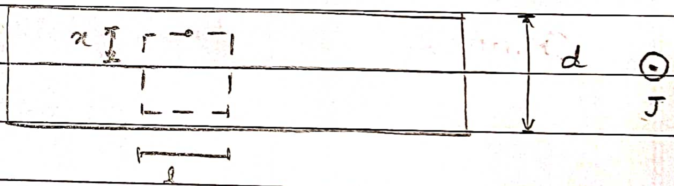
Thin Sheet

$$\oint_{ABCDA} \vec{B}\cdot d\vec{l} = \cancel{\int_{AB} \vec{B}\cdot d\vec{l}}\ (0) + \int_{BC} \vec{B}\cdot d\vec{l} + \cancel{\int_{CD} \vec{B}\cdot d\vec{l}}\ (0) + \int_{DA} \vec{B}\cdot d\vec{l}$$

$$= Bl + Bl = \mu_0 (\lambda l)$$

$$\Rightarrow \boxed{B = \left(\frac{\mu_0 \lambda}{2}\right)}$$

Thick Sheet



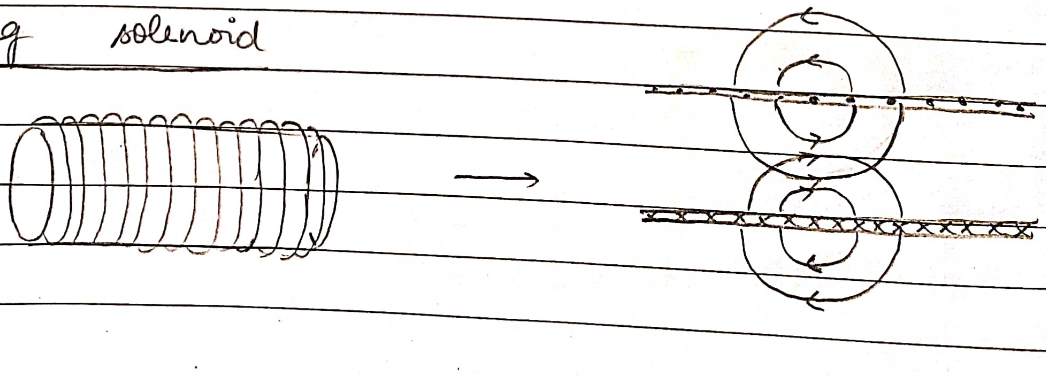
1. $x < d/2$

$$Bl + Bl = \mu_0 (l \times 2x)(J)$$

$$\Rightarrow \boxed{B = \mu_0 J x}$$

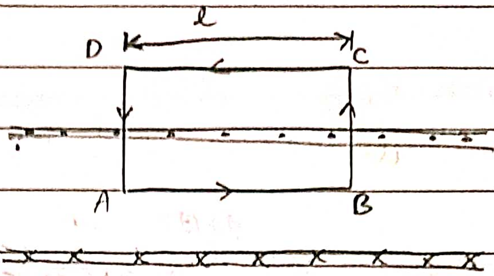
2. $x > d/2 \Rightarrow B = \frac{\mu_0 J d}{2} = \frac{\mu_0 \lambda}{2}$

- Long solenoid



Inside the solenoid, fields add, however outside, they cancel out.

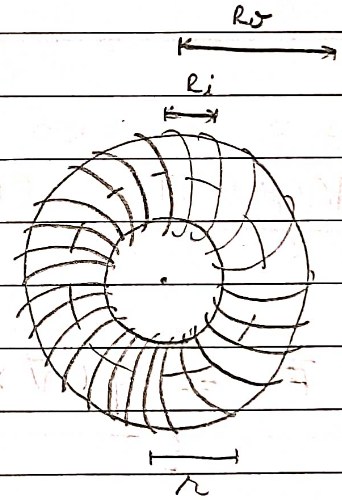
Applying AL,



$$\oint_{ABCDA} \vec{B}\cdot d\vec{l} = \underbrace{Bl}_{\left(\int_{AB} \vec{B}\cdot d\vec{l}\right)} + 0+0+0$$

$$= \mu_0 \underbrace{(nl)}_{(\#\text{ loops})} i \quad \Rightarrow \quad \boxed{B = \mu_0 n i}$$

• **Toroid** (Circular Solenoid)



1. Inside

$$B\,(2\pi r) = \mu_0 N i$$

$$\Rightarrow \boxed{B = \mu_0 \left(\frac{N}{2\pi r}\right) i}$$

2. Outside ($r < R_i$ or $r > R_o$)

$$\underline{B = 0}$$ (Since no current enclosed & both currents enclosed respectively — (opp. dirn))

NOTE: For a charge (q) moving with $\vec{v}$

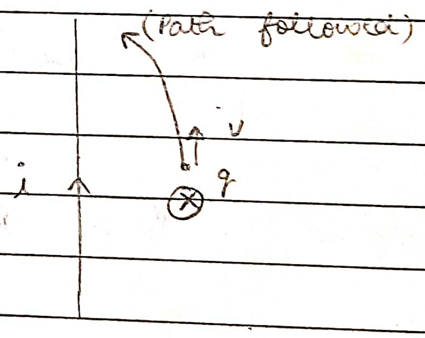
$$i = \frac{dq}{dt}$$

$$d\vec{B} = \left(\frac{\mu_0}{4\pi}\right) \frac{\overset{dq/dt}{i}\,(d\vec{l} \times \vec{r})}{r^3} = \left(\frac{\mu_0}{4\pi}\right) \frac{dq}{r^3}\left(\frac{d\vec{l}}{dt} \times \vec{r}\right)$$

$$= \boxed{\left(\frac{\mu_0}{4\pi}\right) dq \frac{(\vec{v} \times \vec{r})}{r^3}}$$

MAGNETIC FORCE ON A MOVING CHARGE

$$\boxed{\vec{F} = q(\vec{v} \times \vec{B})}$$



$\Rightarrow$ $\vec{F} \perp \vec{v}$ & $\vec{F} \perp \vec{B}$

$\Downarrow$

$\vec{F} \cdot \vec{v} = 0$

$\Rightarrow$ (Work done by mag. force on charged particle will <u>always</u> be <u>0</u>.)

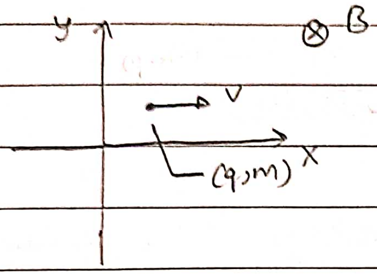
Hence, <u>speed</u> remains <u>const.</u> if <u>only magnetic field</u> exists.

Date
Page

(I) $\vec{v} \parallel \vec{B} \Rightarrow \underline{\vec{F} = 0} \Rightarrow$ particle unaffected.

(II) $\vec{v} \perp \vec{B} \Rightarrow$ circular path followed

Proof:



$$\vec{B} = \langle 0 \quad 0 \quad B_0 \rangle$$
$$\vec{v_0} = \langle v_0 \quad 0 \quad 0 \rangle$$

$$\vec{v} = \langle v_x \quad v_y \quad 0 \rangle$$

Since no work done on particle,
Therefore $v_x^2 + v_y^2 = v_0^2$

$$\vec{F} = q(\vec{v} \times \vec{B}) = q(\langle v_x \quad v_y \quad 0 \rangle \times \langle 0 \quad 0 \quad B_0 \rangle)$$
$$= qB_0 \langle -v_y \quad v_x \quad 0 \rangle$$

$$\vec{a} = \left(\frac{qB_0}{m}\right) \langle -v_y \quad v_x \quad 0 \rangle$$

$$\frac{d\vec{a}}{dt} = \left(\frac{qB_0}{m}\right) \left\langle -a_y \quad a_x \quad 0 \right\rangle = \left(\frac{qB_0}{m}\right)^2 \left\langle -v_x \quad -v_y \quad 0 \right\rangle$$

$$\Rightarrow \frac{d^2 v_i}{dt} = -\left(\frac{qB_0}{m}\right)^2 v_i \qquad (i = x, y)$$

$$\Rightarrow v_i = v_{max(i)} s(\omega t + \varphi) \qquad \boxed{\omega = qB_0/m}$$

For $t = 0$, $\quad v_x = v_0$, $v_y = 0 \quad \Rightarrow \quad v_x = v_0 \cos\omega t$, $v_y = v_0 \sin\omega t$

$$\vec{x}(t) - \vec{x}(0) = \left\langle \frac{v_0}{\omega} \sin\omega t \quad \frac{v_0}{\omega}(1 - \cos\omega t) \right\rangle$$

If $\vec{r}(0) = \langle 0 \quad 0 \rangle \Rightarrow \left(\frac{\omega x}{V_0}\right)^2 + \left(1 - \frac{\omega y}{V_0}\right)^2 = 1$

$$\Rightarrow x^2 + \left(y - \frac{V_0}{\omega}\right)^2 = \frac{V_0^2}{\omega^2}$$

Eq^n of circle

So here, mag. force acts as centripetal force

$$\Rightarrow \underbrace{F}_{qvB} = \frac{mv^2}{R} \Rightarrow R = \frac{mv}{qB}$$

$$t = \frac{2\pi R}{V} = \frac{2\pi}{(qB/m)} = \frac{2\pi}{\omega} \; ; \quad \boxed{\omega = \frac{qB}{m}}$$

Q. A proton & α-particle are projected $\perp$ to uniform B. They describe circles of radius R_p & R_α respectively. Then find R_p/R_α if

a) they are projected with same speed
b) they are projected with same momentum
c) they are projected with same K.E.

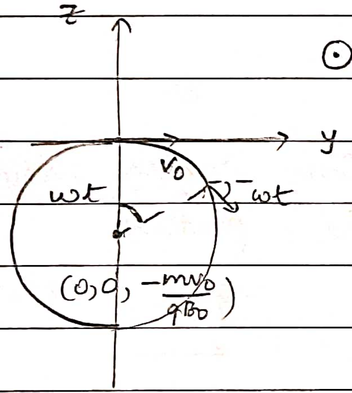
A. a) $\frac{R_p}{R_\alpha} = \left(\frac{m_p}{m_\alpha}\right)\left(\frac{V_p}{V_\alpha}\right)\left(\frac{q_\alpha}{q_p}\right) = \frac{2}{4} = 1/2$

b) $\frac{R_p}{R_\alpha} = \left(\cancel{\frac{m_p v_p}{m_\alpha v_\alpha}}\right)\left(\frac{q_\alpha}{q_p}\right) = 2$

c) $\dfrac{R_p}{R_\alpha} = \left(\dfrac{\sqrt{\cancel{2K_0}E_p}}{\sqrt{\cancel{2KE_\alpha}}}\right)\left(\dfrac{q_\alpha}{q_p}\right)\left(\sqrt{\dfrac{m_p}{m_\alpha}}\right) = 1$

16/06/2023

Q. A particle (m, q) is projected from origin with a velocity $\vec{v} = v_0\hat{j}$.
In space, $\exists$ uniform $\vec{B} = B_0\hat{i}$. Find (x, y, z) coordinate of the particle as a f^n of time.

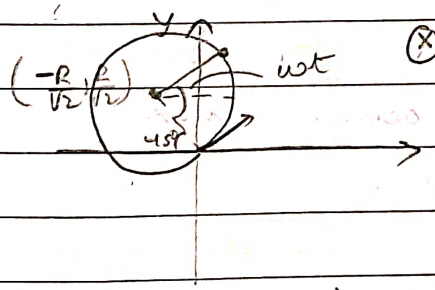
A. 

$\odot\ B_0$ $\qquad \omega = \dfrac{qB_0}{m} \qquad R = \dfrac{v_0}{\omega} = \dfrac{mv_0}{qB_0}$

$\vec{r}(t) = \langle 0 \quad R\sin\omega t \quad -R(1-\cos\omega t)\rangle$

$\vec{v}(t) = \langle 0 \quad v_0\cos\omega t \quad -v_0\sin\omega t\rangle$.

Q. In space, $\exists\ \vec{B} = -B_0\hat{k}$
A particle (q, m) is projected from origin at $t=0$ with $\vec{V}(0) = \langle v_0 \quad v_0 \quad 0\rangle$
find (x, y, z) coordinates of the particle after time 't'.

A. 

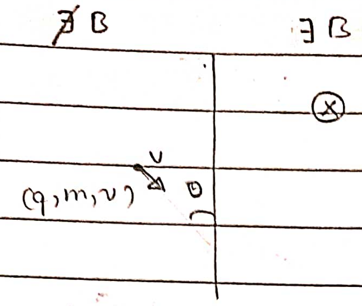
$\otimes\ B_0$ $\qquad R = \dfrac{\sqrt{2}v_0}{\omega} = \dfrac{\sqrt{2}mv_0}{qB_0}$

$C = \left\langle \dfrac{-R}{\sqrt{2}} \quad \dfrac{R}{\sqrt{2}} \quad 0\right\rangle$

$\vec{r}(t) - \vec{r}_c = \langle R\sin(\omega t - 45^\circ) \quad R\cos(\omega t - 45^\circ) \quad 0\rangle$

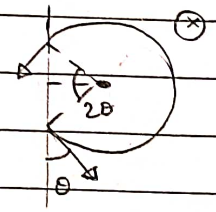
$\Rightarrow \vec{r}(t) = \left\langle R\left(\sin(\omega t - 45^\circ) - \dfrac{1}{\sqrt{2}}\right) \quad R\left(\cos(\omega t - 45^\circ) + \dfrac{1}{\sqrt{2}}\right) \quad 0\right.$

Q



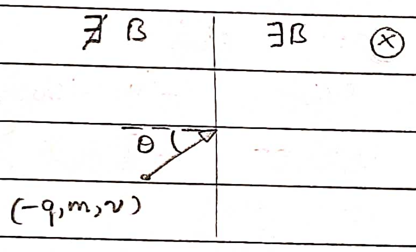
Find time after which the particle exists region with field.

