Tutorial 4

In this ~~assign~~ tutorial do all computations in ~~both~~ six significant digits.

(1) $f(x) = \tan x$

$a = 0.5$

$$f'(a) \approx \frac{f(a+h) - f(a)}{h} =: N_1(h)$$

Find $N_1(0.1)$, $N_1(0.05)$, $N_1(0.025)$ & $N_1(0.0125)$

Then fill the Richardson extrapolation table

(2) $f(x) = \tan x$

$a = 0.5$

$$f'(a) \approx \frac{f(a+h) - f(a-h)}{2h} =: N_1(h)$$

Find $N_1(0.1)$, $N_1(0.05)$, $N_1(0.025)$ and $N_1(0.0125)$

Then fill the Richardson extrapolation table

3) $$I = \int_0^1 e^{-x^2}\, dx$$

Do composite Trapezoidal rule with

$N=1$, $N=2$, $N=4$, $N=8$

Then fill the Romberg integration table.

4) Suppose $P(x)$ is a polynomial of degree ≤ 3

Show that if we compute T'_{2N} in Romberg integration then we get exact value of integral.

5) ~~$I = \int_0^{\pi/2} \sin$~~

5) $I = \int_0^1 \sin(x^2)\,dx$

By composite Trapezoidal Trapezoidal rule we get the following data

N	T_N
1	4.20735 E-1
2	3.34070 E-1
4	3.15976 E-1
8	3.11681 E-1
16	3.10621 E-1

find the Ration R'_h, find T^2_N, R^2_h

~~T^3_N, R^3_h~~

Find best approximation to the integral (via Romberg integration)

Solutions to Tutorial 4

1) $f'(a) = N_1(h) + K_1 h + K_2 h^2 + K_3 h^3 + \cdots + K_n h^n + \cdots$
h suff small

h	$N_1(h)$	$N_2(h)$	$N_3(h)$	$N_4(h)$
0.1	1.37834			
0.05	1.33605	1.29376		
0.025	1.31670	1.29735	1.29855	
0.0125	1.30744	1.29818	1.29846	1.29845

$$\frac{d}{dx}\tan x\Big|_{0.5} = \sec^2 x\Big|_{0.5} = 1.29845 \text{ in 6 sig digits}$$

2) In this case
$f'(a) = N_1(h) + K_2 h^2 + K_4 h^4 + K_6 h^6 + \cdots$
So the formulas for $N_2(h)$ $N_3(h)$, $N_4(h)$ are different

h	$N_1(h)$	$N_2(h)$	$N_3(h)$	$N_4(h)$
0.1	1.30672			
0.05	1.30050	1.29843	1.29845	
0.025	1.29896	1.29845	1.29844	1.29844
0.0125	1.29857	1.29844		

Ratio give this to be accurate

Ratios give error here

Ratio is 2 ideally should be 16

(3) $I = \int_0^1 e^{-x^2} dx$ (2)

$T_1 = \frac{1}{2} 1 (1 + 3.67879)$

$= \frac{1}{2} (1.36788)$

$= 6.83940 \quad E-1$

$T_2 = \frac{1}{2}\frac{1}{2} (f(0) + 2f(0.5) + f(1))$

$= \frac{1}{4} (1.36788 + 2 \times 7.78801\ E-1)$

$= \frac{1}{4} (2.92548)$

$= 7.31371\ E-1$

$T_4 = \frac{1}{2}\frac{1}{4} [f(0) + 2f(0.25) + 2f(0.5) + 2f(0.75) + f(1)]$

$= \frac{1}{8} (2.92548 + 2 \times 9.39413\ E-1 + 2 \times 5.69783\ E-1)$

$= \frac{1}{8} (5.94387)$

$= 7.42984 \quad E-1$

$T_8 = \frac{1}{2}\frac{1}{8} [f(0) + 2f(0.125) + 2f(0.25) + 2f(0.375) + 2f(0.5) + 2f(0.625) + 2f(0.75) + 2f(0.875) + f(1)]$

$= \frac{1}{16} [5.94387 + 2 \times 9.84496\ E-1 + 2 \times 8.68815\ E-1 + 2 \times 6.76634\ E-1 + 2 \times 4.65043\ E-1]$

$$T_8 = \frac{1}{16}\left[1.19338\ E1\right)$$

(3) (4)

$$= 7.45865\ E-1$$

$I = T_N + k_2 h^2 + k_4 h^4 + k_6 h^6 + \cdots$ (h subb small)

