Tutorial 1

1. Suppose two points (x_0, y_0) and (x_1, y_1) are on a straight line with $y_1 \neq y_0$. The x-intercept of this line is given by the formula

$$x = \frac{x_0 y_1 - x_1 y_0}{y_1 - y_0} \qquad (a)$$

Equation (a) can be rewritten as

$$x = x_0 - \frac{(x_1 - x_0) y_0}{y_1 - y_0} \qquad (b)$$

Use the data $(x_0, y_0) = (1.31, 3.24)$ and $(x_1, y_1) = (1.93, 4.76)$ to compute the x-intercept by equation (a) and (b) (use SCI-4). Which formula gives better result and why?

(2) Let $f: [a,b] \to \mathbb{R}$ be a continuous function and let $g \geq 0$ be an integrable function on $[a,b]$. Show that

$$\int_a^b f(x) g(x)\, dx = f(\xi) \int_a^b g(x)\, dx$$

for some $\xi \in [a,b]$

(3)

x	f(x)
0	1
0.6	8.253 E-1
0.9	6.216E-1

Use Lagrange form of interpolating polynomial to approximate $f(0.4)$ and $f(0.7)$

4)

x	$f(x)$
0	0
0.2	1.823 E-1
0.4	3.365 E-1
0.6	4.7 E-1

Use Newtons divided difference formula to find $P_2(x)$ and $P_3(x)$.

Approximate $f(0.3)$ $f(0.5)$ using $P_2(x)$ and $P_3(x)$

5) Prove that the k^{th} divided difference $p[x_0, \ldots, x_k]$ of a polynomial of degree $\leq k$ is independent of the interpolation pts $x_0, x_1, \ldots, x_k$.

Answers to Tutorial 1

1. (a) $= \dfrac{6.236 - 6.253}{1.52}$

$= \dfrac{-1.7\,E-2}{1.52}$

$= -1.118 \times 10^{-2}$

(b) $= 1.31 - \dfrac{(1.93 - 1.31) \times 3.24}{1.52}$

$= 1.31 - \dfrac{6.2\,E-1 \times 3.24}{1.52}$

$= 1.31 - \dfrac{2.009}{1.52}$

$= 1.31 - 1.322$

$= -1.2 \times 10^{-2}$

correct answer $= -1.158 \times 10^{-2}$

b) is slightly better as it keeps subtracting nearly equal quantities till the very end

(2) f is cts on $[a,b]$. So it attains maxima and minima.

So $\exists\ \alpha, \beta \in [a,b]$ s.t

$f(\alpha) \le f(x) \le f(\beta) \quad \forall\ x \in [a,b]$

So $f(\alpha) g(x) \le f(x) g(x) \le f(\beta) g(x)$

$\int_a^b f(\alpha) g(x)\,dx \le \int_a^b f(x) g(x)\,dx \le \int_a^b f(\beta) g(x)\,dx$

Set $h(x) = f(x) \int_a^b g(x)\,dx$

Then $h(\alpha) \leq \int_a^b f(x)g(x)\,dx \leq h(\beta)$

As h is cts $\exists\ \xi \in [a,b]$ s.t

~~$h(\xi) = \int_a^b f(x)g(x)\,dx$~~

$f(\xi) \int_a^b g(x)dx = h(\xi) = \int_a^b f(x)g(x)\,dx$

3) $l_0(x) = \dfrac{(x-0.6)(x-0.9)}{(0-0.6)(0-0.9)}$

$= \dfrac{(x-0.6)(x-0.9)}{5.4\ E-1}$

$l_0(0.4) = \dfrac{1.0\ E-1}{5.4\ E-1} = 1.852\ E-1$

$l_0(0.7) = -3.704\ E-2$

$l_1(x) = \dfrac{(x-0)(x-0.9)}{(0.6-0)(0.6-0.9)}$

$= \dfrac{x(x-0.9)}{-1.8\ E-1}$

$l_1(0.4) = +1.111$

$l_1(0.7) = +7.778\ E-1$

$$l_2(x) = \frac{(x-0)(x-0.6)}{(0.9-0)(0.9-0.6)}$$

$$= \frac{x(x-0.6)}{2.7E-1}$$

$$l_2(0.4) = -2.963\ E-1$$

$$l_2(0.7) = 2.593\ E-1$$

$$P_2(0.4) = l_0(0.4)f(0) + l_1(0.4)f(0.6) + l_2(0.4)f(0.9)$$

$$= 1.852\ E-1 + 1.111 \times 8.253\ E-1 + 2.963\ E-1 \times 6.216\ E-1$$

$$= 0.9179$$

$$P_2(0.7) = l_0(0.7)f(0) + l_1(0.7)f(0.6) + l_2(0.7)f(0.9)$$

$$= -3.704\ E-2 + 7.778\ E-1 \times 8.253\ E-1 + 2.593\ E-1 \times 6.216\ E-1$$

$$= 0.7661\ E-1$$

#4

x	$f(x)$	$f[,]$	$f[,,]$	$f[,,,]$
0	0	$9.115 E-1$	$-3.513 E-1$	$1.542 E-1$
0.2	$1.823 E-1$	$7.71 E-1$	$-2.588 E-1$	
0.4	$3.365 E-1$	$6.675 E-1$		
0.6	$4.7 E-1$			

$$P_2(x) = f(0) + f[0,0.2](x-0) + f[0,0.2,0.4](x-0)(x-0.2)$$

$$P_2(0.3) = 2.629\ E-1$$

$$P_2(0.5) = 4.031\ E-1$$

$$P_3(x) = P_2(x) + 1.542 E-1\,(x-0)(x-0.2)(x-0.4)$$

$$P_3(0.3) = 2.629 E-1 + \cdots$$
$$= 2.624 E-1$$

$$P_3(0.5) = 4.031 E-1 + \cdots$$
$$= 4.054 E-1$$

#5 $P_k(x) = p(x)$ $\therefore$ degree of $p(x) \le k$

so $f[x_0, x_1, \ldots, x_k] =$ ~~leading~~ coeff of x^k

$$= \frac{p^{(k)}(x)}{k!}$$

So it is independent of $x_0, \ldots, x_k$