A



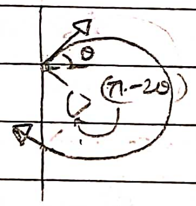
$$t_{(2\pi - 2\theta)} = \frac{2\pi - 2\theta}{(qB/m)}$$

Q



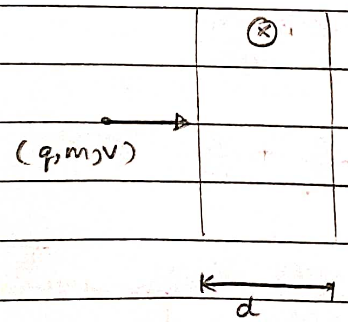
Find time after which the particle exits the region with field

A.



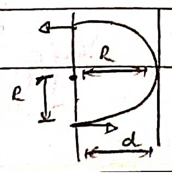
$$t_{(\pi + 2\theta)} = \frac{(\pi + 2\theta)}{(qB/m)}$$

Q



Find v for which particle comes back.

A.

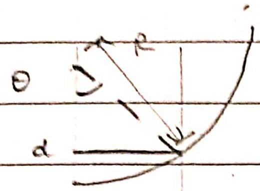


For returning $d \geq R$

$$\Rightarrow v \leq \frac{qBd}{m}$$

Q. In the above Q, find deflection of particle if it doesn't return

A.

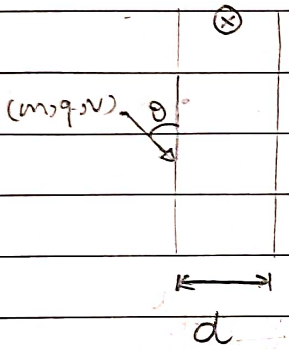


Deflection $= R(1-c\theta)$

$$= R\left(1-\frac{\sqrt{R^2-d^2}}{R}\right)$$

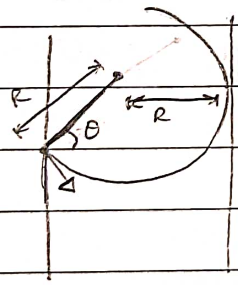
$$s\theta = \left(\frac{d}{R}\right)$$

Q.



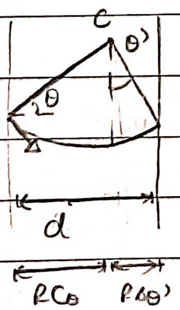
Find time after which particle exits region with B & thickness for which particle returns.

A.



For returning, $d \geq R(1+c\theta)$

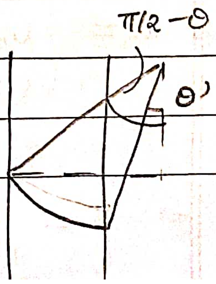
if $d < R(1+c\theta)$



$$Rc\theta + Rs\theta' = d$$

$$\Rightarrow s\theta' = \left(\frac{d-Rc\theta}{R}\right)$$

$$t_{(\frac{\pi}{2}-\theta+\theta')} = \left(\frac{\pi/2-\theta+\theta'}{qB/m}\right)$$

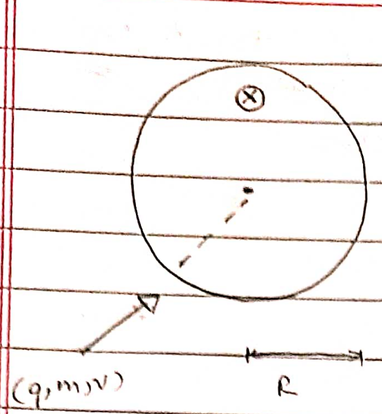


Even if C lies outside the region,

$\theta' \rightarrow -\theta'$

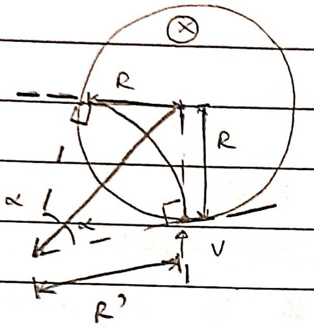
$$t_{(\frac{\pi}{2}-\theta-\theta')} = \left(\frac{\pi/2-\theta-\theta'}{qB/m}\right)$$

Q.



After what time will particle leave circular region

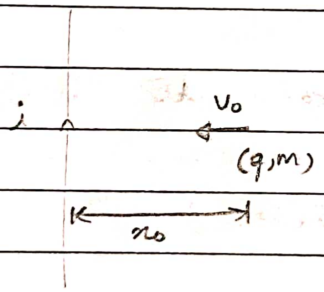
A.



$$t_{2\alpha} = \frac{2\alpha}{(qB/m)} = \frac{2\tan^{-1}\left(\frac{R}{R'}\right)}{(qB/m)}$$

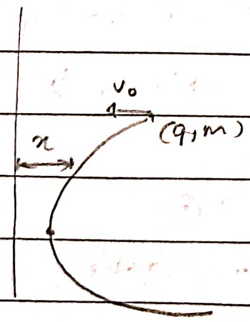
$$t_\alpha = \frac{R}{R'}$$

☆ Q.



What will be the min. dist. of particle from wire?

A.



Initially $v_x = -v_0$
$v_y = 0$

☆ Finally $v_x = 0$
$v_y = -v_0$ $\quad (W_B = 0)$

$$\vec{v} = \langle v_x \quad v_y \rangle$$

$$v_x^2 + v_y^2 = v_0^2 \quad (W_B = 0)$$

$$\vec{B} = \left(\frac{\mu_0 i}{2\pi x}\right)(-\hat{k})$$

$$\vec{F} = q\,\vec{v}\times\vec{B} = q\,\langle v_x \quad v_y \quad 0\rangle \times \left\langle 0 \quad 0 \quad -\frac{\mu_0 i}{2\pi x}\right\rangle$$

$$\Rightarrow \vec{a} = \frac{\vec{F}}{m} = \left(\frac{q\mu_0 i}{2\pi m x}\right)\langle -v_y \quad v_x \quad 0\rangle$$

Let $\lambda = \left(\frac{q\mu_0 i}{2\pi m}\right)$ $\Rightarrow$ $a_x = -\frac{\lambda v_y}{x}$ $\Rightarrow$ $v_x \frac{dv_x}{dx} = -\frac{\lambda v_y}{x}$

$a_y = \frac{\lambda v_x}{x}$ $\Downarrow$

$\because$ $v_x^2 + v_y^2 = v_0^2$

$\Rightarrow 2v_x \frac{dv_x}{dx} + 2v_y \frac{dv_y}{dx} = 0$

$$\cancel{v_y} \frac{dv_y}{dx} = \frac{\lambda \cancel{v_y}}{x}$$

$$\Rightarrow dv_y = \lambda \frac{dx}{x}$$

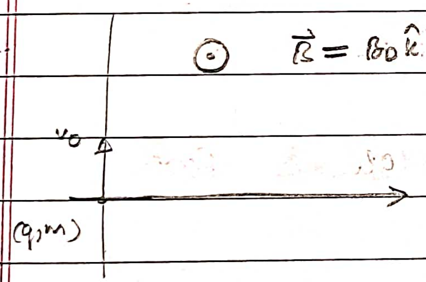
$$\Rightarrow \int_0^{-v_0} dv_y = \int_{x_0}^{x} \lambda \frac{dx}{x}$$

$$\Rightarrow -v_0 = \lambda \ln\left(\frac{x}{x_0}\right)$$

$$\Rightarrow \underline{x = x_0 e^{\left(\frac{-2\pi v_0 m}{q\mu_0 i}\right)}}$$

NOTE: for max dist. $v_y : 0 \rightarrow v_0$ $\Rightarrow$ $\underline{x = x_0 e^{\left(\frac{2\pi v_0 m}{q\mu_0 i}\right)}}$

★ Q.



$\vec{F}_{\text{Resistive}} = -\alpha \vec{v}$ acts on particle besides $\vec{F}_B$.
Find x-coordinate of the pt. where particle ultimately comes to rest.

A. Let $\vec{v} = \langle v_x \;\; v_y \;\; 0 \rangle$ $\Rightarrow$ $\vec{F}_B = q\vec{v} \times \vec{B}$

$$= qB_0 \langle v_y \;\; -v_x \;\; 0 \rangle$$

$$\vec{F}_R = \langle -\alpha v_x \;\; -\alpha v_y \;\; 0 \rangle$$

$$\Rightarrow \vec{F}_{net} = \vec{F}_B + \vec{F}_R = \langle qB_0 v_y - \alpha v_x \;\; -\alpha v_y - qB_0 v_x \;\; 0 \rangle$$

When particle comes to rest $qB_0 v_y - \alpha v_x = 0$, $\alpha v_y + qB_0 v_x = 0$ $\Rightarrow$ $\underline{v_x = v_y = 0}$

$\Rightarrow$ $\underline{v_x} : 0 \rightarrow 0$

$\underline{v_y} : v_0 \rightarrow 0$

$$\Rightarrow \vec{a} = \left\langle \underbrace{\frac{qB_0}{m}v_y - \frac{\alpha}{m}v_x} \quad \underbrace{-\frac{qB_0}{m}v_x + \frac{\alpha}{m}v_y} \quad 0 \right\rangle$$

$$a_x = \frac{qB_0}{m}v_y - \frac{\alpha}{m}v_x$$

$$\Rightarrow \int_0^0 dv_x = \int_0^y \frac{qB_0}{m}\underbrace{v_y\,dt}_{dy} - \int_0^x \frac{\alpha}{m}\underbrace{v_x\,dt}_{dx}$$

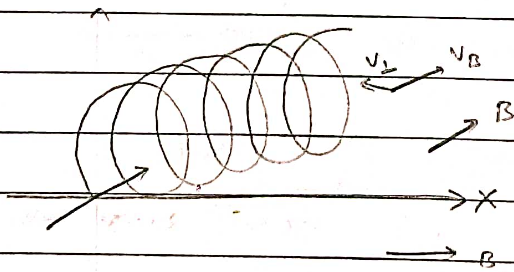
(similarly)

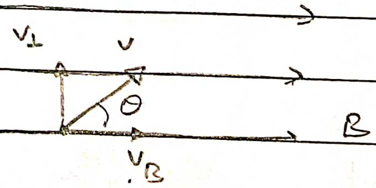
$$\Rightarrow \frac{qB_0}{m}y - \frac{\alpha}{m}x = 0 \qquad V_0 = \frac{qB_0}{m}x + \frac{\alpha}{m}y$$

$$\Rightarrow V_0 = \frac{qB_0}{m}x + \left(\frac{\alpha}{m}\right)\left(\frac{m}{qB_0} \times \frac{\alpha x}{\cancel{m}}\right) = \left(\frac{q^2B_0^2 + \alpha^2}{m}\right)x$$

$$\Rightarrow x = \frac{mV_0}{(q^2B_0^2 + \alpha^2)}$$

(III) $\vec{v}$ & $\vec{B}$ enclose θ — Helical Path





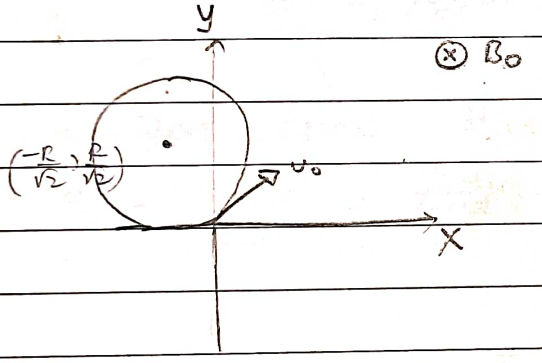
Radius of helix $= \dfrac{V_\perp}{\omega} = \dfrac{mV_\perp}{qB}$

Pitch of helix $= TV_B = \left(\dfrac{2\pi m}{qB}\right)V_B$
(Disp. along B in one time period)

NOTE: Particle touches the line || to B after every T.
Hence dist b/w any intersections of path & line is an integral multiple of pitch

Q. In space, ∃ $\vec{B} = -B\hat{k}$. A particle (q, m) projected from origin with $\vec{v} = \langle v_0 \ \ v_0 \ \ v_0 \rangle$
Find (x, y, z) coordinates of the particle after time t.
The particle passes through $(0, 0, z_0)$,
find min value of z_0

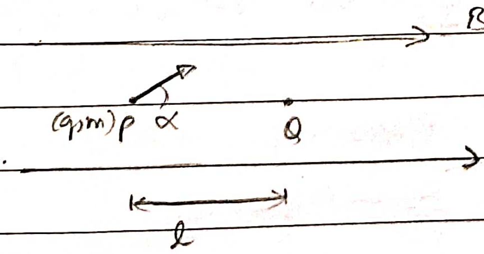
A.



$$\vec{r}(t) = \left\langle R\left(\sin(\omega t + 45^\circ) - \frac{1}{\sqrt{2}}\right) \quad R\left(\cos(\omega t - 45^\circ) + \frac{1}{\sqrt{2}}\right) \quad v_0 t \right\rangle$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi m}{qB_0}$$ so, $$(z_0)_{min} = v_0 T = \frac{2\pi m v_0}{qB_0}$$

Q.



It was observed that the particle passes through Q when $B = B_1$ or B_2 $(B_1 < B_2)$ but not for any value in b/w.

Find speed of projection.