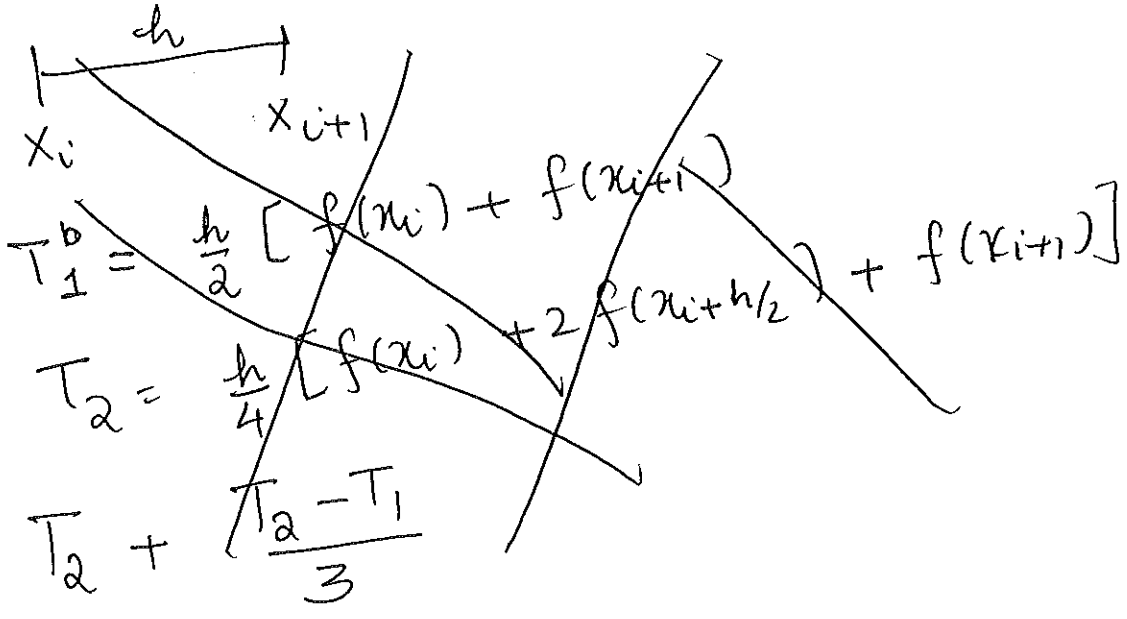
N	T_N	T_N^1	T_N^2	T_N^3
1	6.83940 E-1			
2	7.31371 E-1	7.47181 E-1		
4	7.42984 E-1	7.46855 E-1	~~7.46746~~ 7.46833 E-1	
8	7.45865 E-1	7.46825 E-1	7.46823 E-1	7.46823 E-1

$\int_0^1 e^{-x^2} dx$ $\overset{\text{Exact value upto 8 decimals}}{=\!=}$ 7.4682413 E-1

Our answer is correct upto 5 sig digits

---

4)

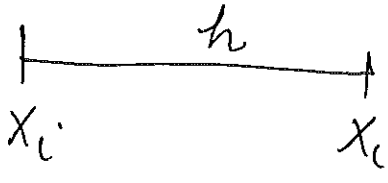


$$T_1^b = \frac{h}{2}\left[f(x_i) + f(x_{i+1})\right]$$

$$T_2 = \frac{h}{4}\left[f(x_i) + 2f(x_i + h/2) + f(x_{i+1})\right]$$

$$T_2^1 = T_2 + \frac{T_2 - T_1}{3}$$

(4)



$$T_1 = \frac{h}{2}\left[f(x_i) + f(x_{i+1}) \right]$$

$$T_2 = \frac{h}{4}\left[f(x_i) + 2f(x_i + h/2) + f(x_{i+1}) \right]$$

$$T_1' = T_2 + \frac{T_2 - T_1}{3}$$

check

$$= \frac{h}{6}\left[f(x_i) + 4f(x_i + h/2) + f(x_{i+1}) \right]$$

Thus T_{2N}' gives simpsons rule which is exact for cubic polynomials

(5) (5)

$$I = \int_0^1 \sin(x^2)\,dx$$

N	T_N	R_h	T_N'	R_h^2
1	4.20735 E-1			
2	3.34070 E-1	4.78	3.05182 E-1	15.6
4	3.15976 E-1	4.21	3.09945 E-1	16
8	3.11681 E-1	4.05	3.10249 E-1	–
16	3.10621 E-1		3.10268 E-1	

T_N^2
3.10263 E-1
3.10269 E-1