Tutorial 10

Consider the differential equation

1. $\frac{dy}{dx} = y - x^2 + 1$ $\quad 0 \le x \le 2$, $y(0) = 0.5$, $h = 0.2$

1. Solve by RK-Method order 2
find $y(0.2)$, $y(0.4)$, $y(0.6)$, $y(0.8)$

2) By RK-method of order 4
find $y(0.2)$, $y(0.4)$

3) By Adam Bashforth method of order 2 find $y(0.4)$, $y(0.6)$, $y(0.8)$, $y(1.0)$ (use $y(0.2)$, $y(2)$)

4) By Adam-Bashforth method of order 4 find ~~$y(0.6)$~~, $y(0.8)$, $y(1.0)$ (use $y(0.2)$, $y(0.4)$ of (2) and $y(0.6)$ from Q 3)

(5) ~~By~~ Use Euler-Method predictor and Trapezoidal corrector find $y(0.2)$, $y(0.4)$, $y(0.6)$
(Use Corrector only once)

(6) Use Adam Bashforth as order 4 as predictor and Adam-Moulton method as corrector to find ~~$y(0.6)$~~, $y(0.8)$, $y(1.0)$ (Use corrector only once.) [Use $y(0.2)$, $y(0.4)$, $y(0.6)$ from Q(4)]

Answers to Tutorial 10

1) RK-Method of order 2

$$y_{n+1} = y_n + \frac{1}{2}(k_1 + k_2)$$

$$k_1 = h f(x_n, y_n)$$

$$k_2 = h f(x_n + h, y_n + k_1)$$

x	k_1	k_2	y
0	-	-	0.5
0.2	3.000 E-1	3.520 E-1	8.260 E-1
0.4	3.572 E-1	4.046 E-1	1.207
0.6	4.094 E-1	4.513 E-1	1.637
0.8	4.554 E-1	4.905 E-1	2.110

2) RK - Method of order 4

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = h f(x_n, y_n)$$

$$k_2 = h f(x_n + \frac{h}{2}, y_n + \frac{1}{2}k_1)$$

$$k_3 = h f(x_n + \frac{h}{2}, y_n + \frac{1}{2}k_2)$$

$$k_4 = h f(x_n + h, y_n + k_3)$$

x	K_1	K_2	K_3	K_4	y
0	–	–	–	–	0.5
0.2	3.000 E-1	3.280 E-1	3.308 E-1	3.582 E-1	8.293 E-1
0.4	3.579 E-1	3.837 E-1	3.862 E-1	4.111 E-1	1.214

3) Adam - Bashforth method of order 2

Given x_0, y_1

$n \geq 2 \quad y_{n+1} = y_n + \frac{h}{2}\left(3f(x_n, y_n) - f(x_{n-1}, y_{n-1})\right)$

x	y	$f(x, y)$
0	0.5	1.5
0.2	8.293 E-1	1.789
0.4	1.216	2.056
0.6	1.654	2.294
0.8	2.137	2.497
1.0	2.657	

4) Adam-Bashforth Method of order 4

given y_0, y_1, y_2, y_3

for $n \geq 3$

$$y_{n+1} = y_n + \frac{h}{24}(55 f(x_n, y_n) - 59 f_{n-1} + 37 f_{n-2} - 9 f_{n-3})$$

x	y	f(x,y)
0	0.5	1.5
0.2	8.293E-1	1.789
0.4	1.214	2.054
0.6	1.654	2.294
0.8	~~2.775~~ 2.135	2.495
~~1.0~~		
1.0	2.65	

---

5) Euler-Method predictor

Trapezoidal rule corrector

$$y_{n+1}^{(0)} = y_n + h f(x_n, y_n)$$

$$y_{n+1} = y_{n+1}^{(1)} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_{n+1}, y_{n+1}^{(0)})\right]$$

x	$y^{(0)}$	$y^{(1)} = y$
0	0.5	0.5
0.2	0.8	8.260 E-1
0.4	1.183	1.207
0.6	1.616	1.637

(6) A-B predictor

y_0, y_1, y_2, y_3 given

$n \geq 3$ $\quad y_{n+1}^{(0)} = y_n + \frac{h}{24}(55 f_n - 59 f_{n-1} + 37 f_{n-2} - 9 f_{n-3})$

A-B corrector

y_0, y_1, y_2, y_3 given

$y_{n+1} = y_{n+1}^{(1)} = y_n + \frac{h}{24}(9 f(x_{n+1}, y_{n+1}^{(0)}) + 19 f_n - 5 f_{n-1} + f_{n-2})$

x	$y^{(0)}$	$y^{(1)} = y$	$f(x, y^{(0)})$	$f(x, y)$
0	0.5	0.5	1.5	1.5
0.2	8.293 E-1	8.293 E-1	1.789	1.789
0.4	1.214	1.214	2.054	2.054
0.6	1.654	1.654	2.294	2.294
0.8	2.135	2.134	2.495	2.494
1.0	2.648	2.649	2.648	