A. $v_B = v C\alpha$

$v_\perp = v S\alpha$

$l = n v_B T = (n C\alpha)\left(\frac{2\pi m v}{qB_1}\right)$

$l = (n+1) v_B T = (n+1) C\alpha \left(\frac{2\pi m v}{qB_2}\right)$

$\Rightarrow \quad \frac{n+1}{B_2} = \frac{n}{B_1} \quad \Rightarrow \quad n = \left(\frac{B_1}{B_2 - B_1}\right)$

$\Rightarrow \quad v = \frac{q(B_2 - B_1)\, l}{2\pi m C\alpha}$

20/06/2023

→ Motion of particle under both $\vec{B}$ & $\vec{E}$

(I) $\vec{B} \parallel \vec{E}$ ⇒ Helical path with const. radius & varying pitch

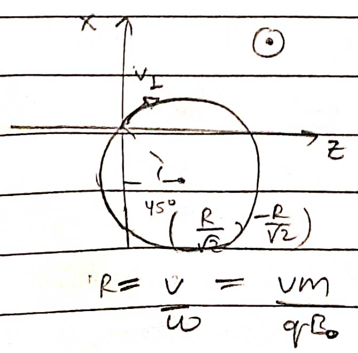
Q. In space, ∃ $\vec{E} = E_0 \hat{j}$ & $\vec{B} = B_0 \hat{j}$.
A particle (q, m) projected from origin with $\vec{v} = v_0\langle 1 \ 1 \ 1\rangle$
Find coordinates of particle as a f^n of time.

A. $\vec{v}_{\parallel} = \langle 0 \quad v_0 \quad 0\rangle \Rightarrow a_y = \left(\frac{qE_0}{m}\right) \Rightarrow y = v_0 t + \frac{1}{2}\left(\frac{qE_0}{m}\right) t^2$

$\vec{v}_\perp = \langle v_0 \quad 0 \quad v_0 \rangle$

$x = R\, S(\omega t + \pi/4) - R/\sqrt{2}$

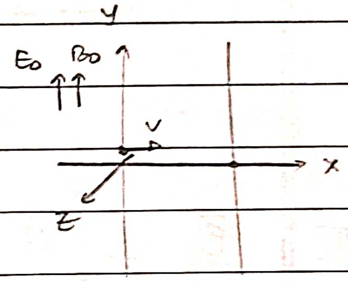
$z = \frac{R}{\sqrt{2}} - R\, C(\omega t + \frac{\pi}{4})$



$R = \frac{v}{\omega} = \frac{vm}{qB_0}$

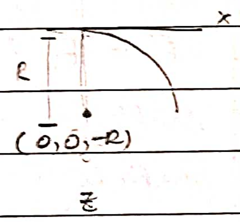
Q. $\vec{E} = E_0 \hat{j}$, $\vec{B} = B_0 \hat{j}$. Identical particles (q, m) projected along +ve x-dirⁿ with diff speeds. A plane screen ∥ to y-z plane is placed at a dist l from origin. Find locus of the pts. on screen where these particles hit the screen.

A. E produces const. acc. $= \left(\frac{qE_0}{m}\right)$



P_1: $\vec{x}_1 = \left\langle R_1 \sin\omega t \quad \frac{1}{2}\left(\frac{qE_0}{m}\right)t^2 \quad R_1 - R_1\cos\omega t \right\rangle$

P_2: $\vec{x}_2 = \left\langle R_2 \sin\omega t \quad \frac{1}{2}\left(\frac{qE_0}{m}\right)t^2 \quad R_2 - R_2\cos\omega t \right\rangle$



For both particles,

$$\frac{z}{l} = \frac{1-\cos\omega t}{\sin\omega t} = \tan\left(\frac{\omega t}{2}\right) \Rightarrow t = \frac{2}{\omega}\tan^{-1}\left(\frac{z}{l}\right)$$

$$y = \frac{1}{2}\left(\frac{qE_0}{m}\right)\left(\frac{2}{\omega}\tan^{-1}\left(\frac{z}{l}\right)\right)^2$$

Q. In the above Q, if $z \ll l$, prove that the locus is a parabola.

(II) $\vec{E}$ & $\vec{B}$ not $\parallel$

To determine speed as a fxn of posn.
use WETh

($\because$ $W_B = 0$ & $\vec{E}$ is a conservative field)

(III) $\vec{E} \perp \vec{B}$

Q. $\vec{E} = E_0 \hat{j}$, $\vec{B} = B_0 \hat{k}$. If particle (q, m) released from origin, find coordinates of particle as a fxn of time.

A. $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

Let $\vec{v}(t) = \langle v_x \quad v_y \quad 0 \rangle$; $\vec{v}(0) = \langle 0 \quad 0 \quad 0 \rangle$

$\Rightarrow \vec{F} = qE_0\hat{j} + q\langle v_x \quad v_y \quad 0\rangle \times B_0\hat{k}$
$= \langle qB_0 v_y \quad qE_0 - qB_0 v_x \quad 0 \rangle$

$\vec{a} = \left\langle \frac{q}{m}B_0 v_y \quad \frac{qE_0}{m} - \frac{qB_0}{m} v_x \quad 0 \right\rangle$

$a_x = \frac{q}{m} B_0 v_y$

$a_y = \frac{qE_0 - qB_0 v_x}{m} \Rightarrow \frac{d(a_y)}{dt} = -\left(\frac{qB_0}{m}\right)\left(\frac{dv_x}{dt}\right) = -\left(\frac{qB_0}{m}\right)^2 v_y$

$\Rightarrow \frac{d^2}{dt^2}(v_y) = -\omega^2 v_y \qquad \left\{ \omega = \frac{qB_0}{m} \right\}$

$$\Rightarrow \quad v_y = v_0 \sin(\omega t + \varphi)$$

At $t=0$, $v_y = 0 \Rightarrow v_0 \sin(\varphi) = 0 \Rightarrow \varphi = 0$ $(\because v_0 \neq 0)$

$$\Rightarrow \quad \underline{v_y = v_0 \sin\omega t}$$

$$a_y = v_0 \omega \cos\omega t \qquad \because a_y(0) = \frac{qE_0}{m} \Rightarrow \frac{qB_0 v_0}{m} = \frac{qE_0}{m}$$

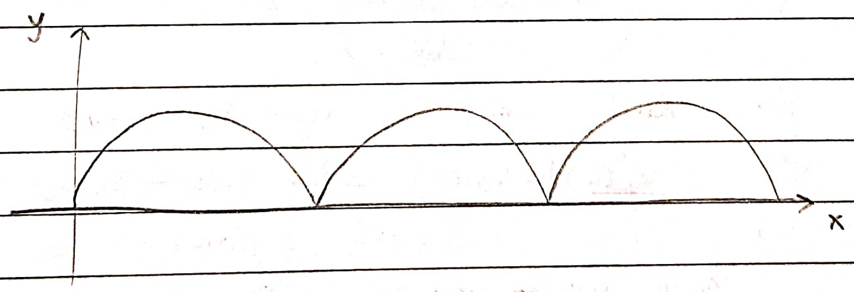
$$\Rightarrow \underline{v_0 = \frac{E_0}{B_0}}$$

$$\Rightarrow \quad v_y = \frac{E_0}{B_0} \sin\omega t$$

$$\Rightarrow \int_0^y dy = \int_0^t \left(\frac{E_0}{B_0}\right) \sin\omega t \, dt \quad \Rightarrow \quad \underline{y = \frac{E_0}{B\omega}[1 - \cos\omega t]}$$

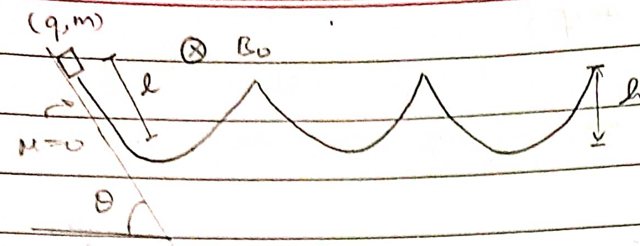
$$v_x = v_0 - \frac{a_y}{\omega} = v_0 - v_0 \cos\omega t \quad \Rightarrow \int_0^x dx = \int_0^t v_0 - v_0 \cos\omega t \, dt$$

$$\Rightarrow \quad \underline{x = \left(\frac{E_0}{B_0}\right)\left(t - \frac{\sin\omega t}{\omega}\right)}$$



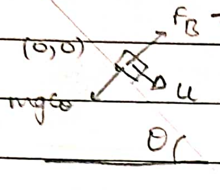
Cycloidal path

(Path followed by topmost pt. of rolling circle)

Q 

Find h.

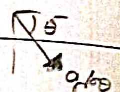
(CoE) $u = \sqrt{2gl\sin\theta}$

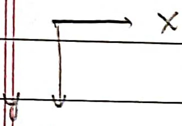
A. 

(Eq) $mg\cos\theta = quB_0 \Rightarrow \underline{\omega = \dfrac{g\cos\theta}{u}}$

Initial condⁿ (IC) : $v_x = u\cos\theta \quad v_y = u\sin\theta$

$a_x = g\sin\theta\cos\theta \quad a_y = g\sin^2\theta$





Let $\vec{v} = \langle v_x \quad v_y \rangle$

$\vec{B} = \langle 0 \quad 0 \quad B_0 \rangle$

$\Rightarrow \vec{F_B} = \langle qB_0 v_y \quad -qB_0 v_x \rangle$

$$\vec{F} = \langle qB_0 v_y \quad mg - qB_0 v_x \rangle$$

$$\vec{a} = \left\langle \frac{qB_0}{m} v_y \quad g - \frac{qB_0}{m} v_x \right\rangle$$

Similar to last Q, $v_y = v_0 \sin(\omega t + \varphi)$

$v_y(0) = v_0 \sin\varphi \Rightarrow u\sin\theta = v_0 \sin\varphi$

$a_y = v_0\omega\cos(\omega t + \varphi) \Rightarrow \underbrace{a_y(0)}_{g\sin^2\theta} = v_0\omega\cos\varphi$

$$v_0 = \sqrt{u^2\sin^2\theta + \frac{g^2\sin^4\theta}{\omega^2}} = \sqrt{u^2\sin^2\theta + \frac{u^2\sin^4\theta}{\cos^2\theta}} = \underline{u\tan\theta}$$

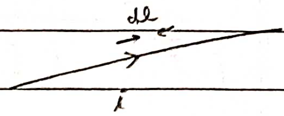
$$\int_0^y dy = \int_0^t u\tan\theta \sin(\omega t + \varphi)\, dt \Rightarrow y = \frac{u\tan\theta}{\omega}\left(\cos\varphi - \cos(\omega t + \varphi)\right)$$

$$(\Delta y)_{max} = h = \frac{2u\tan\theta}{\omega} = \frac{2u^2\tan\theta}{g\cos\theta} = \underline{4l\tan^2\theta}$$

21/06/2023

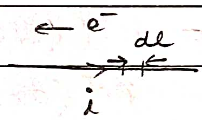
FORCE ON CURRENT CARRYING WIRE

$$d\vec{F} = q(\vec{v} \times \vec{B})$$
$$= (i\,dt)(\vec{v} \times \vec{B})$$
$$= i(\vec{v}\,dt \times \vec{B})$$
$$= \boxed{i(d\vec{l} \times \vec{B})}$$



Alternate Derivation –

$$i = neAv_d$$



$$d\vec{F}_e = (-e)(\vec{v}_d \times \vec{B})$$

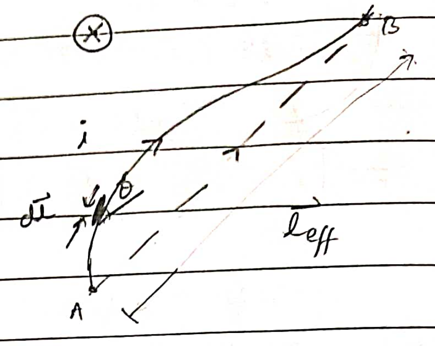
since wire is thin, this force on e^- is unable to drift it and is thus transferred to wire.

$$d\vec{F} = (nA\,dl)(d\vec{F}_e) = (nA\,dl)(-e)(\vec{v}_d \times \vec{B})$$
$$= (nAev_d)\left(\frac{-\vec{v}_d\,dl \times \vec{B}}{v_d}\right)$$

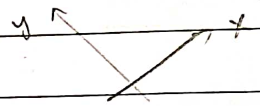
Let us take $d\vec{l}$ in dirn of $-\hat{v}_d$

$$= (nAev_d)(d\vec{l} \times \vec{B})$$
$$= i(d\vec{l} \times \vec{B})$$

- In uniform field

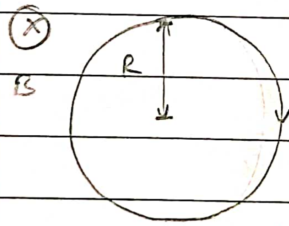


$$\boxed{\vec{F} = i\,(\vec{l}_{eff} \times \vec{B})}$$

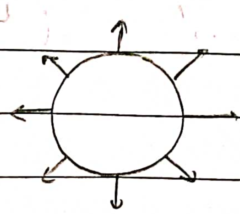


Proof:
$$d\vec{F} = i(d\vec{l} \times \vec{B})$$
$$= i\,(\langle dl\cos\theta \;\; dl\sin\theta\rangle \times \vec{B})$$
$$= i\,(\langle dx \;\; dy\rangle \times \vec{B})$$

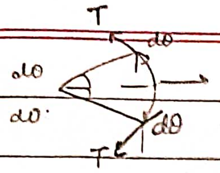
$$\vec{F} = \int d\vec{F} = i\left[\int (dx\,\hat{i}) \times \vec{B} + \int (dy\,\hat{j}) \times \vec{B}\right]$$
$$= i\left[B\int dx\,(\hat{i}\times\hat{B}) + (B\underbrace{\int dy}_{0})(\hat{j}\times\hat{B})\right]$$
$$= \underline{i(\vec{l}_{eff} \times \vec{B})}$$

Q. 

