Tutorial 2

1. ~~Suppose~~ let $x_0, x_1, \ldots, x_m$ be not necessarily distinct pts. Let f and g be two polynomials s.t f and g agree ~~upon~~ on $\underline{x} = x_0, x_1, \ldots, x_m$. Also $\deg f \leq m$ and $\deg g \leq m$. Show $f = g$

2. $f(0) = 1 \quad f'(0) = 0.5 \quad f''(0) = 2$

$f'''(0) = 3.$

Find a polynomial of degree ≤ 3 which agrees with $f(x)$ at $0, 0, 0, 0$.

3. $f(0.2) = 1.823\ E{-}1$

$f'(0.2) = 8.333\ E{-}1$

$f(0.4) = 3.365\ E{-}1$

$f'(0.4) = 7.143\ E{-}1$

a) Construct the divided difference table

b) Approximate $f(0.3)$

(2)

4. A function $f(x)$ has a double zero at z_1 and a triple zero at z_2. Determine the form of the polynomial of degree ≤ 5 which interpolates $f(x)$ twice at z_1, three times at z_2 and once at some other point z_3

(5)

$f(0.2) = 1.987\ E-1$

$f'(0.2) = 9.801\ E-1$

$f''(0.2) = -1.987\ E-1$

$f(0.3) = 3.894\ E-1$

Construct the divided difference table and approximate $f(0.3)$

Solutions to Tutorial 2

1. Suppose $z_1, z_2, \ldots, z_r$ be the distinct pts among $\underline{x} = x_0, x_1, \ldots, x_m$. Assume z_i is repeated c_i times.

Then $\sum_{i=0}^{r} c_i = m+1$.

$p(x) = f(x) - g(x)$

$p(z_1) = 0 \quad p'(z_1) = 0, \ldots, \quad p^{(c_1-1)}(z_1) = 0$

So $(x - z_1)^{c_1} \mid p(x)$

Similarly $(x - z_2)^{c_2} \mid p(x)$

$\vdots$

$(x - z_r)^{c_r} \mid p(x)$

as $z_1, \ldots z_r$ are distinct

we get $\prod_{i=0}^{r} (x - z_i)^{c_i} \mid p(x)$

$\deg p(x) \leq m$ and it is divisible by a deg $m+1$ polynomial.

So $p(x) = 0$.

2) The Taylor polynomial agree with f at 0, 0, 0, 0

$$P_3(x) = 1 + 0.5x + x^2 + \frac{1}{2}x^3$$

3)

x	$f(x)$	$f[,]$	$f[,,]$	$f[,,,]$
0.2	1.823E-1	8.333 E-1	-3.115 E-1	1.4 E-1
0.2	1.823E-1	7.71 E-1	-2.835 E-1	
0.4	3.365E-1	7.143 E-1		
0.4	3.365 E-1			

$$P_3(x) = 1.823\,E\text{-}1 + 8.333\,E\text{-}1\,(x-0.2) - 3.115E\text{-}1\,(x-0.2)^2 + 1.4E\text{-}1\,(x-0.2)^2(x-0.4)$$

$$P_3(0.3) = 2.624\ E\text{-}1$$

$f(x) = \log(1+x)$

Note $f(0.3) = 2.624\ E\text{-}1$ correct upto 4 sig digits

4) $P_5(x) = f[z_1, z_1, z_2, z_2, z_2, z_3](x-z_1)^2(x-z_2)^3$

$P_5(z_3) = f(z_3)$

So $f[z_1, z_1, z_2, z_2, z_2, z_3] = \dfrac{f(z_3)}{(z_3-z_1)^2(z_3-z_2)^3}$

5)

X	f(x)	f[,]	f[,,]	f[,,,]
0.2	1.987 E-1	9.801 E-1	-9.935 E-2	-1.683 E-1
0.2	1.987 E-1	9.801 E-1	-1.330 E-1	
0.2	1.987 E-1	9.535 E-1		
0.4	3.894 E-1			

$P_3(x) = 1.987\,E\text{-}1 + 9.801\,E\text{-}1\,(x-0.2) - 9.935\,E\text{-}2\,(x-0.2)^2 + 1.683\,E\text{-}1\,(x-0.2)^3$

$P_3(0.3) = 2.955\,E\text{-}1$

$f(x) = \sin x$

$f(0.3) = 2.955\,E\text{-}1$ upto 4 sig digits