Find
a) net force on ring
b) tension on the ring due to F_B
c) if cross-sectional area of ring is A & Young's modulus Y, find increment in radius of ring ($\Delta R << R$)

A. a) 

Forces on diametrically opp. pts. cancel out.
Hence $F_{net} = 0$

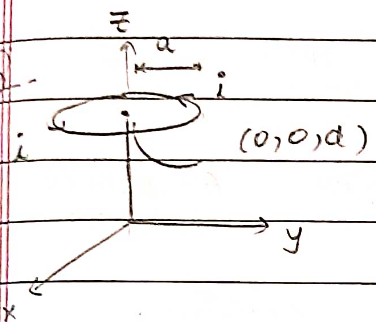
b) 

$\longrightarrow iB(2R\,d\theta)$

c) $\Delta R = \dfrac{TR}{AY} = \left(\dfrac{iR^2B}{AY}\right)$

$2T\,d\theta = iBR(2d\theta)$

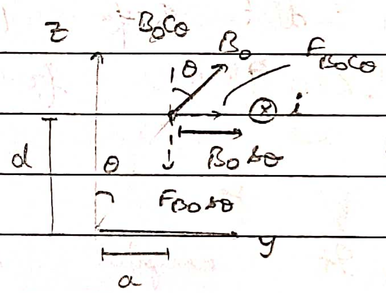
$\Rightarrow \quad \underline{T = iRB}$

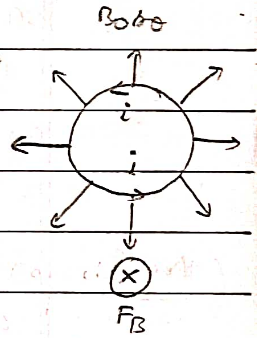
Q. 

$\vec{B} = B_0\hat{r}$, where $\hat{r}$ is unit vector drawn from origin to pt. in the space.

Find F_B acting on the ring.

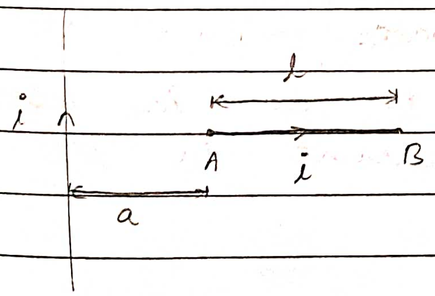
A. $\forall$ pts on ring.

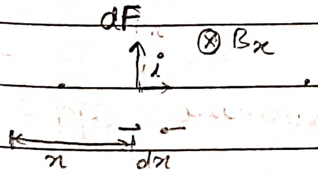




$$F_B = \int B_0 \sin\theta \; i\,dl$$
$$= B_0 \sin\theta \; i\,(2\pi a)$$
$$= \underline{\dfrac{2\pi i B_0\, a^2}{\sqrt{a^2+d^2}}}$$

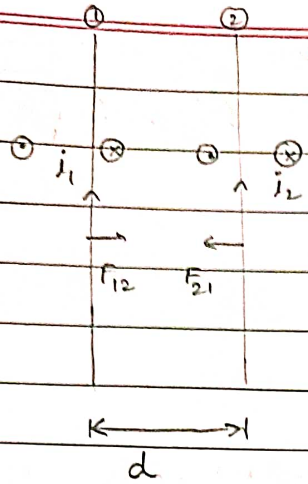
Q. Find F_B on AB.



A 

$B_x = \left(\dfrac{\mu_0}{4\pi}\right)\left(\dfrac{i}{x}\right)(2) = \left(\dfrac{\mu_0 i}{2\pi x}\right)$

$dF = i\left(\dfrac{\mu_0 i}{2\pi x}\right)dx \Rightarrow \underline{F = \left(\dfrac{\mu_0 i^2}{2\pi}\right) l\left(\dfrac{l}{a}\right)}$



Current in same dirⁿ ⇒ wires attract

$$\left(\text{Force / length b/w wires}\right) = \frac{\mu_0 i_1 i_2}{2\pi d}$$

This config. is used to define Ampere

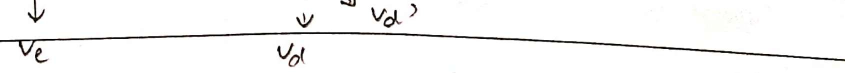
Def: 1 A current is the current passed through two long || wires kept at a dist. of 1 m which experience 2×10^{-7} N force

NOTE: It might seem that since wires are acc. towards, work is being done by $\vec{B}$.

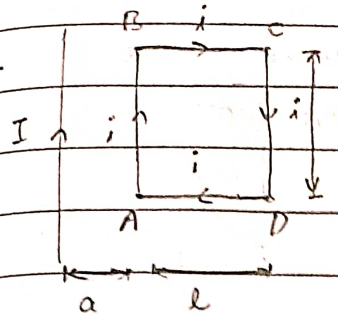
However, this is not the case.

The K.E is being provided by the extra work that the battery (source of current) has to do in order to maintain the same current in the wire.

Initially v_d of e was only in one dirⁿ but due to acc., another component appears.



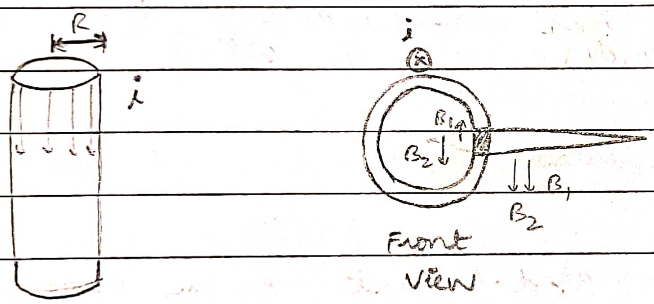
Hence, extra work has to be done by battery.

Q. 

Find net force acting on loop.

A. $\vec{F}_{on\ BC} + \vec{F}_{on\ DA} = 0$ (By sym) $\Rightarrow F = \left(\frac{\mu_0}{2\pi}\right)(I)(ib)\left[\frac{1}{a} - \frac{1}{(l+a)}\right]$ $(\leftarrow)$

$$= \frac{\mu_0 i I b l}{2\pi a(l+a)}$$



$B_1 \rightarrow$ (Field due to slit)

$B_2 \rightarrow$ (Field due to rest of cylinder).

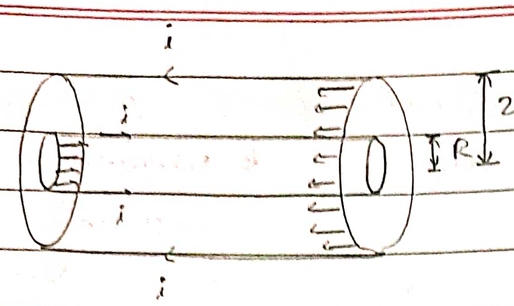
$$B_1 + B_2 = \frac{\mu_0 i}{2\pi R} \Rightarrow B_2 = \left(\frac{\mu_0 i}{4\pi R}\right)$$

$$B_1 - B_2 = 0$$

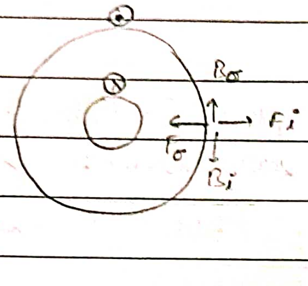
$$dF_{slit/B_2} = \left(\frac{\mu_0 i}{4\pi R}\right)(l)\left(\frac{i\,R\,d\theta}{2\pi R}\right) \Rightarrow \frac{dF_{slit/B_2}}{R\,l\,d\theta} = \left(\frac{\mu_0 i^2}{8\pi^2 R^2}\right)$$

($\because$ Force is normal to strip) (Area of strip)

$$\boxed{P = \frac{\mu_0 i^2}{8\pi^2 R^2}}$$

Q 

Find pressure due to magnetic force on outer & inner cylinder

A. 

$$F_o - F_i = \frac{\mu_0 i}{4\pi}\left(\frac{i(2R)d\theta}{2\pi(2R)}\right)\frac{l}{2R} - \frac{\mu_0 i}{2\pi}\left(\frac{i(2R)d\theta}{2\pi(2R)}\right)$$

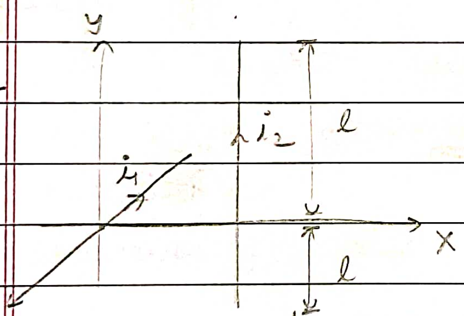
$$\Rightarrow P_o = \frac{F_o - F_i}{l(2R)(d\theta)} = \frac{\mu_0 i^2}{32\pi^2 R^2} - \frac{\mu_0 i^2}{16\pi^2 R^2}$$

$$= -\frac{\mu_0 i^2}{32\pi^2 R^2}$$

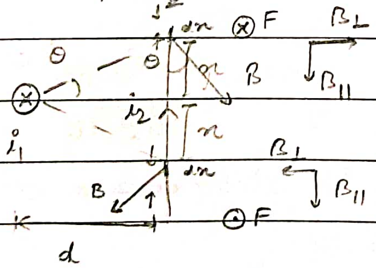
(indicates that P_o is acting outwards)

Since B due to outer cylinder on inner one's surface is zero. Therefore, P only due to B of inner cylinder.

$$\Rightarrow P_i = \frac{\mu_0 i^2}{8\pi^2 R^2}$$

Q. 

Find net torque exerted by wires on each other.

A. 

$$d\tau_x = 2F_x \cdot x$$

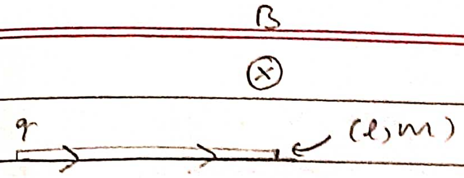
$$= (2 i_2\, dx\, B_\perp)\, x$$

$$= 2 i_2 \underbrace{B}_{\frac{\mu_0 i_1}{2\pi\sqrt{x^2+d^2}}} \sin\theta\, x\, dx$$

$$= \frac{\mu_0 i_1 i_2}{\pi} \frac{x^2}{(x^2+d^2)} dx$$

$$= \frac{\mu_0 i_1 i_2}{\pi}\left(1 - \frac{d^2}{x^2+d^2}\right) dx$$

$$\Rightarrow \tau = \frac{\mu_0 i_1 i_2}{\pi}\left[l - d\tan^{-1}\left(\frac{l}{d}\right)\right]$$



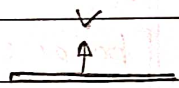
Q. Charge q suddenly passed through conductor. Find max height to which it will rise.

A. Sudden passing of charge causes the conductor to experience an impulsive force, due to which it will acquire an initial velocity.

p: $J = mv$

F: $F = qvB$

$t = \left(\frac{l}{v}\right)$



$J = Ft$ $\qquad$ ($\because$ Suddenly passed, $\Rightarrow$ t very less)

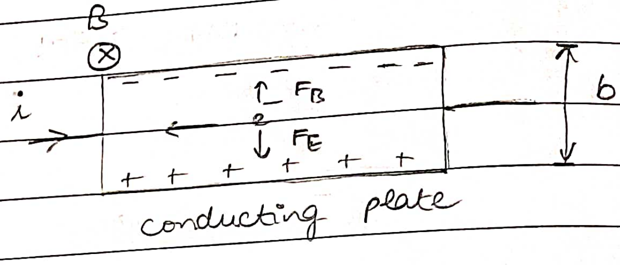
$\Rightarrow \quad m\cancel{v} = (q\cancel{v}B)\left(\frac{l}{v}\right)$

$\Rightarrow \quad v = \left(\frac{qBl}{m}\right)$ $\qquad \Rightarrow \quad h = \frac{v^2}{2g} = \frac{q^2B^2l^2}{2mg}$

21/06/2023

HALL EFFECT

F_B caused e^- to drift $\perp$ to their initial dirxn



This causes accumulation of charge at the ends of plate, which creates a transverse potential.

In steady state, force due to B & the force due to E(due to transverse potential) balance each other.

So, $\vec{F_B} + \vec{F_E} = 0 \Rightarrow e\vec{E} + e(\vec{B} \times \vec{v_d}) = 0$

$\Rightarrow \vec{E} = -(\vec{B} \times \vec{v_d})$

$\because B \perp v_d \Rightarrow E = -Bv_d$

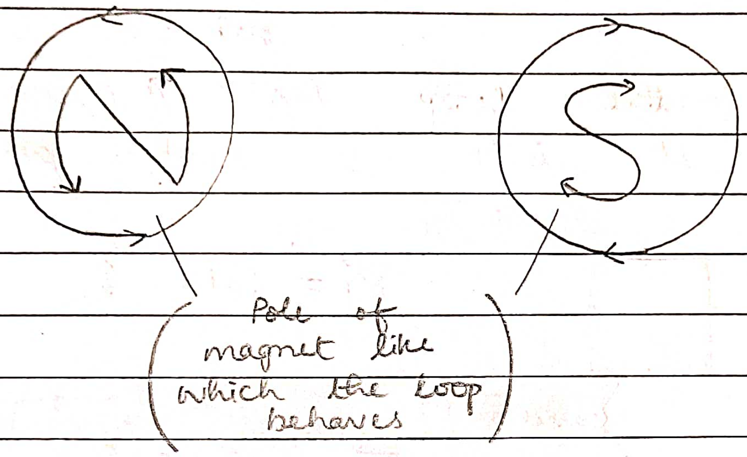
$$\boxed{V_T = Bv_d b}$$

Hall effect is used for :-

1. Finding out polarity of charge carriers
2. Determining free e^- density of conductor
3. Determining v_d

MAGNETIC DIPOLE MOMENT

A current loop behaves as a magnetic dipole.



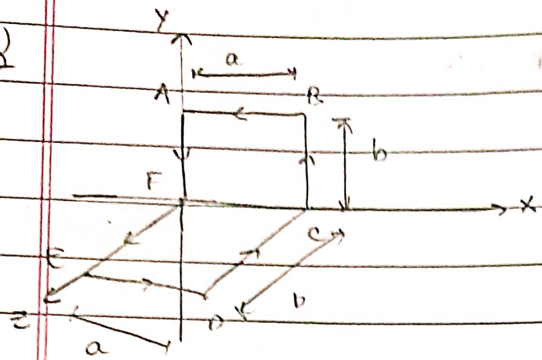
For a plane loop, $\boxed{\vec{M} = (NiA)\,\hat{n}}$

(# turns) — N; (Normal vector to plane of loop) — $\hat{n}$

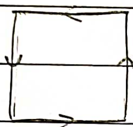
* Dirⁿ is specified by Right hand thumb rule.

NOTE: ① The formula is valid only for a closed planar loop & is independent of its shape

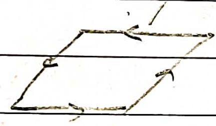
② For multi planar loops, we break them into many closed planar loops & add their $\vec{M}$, making sure the currents that we assume to complete such loops cancel each other.

Q.  Find $\vec{M}$

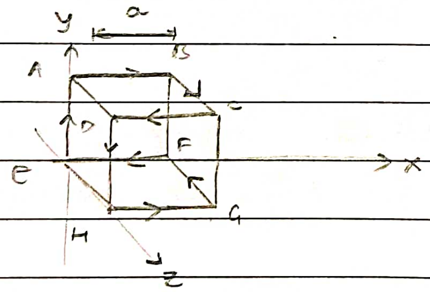
A. Since the loop isn't planar, we divide it into 2 loops which are

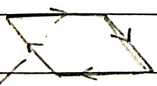
ABCF :  $\vec{M_1} = (abi)\hat{k}$

(cancel out)

FEDC :  $\vec{M_2} = (abi)\hat{j}$

$$\vec{M} = \vec{M_1} + \vec{M_2} = \langle 0 \quad abi \quad abi \rangle$$

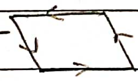
Q.  Find $\vec{M}$

A. ABCD :  $\vec{M_1} = a^2 i(-\hat{j})$

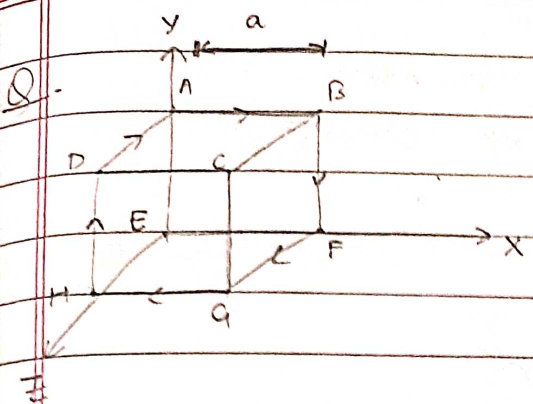
(cancel out)

ADHE:  $\vec{M_2} = a^2 i \ \hat{i}$

(cancel out)

EFGH :  $\vec{M_3} = a^2 i \ (\hat{j})$

$$\vec{M} = \vec{M_1} + \vec{M_2} + \vec{M_3} = a^2 i \ \hat{i}$$

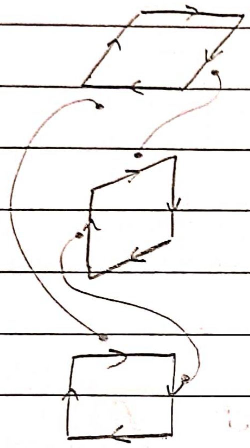
Q. 

Find $\vec{M}$

A. ABCD :

BFGC :

DCGH :



$\vec{M_1} = a^2 i \;(-\hat{j})$

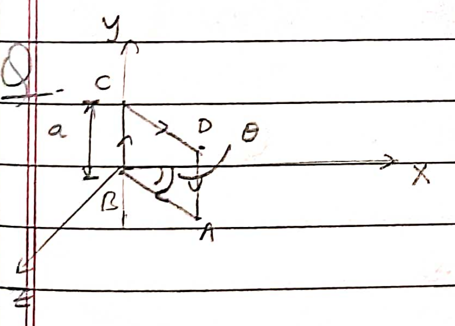
$\vec{M_2} = a^2 i \;(-\hat{i})$

$\vec{M_3} = a^2 i \;(-\hat{k})$

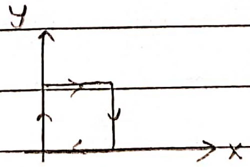
— - cancel out

$\vec{M} = \vec{M_1} + \vec{M_2} + \vec{M_3}$

$= \underline{\langle -a^2 i \quad -a^2 i \quad -a^2 i \rangle}$

Q. 

Find $\vec{M}$

A. ① If it were 

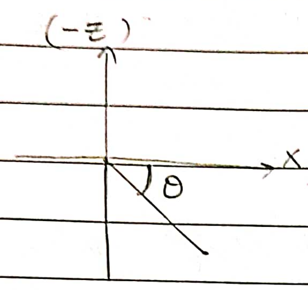
$\vec{M} = a^2 i \;\hat{k}$

Since loop rotates. $\theta \Rightarrow$ Normal will rotate θ

$\Rightarrow \hat{n} = \langle -s\theta, \; 0 \;\; c\theta \rangle$

$\Rightarrow \vec{M} = a^2 i \;\langle -s\theta \quad 0 \quad c\theta \rangle$

(2) Take cross product of 2 sides
(sides taken in dirn of current)

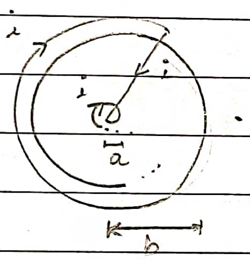


$\vec{S_1} = -b\hat{j}$ $\vec{S_2} = a\,\langle c\theta \;\; 0 \;\; s\theta \rangle$

$\vec{M} = i(\vec{S_1} \times \vec{S_2})$

$= a^2 i \,\langle -s\theta \;\; 0 \;\; c\theta \rangle$

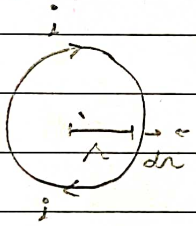
Q.



Plane spiral has total N turns which are very close.

Find $\vec{M}$

A.

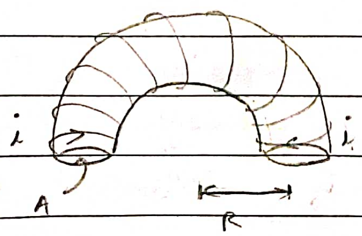


$d\vec{m} = \left(\frac{N}{b-a}\right) dr \, (i)(\pi r^2) \otimes$

$= \frac{N i \pi}{(b-a)} r^2 \, dr \otimes$

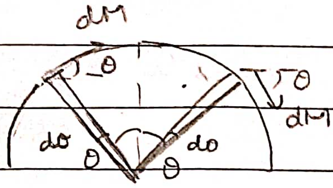
$\vec{M} = \frac{N i \pi}{3} (b^2 + ab + b^2) \otimes$

Q.



Half-toroid

Find $\vec{M}$

A. 

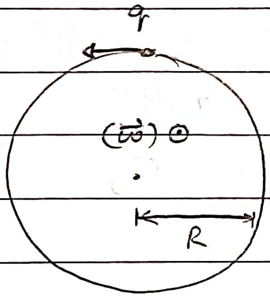
$$dM_{net} = 2\, dM\, c\theta$$

$$M_{net} = \int_0^{\pi/2} (2c\theta)\left(\frac{N}{\pi R} \times R\,d\theta\right) iA$$

$$= \frac{2NiA}{\pi}\int_0^{\pi/2} c\theta\, d\theta$$

$$= -\left(\frac{2NiA}{\pi}\right)$$

→ Orbiting charge



$$t = \left(\frac{2\pi}{\omega}\right)$$

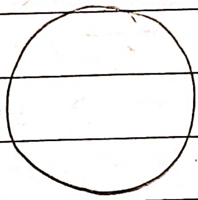
$$i = \left(\frac{q}{t}\right) = \left(\frac{q\omega}{2\pi}\right)$$

$$\vec{M} = iA = \left(\frac{q\vec{\omega}}{2\pi}\right)(\pi R^2) = \frac{\overbrace{(mR^2)}^{I} q\vec{\omega}}{2m}$$

$$\Rightarrow \boxed{\vec{M} = \frac{q}{2m}\vec{L}} \qquad (\because \vec{L} = I\vec{\omega})$$

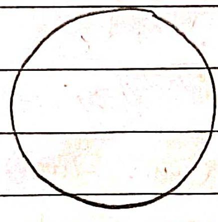
This formula is valid if charge distr. = mass distr

eg -



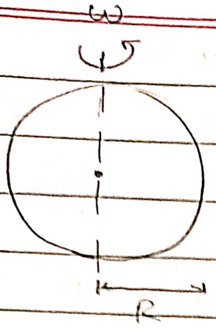
(Solid sphere)

$$\vec{M} = \left(\frac{q}{2m}\right)\left(\frac{2mR^2}{5}\right)(\vec{\omega})$$



(Conducting sphere)

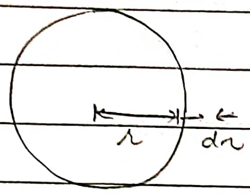
Formula not valid since q only on surface, but mass throughout volume.

Q. 

(Disc)

$\sigma = \sigma_0 (1 + r/R)$, r — dist. from centre

Find $\vec{M}$

A. 

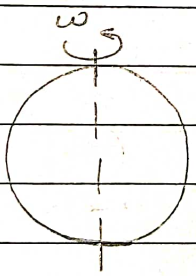
$$d\vec{M} = \left(\frac{dq}{2m}\right) \underbrace{I}_{\left(\frac{mr^2}{2}\right)} \vec{\omega}$$

$$= \left(\frac{\pi \sigma_0 \vec{\omega}}{2}\right)\left(r^3 + \frac{r^4}{R}\right) dr$$

$$dq = \sigma (2\pi r)\, dr$$

$$= 2\pi\sigma_0 \left(r + \frac{r^2}{R}\right) dr$$

$$\Rightarrow \quad d\vec{M} = \frac{9\pi \sigma_0 R^4 \omega}{40}$$

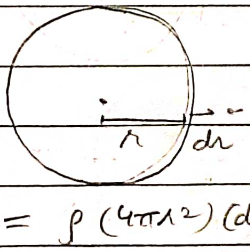
Q. 

(solid sphere)

$\rho = \frac{\rho_0 r}{R}$

Find $\vec{M}$

Using this result, find $\vec{M}$ if sphere is rotating about a tangent.

A. 

$$d\vec{M} = \left(\frac{dq}{2m}\right) \underbrace{(I)}_{\frac{2}{3}mr^2} \vec{\omega}$$

$$= \frac{4\pi \rho_0 \vec{\omega}}{3R} r^5\, dr$$

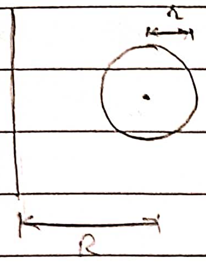
$$dq = \rho (4\pi r^2)(dr)$$

$$= \frac{4\pi\rho_0}{R} r^3\, dr$$

$$\vec{M} = \frac{2\pi\rho_0 R^5 \vec{\omega}}{9}$$

* Generally τ is axis dependant.
But for a couple, τ_{net} is independent of axis of rotation

If sphere rotating about its tangent,



$$I' = \frac{2mr^2}{3} + mR^2$$

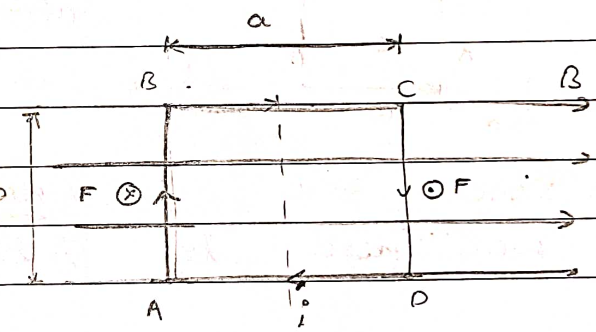
$$d\vec{M'} = \frac{4\pi \rho_0 r^2}{2Rm}\left(\frac{2mr^2}{3} + mR^2\right) dr$$

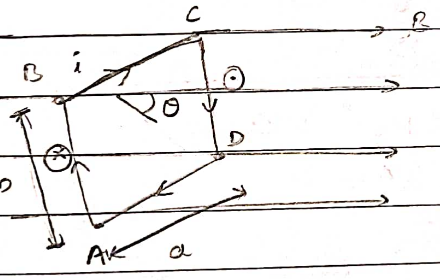
$$\vec{M'} = \frac{2\pi\rho_0 R^5 \vec{\omega}}{9} + \frac{2\pi\rho_0 R}{} \int_0^R r^3\, dr$$

$$= \frac{13}{18}\pi\rho_0 R^5 \vec{\omega}$$

→ Torque on current carrying loop

$$\tau = Bib\cdot a$$
$$= MB$$





$$\tau = (Bib)(a\sin\theta)$$
$$= |\vec{M}\times\vec{B}|$$

$$\Rightarrow \boxed{\vec{\tau} = \vec{M}\times\vec{B}}$$

NOTE: Torque on current carrying loop due to uniform B is independent of axis *

27/06/2025

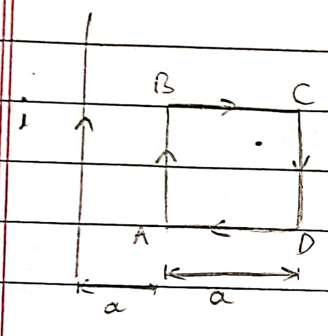
Work done in rotating dipole $= \int \tau \, d\theta$

$$= \int_{\theta_1}^{\theta_2} MB \sin\theta \, d\theta$$

$$= -MB(\cos\theta_2 - \cos\theta_1)$$

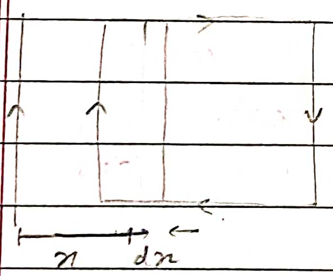
This work is stored as U.

So, let $\boxed{U = -\vec{M} \cdot \vec{B}}$

Q. 

Find work done in rotating loop by 180° about CD

A. Since B is not uniform, we need to integrate dU of elems



$$dU_i = -\vec{M} \cdot \vec{B}$$

$$= -\left(\frac{\mu_0 i}{2\pi x}\right) a \, dx \, i$$

$$U_i = \int_a^{2a} -\frac{\mu_0 i^2 a}{2\pi} \frac{dx}{x}$$

$$= -\frac{\mu_0 i^2 a \, l(2)}{2\pi}$$

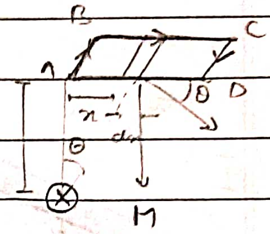
$$U_f = -\int_{2a}^{3a} -\frac{\mu_0 i^2 a}{2\pi} \frac{dx}{x} = \frac{\mu_0 i^2 a}{2\pi} l\left(\frac{3}{2}\right)$$

(~~dirn of~~ $\vec{M}$ is reversed)

$$W = \Delta U = \frac{\mu_0 i^2 a \, l(3)}{2\pi}$$

Q. In the above Q, find work done if loop rotated about AB by $90°$

A. $U_i = \frac{\mu_0 i^2 a}{2\pi} \ln(2)$



$dU_f = \left(\frac{\mu_0 i}{2\pi}\right)\left(\frac{1}{\sqrt{a^2+x^2}}\right)(a\,dx\ i)\cos\theta$

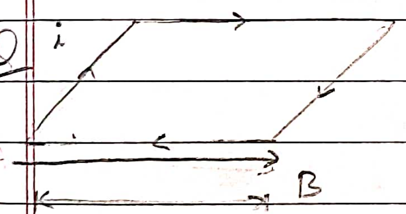
$x = a\tan\theta$

$dx = a\sec^2(\theta)\,d\theta$

$= \left(\frac{\mu_0 i^2}{2\pi}\right)\left(\frac{1}{a\sec\theta}\right)(a^2\sec^2(\theta)\cos\theta\,d\theta)$

$= \frac{\mu_0 i^2 a}{2\pi}\tan\theta\,d\theta$

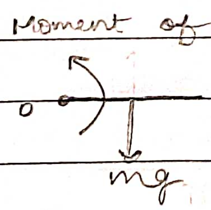
$U_f = \frac{\mu_0 i^2 a}{2\pi}\left[\ln|\sec(\theta)|\right]_0^{\pi/4} = -\frac{\mu_0 i^2 a}{4\pi}\ln(2)$

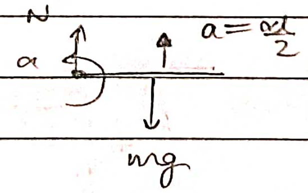
Q. 

N turns

mass M

Find
a) min mag. of B for which the coil will start toppling
b) If B is twice of that reqd in part (a), find the initial angular acc. of coil & also the normal reaction b/w floor & coil.

A. a) 

b) 

Abt O: $\tau_B = \tau_{mg}$

$\Rightarrow BNil^2 = \frac{mgl}{2}$

$\Rightarrow B = \frac{mg}{2Nil}$

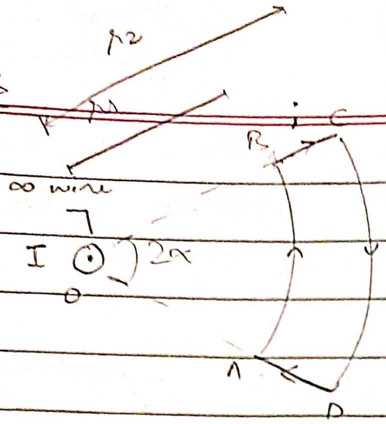
τ: $\tau = I\alpha \Rightarrow \tau_B - \tau_{mg} = I\alpha$

$\Rightarrow \left(\frac{2ml^2}{12} + \frac{2ml^2}{4} + ml^2\right)\alpha = Nil^2B - \frac{mgl}{2}$

$\Rightarrow \alpha = 3g/8l$

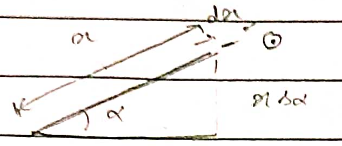
F: $N - mg = \frac{m\alpha l}{2} \Rightarrow N = \frac{19mg}{16}$

29/06/2023



Q. Find net torque acting on the loop.

A. (I)



$$d\tau = 2\left(\frac{\mu_0 I}{2\pi}\right)\left(\frac{1}{\cancel{x}}\right) i\, dx \cdot \cancel{x} s\alpha$$

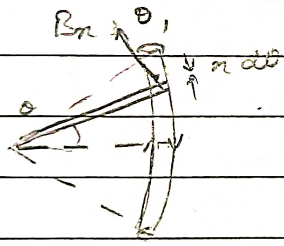
$$\tau = \frac{\mu_0 i I}{\pi} s\alpha \int_{r_1}^{r_2} dx$$

$$= \frac{\mu_0 i I s\alpha (r_2 - r_1)}{\pi}$$

$F_{AB} = F_{CD} = 0 \quad (\vec{B} \parallel i)$

$\Rightarrow$ No τ

(II)



$$d\vec{M} = (x\, d\theta\, dx)(i)\ \hat{k}$$

$$\vec{B}_x = \langle -B_x s\theta \quad B_x c\theta \rangle$$

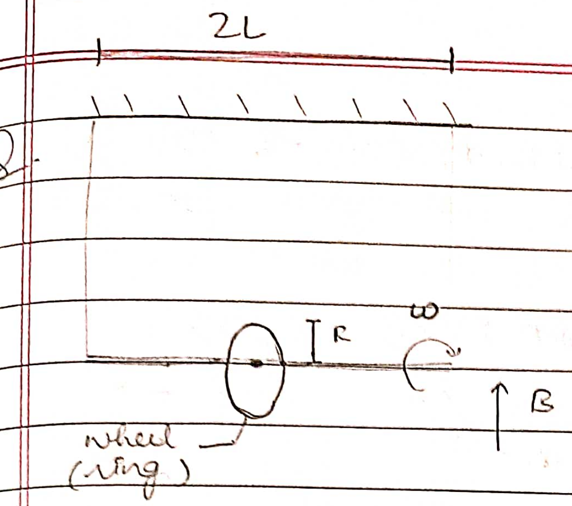
$$d\vec{\tau}_{(x,\theta)} = \vec{B_x} \times d\vec{M}$$

$$= \langle B_x x\, c\theta\, d\theta\, dx\, i \quad B_x x\, s\theta\, d\theta\, dx\, i$$

$$d\vec{\tau}_x = \int_{-\alpha}^{\alpha} d\vec{\tau}_{(x,\theta)} = \left\langle \left(\frac{\mu_0 I}{2\pi}\right)(i)\left(\frac{x\,dx}{x}\right)\int_{-\alpha}^{\alpha} c\theta\, d\theta \quad 0 \quad 0\right\rangle$$

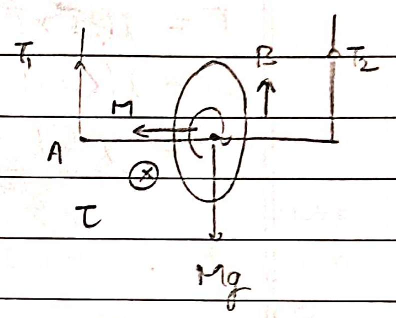
$$= \left\langle \frac{\mu_0 I i}{\pi} s\alpha\, dx \quad 0 \right\rangle$$

$$\tau = \int_{r_1}^{r_2} \frac{\mu_0 I i}{\pi} s\alpha\, dx = \frac{\mu_0 i I s\alpha (r_2 - r_1)}{\pi}$$



Q. Initial tension T_0. Find max ω with which the wheel can be rotated w/o breaking the string if they break at a tension of $3T_0/2$

A. $M = \left(\frac{q\omega}{2m}\right)(mr^2) = \frac{q\omega r^2}{2}$



F_i: $2T_0 = Mg$

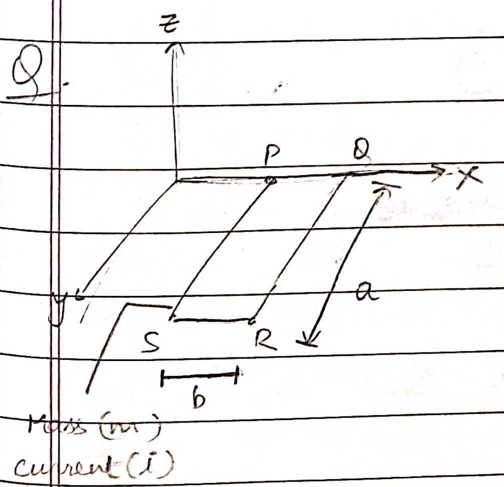
F_f: $T_1 + T_2 = 2T_0$

$\tau(A)$: $MgL + MB = T_2(2L)$

$\Rightarrow 2T_0L + \frac{q\omega r^2 B}{2} = T_2(2L)$

$T_1 < T_2 \Rightarrow$ For breaking $T_2 = \frac{3T_0}{2}$

$\omega = \frac{2T_0L}{qr^2B}$



Q. $\vec{B} = \langle 3 \quad 0 \quad 4 \rangle B_0$

Loop is in eq.

Then

a) Find dirxn of current in PQ

b) F_B (RS)

c) Find i in terms of B_0, a, b & m

Page

A. Assuming $i: P \to Q$,

$\vec{M} = abi\,(-\hat{k})$ $\Rightarrow$ $\vec{\tau}_B = (abB_0 i)\langle 0 \;\; -3 \;\; 0\rangle$

$\vec{B} = B_0\langle 3 \;\; 0 \;\; 4\rangle$

Abt PQ: $\vec{\tau}_B + \vec{\tau}_{mg} = 0 \Rightarrow (abB_0 i)\langle 0 \;\; -3 \;\; 0\rangle + \frac{mga}{2}\langle 0 \;\; 1 \;\; 0\rangle = 0$

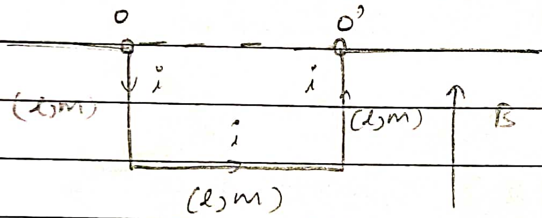
$\vec{F} = i(\vec{l}\times\vec{B})$

$= i(\langle 0 \;\; -b \;\; 0\rangle \times B_0\langle 3 \;\; 0 \;\; 4\rangle)$

$= \underline{iB_0 b\,\langle -4 \;\; 0 \;\; 3\rangle}$

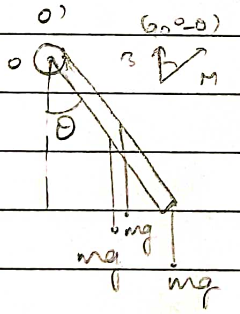
$\Rightarrow 3abB_0 i = \frac{mga}{2}$

$\Rightarrow i = \frac{mg}{6bB_0}$ $\quad i > 0 \Rightarrow \underline{i: P \to Q}$



Q. In eq., plane of frame makes θ with vertical. Find i

A.



* $\vec{\tau}_B = Bl^2 i\, c\theta$ ⊙

$\vec{\tau}_g = \frac{mgl\, s\theta}{2} + \frac{mgl\, s\theta}{2} + mgl\, s\theta$

$= 2mgl\, s\theta$ ⊗

$Bl^2 i\, c\theta = 2mgl\, s\theta \Rightarrow \underline{i = \frac{2mg\, t\theta}{B}}$

* Technically, we have assumed a wire OO' ($\overleftarrow{i}$) to find $\vec{M}$.

We can do this since mg passes through the axis abt which we are calculating torque.

So $\tau_{mg} = 0$

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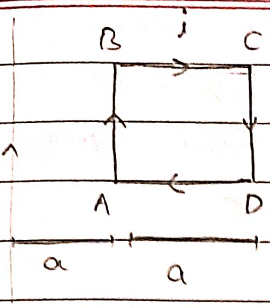
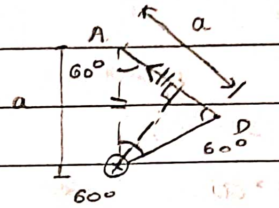


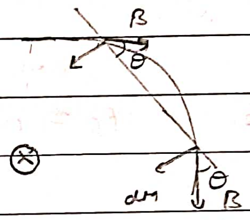
★Q. Find work done to rotate the loop about AB through 120°.

A. $U_i = \frac{\mu_0 i^2 a}{2\pi} l(2)$



Final config.

By symmetry, $\underline{U_f = 0}$



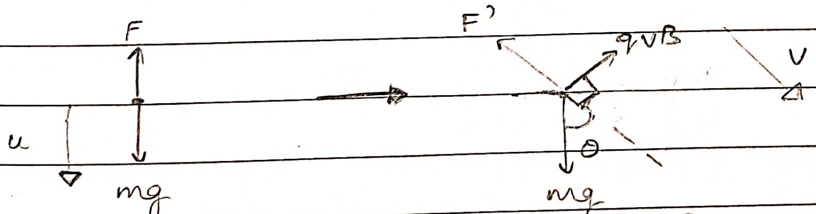
$B\,dM\,c(90+\theta)$
$+ B\,dM\,c(90-\theta)$
$= 0$

$\Rightarrow W = U_f - U_i$

$= \underline{-\frac{\mu_0 i^2 a}{2\pi} l(2)}$

★Q. A small ball (m, q) is falling with terminal velocity u. A horizontal B applied & new terminal vel. is attained by the ball. If power dissipated becomes η times of initial value, find B max.

A



$P_i = Fu$

$P_f = F'v$

$\underline{F = mg}$ $\qquad$ $\underline{mg\cos\theta = qvB}$

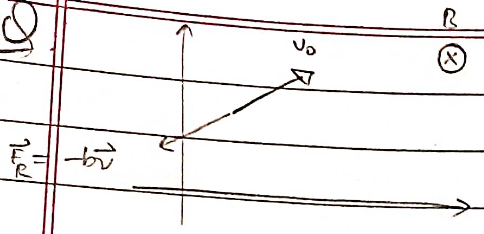
Since $B \perp v$, it does not provide power.

Therefore $\frac{W_{F'}}{t} = \frac{\Delta U}{t} = mgv\cos\theta \Rightarrow \underline{F'v = mgv\cos\theta}$

Given $P_f = \eta P_i \Rightarrow mgv\cos\theta = \eta F u \Rightarrow \underline{v = \eta u/\cos\theta}$

$\Rightarrow mg\cos\theta = q\left(\frac{\eta u}{\cos\theta}\right)B \Rightarrow B = \frac{mg}{2\eta q u}\sin 2\theta \Rightarrow \boxed{B_{max} = \frac{mg}{2\eta q u}}$

☆ **Q.**



Find distance covered by particle in the time the vel. vector rotates by 2π.

A. Here, $\vec{F}_B$ provides centripetal acc. & $\vec{F}_R$ provides tangential acc.

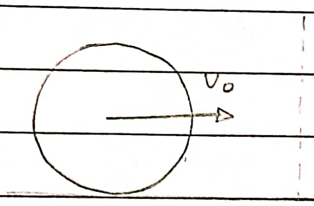
$$\omega = \frac{qB}{m} \Rightarrow t = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

Since $F_R = ma = m\left(\frac{dv}{dt}\right) = -bv \Rightarrow \frac{dv}{v} = -\frac{b}{m}\,dt$

Therefore, $\underbrace{d}_{\text{(Distance)}} = \int \underbrace{v}_{\text{(Speed)}}\, dt = \int_0^{\frac{2\pi m}{qB}} v_0 e^{-bt/m}\, dt \qquad \Rightarrow \underline{v = v_0 e^{-bt/m}}$

$$= \underline{\frac{mv_0}{b}\left(1 - e^{-\frac{2\pi b}{qB}}\right)}$$

☆ **Q.**



(q, m)

Find q for which ring starts rolling after entering the region of B completely.

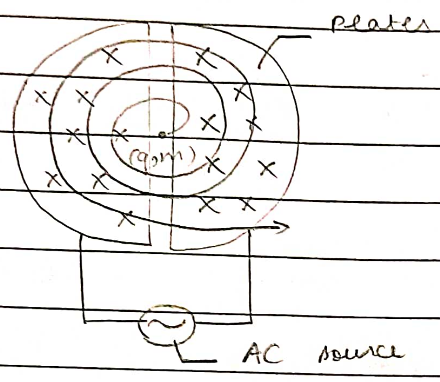
A.

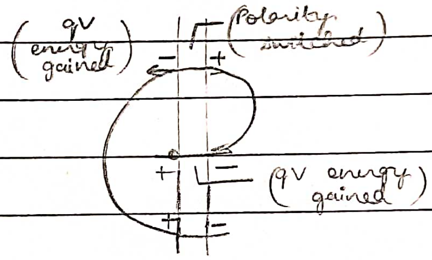
04/07/2025

CYCLOTRON

Charge Accelerator based on the principle that freq. of circular motion of a charge in B is independent of v.

Since $\omega \rightarrow$ const
$\Rightarrow T \rightarrow$ const
So we keep an AC source with time period T





Limitation : Can't accelerate charge upto very high speeds since as $v \rightarrow c$, mass of charge changes.

MAGNETIC PROPERTIES OF MATTER

On the basis of mag. behaviour, matter can be divided into 3 categories :-

Paramagnetic – $\vec{M}_{net} \neq 0$
Diamagnetic – $\vec{M}_{net} = 0$

Ferromagnetic

Paramagnetic substances don't show mag. ppts. unless placed in $\vec{B}_{ext}$.

When it is applied,

$$\vec{B}_{net} = \vec{B}_{applied} + \vec{B}_{magnetization}$$

$$\Rightarrow \frac{\vec{B}_{net}}{\mu_0} = \frac{\vec{B}_{applied}}{\mu_0} + \frac{\vec{B}_{mag.}}{\mu_0}$$

$$= \underbrace{\vec{H}}_{\text{(Magnetic Intensity)}} + \underbrace{\vec{I}}_{\text{(Intensity of magnetisation)}}$$

$$\boxed{\vec{I} = \frac{\vec{M}}{\underbrace{V}_{\text{(vol. of substance)}}}}$$

Experimentally, it is observed that

$$\vec{I} \propto \vec{H} \Rightarrow \boxed{\vec{I} = \underbrace{\chi}_{\text{(Mag. susceptibility)}} \vec{H}}$$

$$\Rightarrow \frac{\vec{B}_{net}}{\mu_0} = \vec{H}\,(\underbrace{1+\chi}_{\text{(Relative permeability)}})$$

$$\Rightarrow \vec{B}_{net} = \underbrace{\mu_0(1+\chi)}_{\text{(Permeability of substance) }[\mu_r]} \vec{H}$$

$$= \underline{\mu \vec{H}} \qquad \boxed{\mu = \mu_0 \mu_r}$$

For paramag. subs, $\underline{\chi > 0}$

In diamag. subs., no μ at atomic level.
But when placed in $\vec{B}_{ext}$, $\exists$ $\vec{\mu}_{induced}$ in dirn^n opp. to applied field.
So for diamag. subs, $\underline{\chi < 0}$

NOTE: (1) Diamag. is a universal ppt i.e $\exists\ \vec{\mu}_{induced}$ in all subs. when placed in $\vec{B}_{ext}$

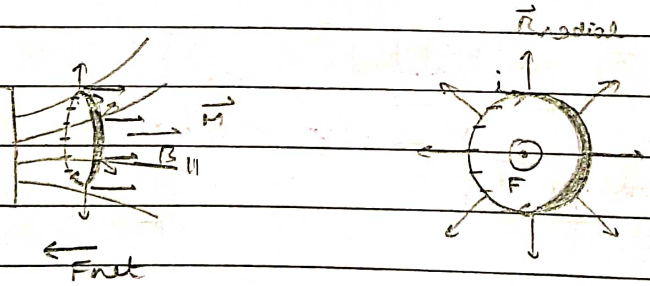
For paramag. subs. $\vec{\mu}$ dominates $\vec{\mu}_{induced}$

(2) For paramag. subs. $\boxed{\chi \propto \frac{1}{T}}$ ← absolute T (in K)

Curie's law

diamag. subs. χ is independent of T.

Paramag. subs. have a tendency to move from weaker field towards stronger field i.e they will be attracted by strong magnets.



$\vec{M} \parallel \vec{B}_{\parallel} \Rightarrow$ No force

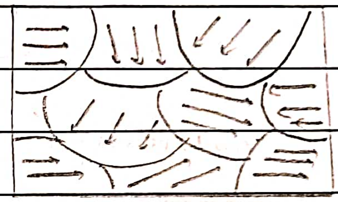
But $\exists$ F due to $\vec{B}_{radial}$ which is towards the magnet.

Hence attraction.

Similarly, diamagnetic subs. experience repulsion.

So, Paramag. subs Diamag subs.

→ Ferromagnetic subs.



Unlike paramag. subs., in ferro subs., ∃ several regions of small vol. called domains in which all atoms are aligned is same dirxn

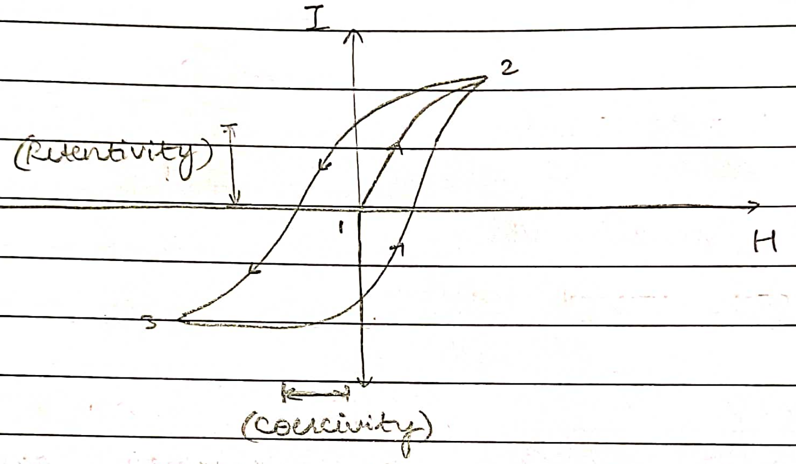
When $\vec{B}_{ext}$ is applied, domain whose $\vec{\mu}$ aligns with $\vec{B}_{ext}$ grows while others shrink.
Under strong enough $\vec{B}_{ext}$, boundaries of domain cease to exist.
Therefore, $\vec{\mu}$ of all atoms is aligned with B_{ext}.

Hence, ferro. subs. have high I.
In such a cond^n, substance is said to be saturated.

- Hysterisys curve –

When $\vec{B}_{ext}$ is removed, $\vec{\mu}$ of domains are not able to align back to their initial dirxn. So, they retain their magnetisation

Hence, another $\vec{B}_{ext}$ in the opp. dirxn is req. to demagnetise the sub.



Retentivity – Amt. of mag. retained by sub.
Coercivity – Amt. of $\vec{B}$ req. to demag. the sub

$\left(\text{Area of Hysterisys Curve}\right) \propto \left(\text{Loss of Energy per cycle}\right)$

For permanent mag., Retentivity high
Coercivity high

electromagnet, Retentivity low
Coercivity low
Area of hysterisys curve low

Page

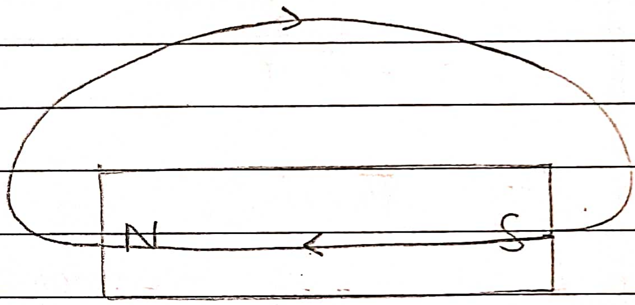
So, we use

Soft iron $\longrightarrow$ Electromagnet
Hard iron $\longrightarrow$ Permanent magnet.

- Critical temp. (T_c) – For $T > T_c$, ferro mag. subs behave as paramag subs.

→ Permanent magnets.

$\vec{M} = ml$
(Pole strength)



* This is sometimes denoted as 'magnetic charge' as this formula is the magnetic analogue of $\vec{p} = qd$.
In that sense, North and South Pole of a magnet behave as + ve & – ve charge

$$m = \frac{\vec{M}}{l} = \frac{Al \cdot i}{l} = iA$$

- Coulomb's law for magnetism – $\exists$ no mag. monopoles.
But if we were to mathematically eq. to electric charges, we can define a coulomb's law for magnetism

$$\boxed{F = \frac{\mu_0}{4\pi}\left(\frac{m_1 m_2}{r^2}\right)}$$

We can thus, treat a short mag. as a mag. dipole.

$$\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{2\vec{M}}{r^3}\right) \quad \text{(axial)}$$

$$= \left(\frac{\mu_0}{4\pi}\right)\left(\frac{\vec{M}}{r^3}\right) \quad \text{(eq.)}$$

$$= \left(\frac{\mu_0}{4\pi}\right)\left(\frac{\vec{M}\sqrt{1+3c^2_\theta}}{r^3}\right) \quad \text{(general)}$$

$$P.E = -\vec{M}\cdot\vec{B}$$

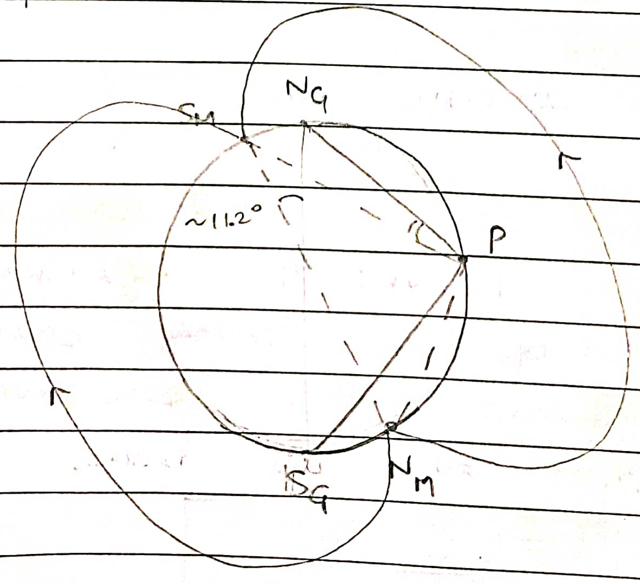
$$\tau = \vec{M}\times\vec{B}$$

$$U = -\vec{M}\cdot\vec{B}$$

06/07/2023

GEOMAGNETIC FIELD

- Geographical meridian — A plane containing geographical axis & a given place



- Magnetic meridian — plane containing magnetic axis & a given place.

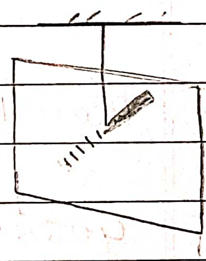
- Angle of Declination — Angle b/w geo. & mag. meridian at a given place.

- Angle of Dip / Inclination — Angle made by Earth's mag. field lines from horizontal.

It is approx 0° at equator & 90° at pole

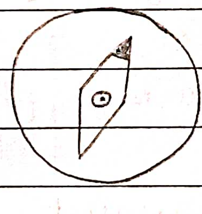
To measure these, we use the following devices.

① Freely suspended bar magnet —



Gives the dirn of $\vec{B}$ at a particular pt.

② Dip circle — Vertical circle

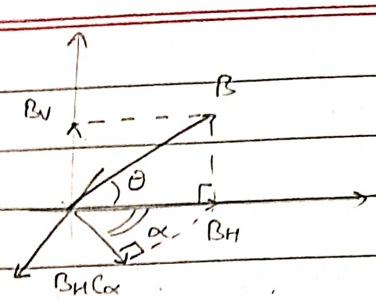


Dip angle measured in any plane is called apparent dip angle.

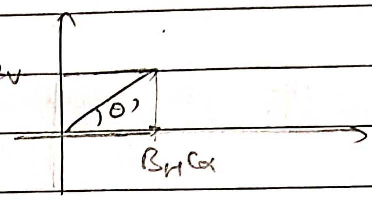
Dip angle measured in magnetic meridian is known as actual dip angle

Least of all angles measured by dip circle is actual dip angle

$t\theta = \frac{B_V}{B_H}$



$t\theta' = \frac{B_V}{B_H C\alpha} = \frac{t\theta}{C\alpha}$



θ — actual dip
θ' — apparant dip
α — ∠ b/w mag. meridian & plane of dip measurement

NOTE: Apparant dip angle will be 90° in a plane ⊥ to mag. meridian

→ Plausible Causes of Geomagnetism

- Gilbert's Model — Giant bar magnet inside earth

But temp. inside earth > Critical temp of ferromagnetic materials.
So this model is unable to explain the phenomenon.

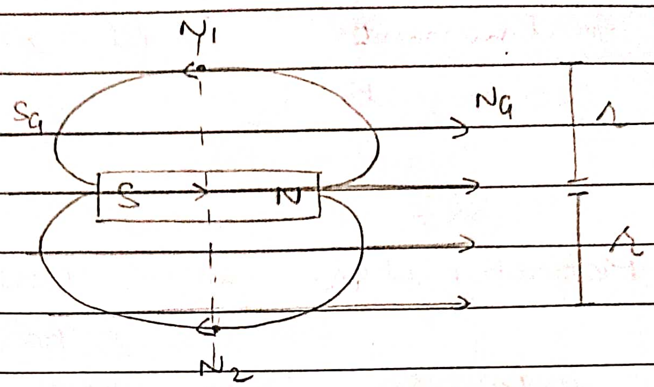
- Current Model — Molten core rotating ⇒ e^- in it revolving ⇒ Current generated

But this model fails to explain why mag. axis is not aligned with geographical axis

→ Null Pt.

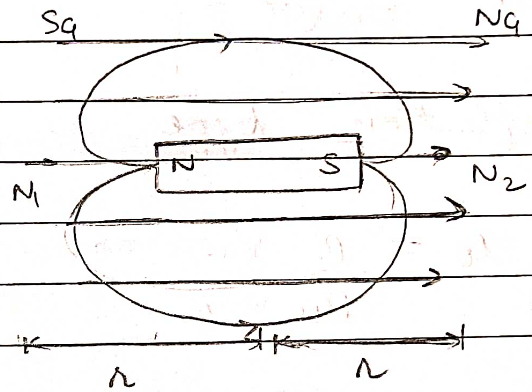
$B_H = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{M}{r^3}\right)$

at N_1 & N_2

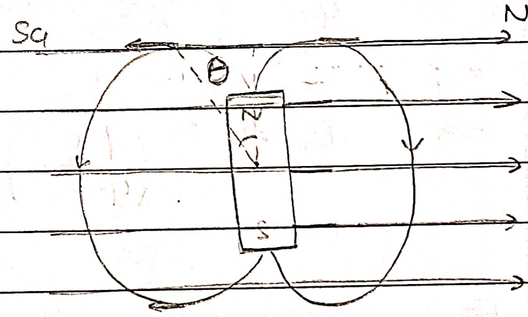


$B_H = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{2M}{r^3}\right)$

at N_1 & N_2



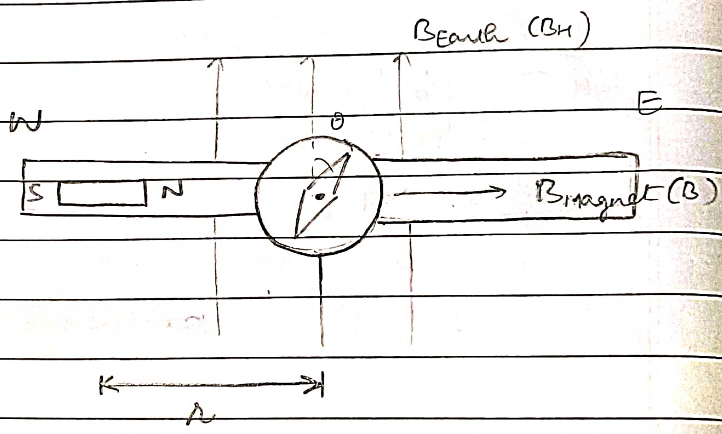
$\theta = \tan^{-1}(\sqrt{2})$



→ Instruments based on geomagnetism

- Magnetometer – used to compare $\vec{M}$ of 2 bar magnets or to determine mag. dipole moment of 1 bar magnet

– Deflection type



$$\tan\theta = \frac{B}{B_H}$$

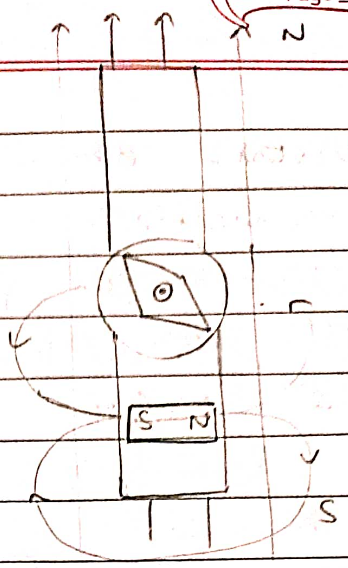
$$B = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{2M}{r^3}\right)$$

(I) tan(A) post.

$$\Rightarrow \quad B_H \tan\theta = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{2M}{r^3}\right)$$

For comparing 2 magnets, we fix one of them & move the other till $\theta = 0$

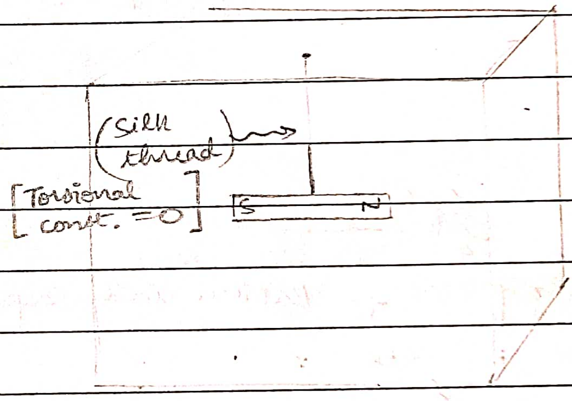
$$\Rightarrow \quad \boxed{\frac{M_1}{r_1^3} = \frac{M_2}{r_2^3}}$$



(II) tan(B) post.

- Oscillation type

An external bar mag. is brought close to the mag. present inside & removed suddenly.



This results in oscillation of the inside mag.

$$\tau = MB_H \sin\theta \Rightarrow I\alpha = MB_H\theta \quad (\theta \text{ very small})$$

$$\Rightarrow \alpha = \left(\frac{MB_H}{I}\right)\theta$$

$$\Rightarrow T = 2\pi\sqrt{\frac{I}{MB_H}}$$

Q. 2 bar mags. tied together & placed in oscillation type MG meter

(i) Similar poles together ⇒ 10 oscillations per min

(ii) Opp. poles together → 6 oscillations per min

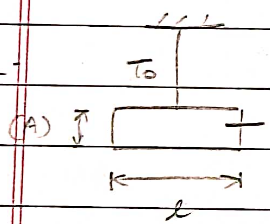
Find M_1/M_2 (M_1/M_2)

A. $$T_{(i)} = 2\pi\sqrt{\left(\frac{I_1+I_2}{M_1+M_2}\right)\left(\frac{1}{B_H}\right)}$$

$$T_{(ii)} = 2\pi\sqrt{\left(\frac{I_1+I_2}{M_1-M_2}\right)\left(\frac{1}{B_H}\right)}$$

$$\Rightarrow \frac{M_1+M_2}{M_1-M_2} = \frac{T_i^2}{T_{ii}^2}$$

$$\Rightarrow \frac{M_1}{M_2} = \frac{T_i^2+T_{ii}^2}{T_i^2-T_{ii}^2} = \frac{25+9}{25-9} = \left(\frac{17}{8}\right)$$

Q. 

Broken into 2 parts

a) 

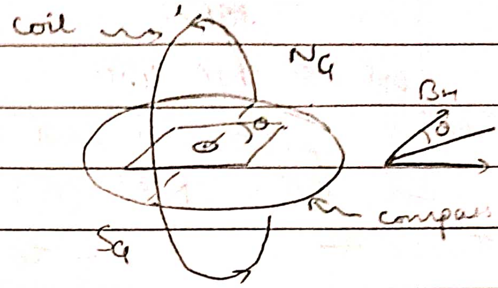
b) 

Find T in each case

A. a) $m \to m/2$, l same $\Rightarrow$ $M \to \frac{M}{2}$, $I \to \frac{I}{2}$ $\Rightarrow$ $\underline{T = T_0}$

b) m same, $l \to l/2$ $\to$ $M \to M/2$, $I \to I/8$ $\Rightarrow$ $\underline{T = T_0/2}$

- Tangent Galvanometer – Used to detect highly sensitive currents. Not used for measurement as reading depends on B_H



$B = B_H \tan\theta$

$\Rightarrow \frac{\mu_0 N i}{2R} = B_H \tan\theta$

$\Rightarrow \boxed{i = \frac{2 R B_H \tan\theta}{\mu_0 N}}$

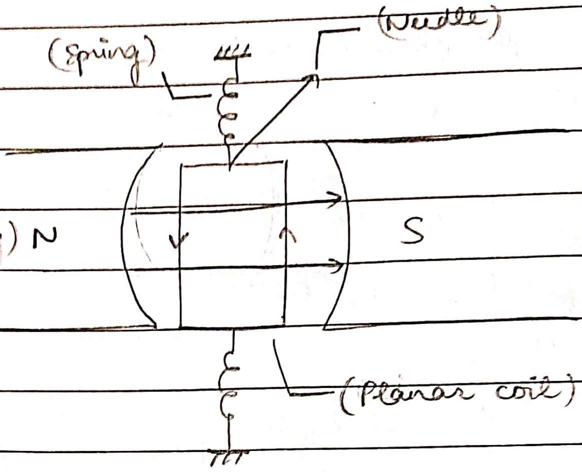
- Current sensitivity – $\frac{d\theta}{di}$

Due to dependence on B_H, it gives diff. values of i for same current at diff. places.

07/07/2022

MOVING COIL GALVANOMETER

$$\tau_B = \vec{M} \times \vec{B}$$
$$= MB$$
$$= (NAi)(B)$$



At eq.

(Torsional const of spring)

$$\Rightarrow \underbrace{k\theta}_{\tau_{spring}} = \underbrace{(NAi)(B)}_{\tau_B}$$

$$\Rightarrow \boxed{i = \left(\frac{k}{NAB}\right)(\theta)}$$

(Galvanometer const)

- Current sensitivity — $\frac{d\theta}{di} = \left(\frac{NBA}{k}\right)$
$$= \frac{1}{G.C}$$