Tutorial 5

1) From the following table determine $f'(0.2)$, $f'(0.3)$, $f'(0.4)$ $f'(0.5)$ by using all three formulas for approximating derivative

x	$f(x)$
0.2	2.027 E-1
0.3	3.093 E-1
0.4	4.228 E-1
0.5	5.463 E-1

2) Consider the boundary value problem

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$$

$y(0) = 0$ $\quad y(1) = 1$

Take $h = 1/4$

approximate $y(0.25)$ $y(0.5)$ & $y(0.75)$

(3) Find root of $x^2 - 3 = 0$ in $[1, 2]$

by (a) Bisection method
(b) Regula-Falsi method
(c) Secant Method
(d) Newton's Method.

|| 7 iterations

(4) Show $g(x) = \pi + 0.5 \sin\left(\frac{x}{2}\right)$ has a unique fixed point on $[0, 2\pi]$.
Find the unique fixed pt of g.

Solution to Tutorial 4

(1)

x	$f(x)$	Formula (1) $f'(x)$	Formula (2) $f'(x)$	Formula (3) $f'(x)$
0.2	2.027 E-1	1.066	—	1.032
0.3	3.093 E-1	1.135	1.101	1.085
0.4	4.228 E-1	1.235	1.185	1.170
0.5	5.463 E-1	1.235	—	1.285

formula 1 $\quad f'(a) \approx \dfrac{f(a+h)-f(a)}{h}$

$h = 0.1$ for $f'(0.2), f'(0.3), f'(0.4)$

$h = -0.1$ for $f'(0.5)$

Formula (2) $\quad f'(a) \approx \dfrac{f(a+h)-f(a-h)}{2h}$

$h = 0.1$ for $f'(0.3), f'(0.4)$

Formula (3) $\quad f'(a) \approx \dfrac{-3f(a)+4f(a+h)-f(a+2h)}{2h}$

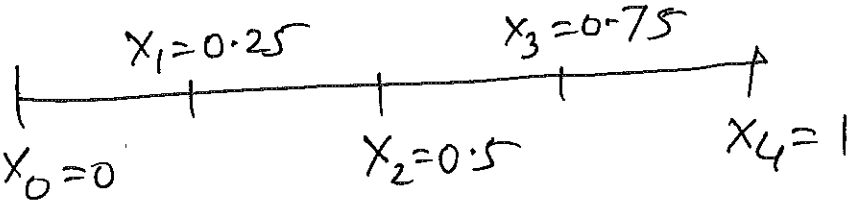
$h = 0.1$ for $f'(0.2), f'(0.3)$

$h = -0.1$ for $f'(0.4), f'(0.5)$

(2) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$

$y(0) = 0 \qquad y(1) = 1$

$h = 1/4$



Note exact $y = \frac{1}{1-e^{-2}}(1 - e^{-2x})$

$$\frac{y_0 - 2y_1 + y_2}{h^2} + 2\left(\frac{y_2 - y_0}{2h}\right) = 0$$

$$\frac{y_1 - 2y_2 + y_3}{h^2} + 2\left(\frac{y_3 - y_1}{2h}\right) = 0$$

$$\frac{y_2 - 2y_3 + y_4}{h^2} + 2\left(\frac{y_4 - y_2}{2h}\right) = 0$$

①, ②, ③ × by h^2 and use $y_0 = 0$, $y_4 = 1$

$$-2y_1 + (1+h)y_2 + 0\,y_3 = 0$$

$$(1-h)y_1 - 2y_2 + (1+h)y_3 = 0$$

$$0\,y_1 + (1-h)y_2 + -2y_3 = -1-h$$

$h = 0.25$ $\quad 1-h = 0.75$ $\quad 1+h = 1.25$

$y_1 = 4.596\ E-1$

$y_2 = 7.353\ E-1$

$y_3 = 9.007\ E-1$

exact $\quad y_1 = 4.551\ E-1$

$y_2 = 7.311\ E-1$

$y_3 = 8.985\ E-1$

Thus our approximation is correct upto 2 sig digits.

(3) find root of $x^2 - 3 = 0$ in $[1, 2]$

(a) Bisection method

	a_n	b_n	$f(a_n)$	$f(b_n)$	w_n
①	1	2	-2	1	1.5
②	1.5	2	-7.5 E-1	1	1.75
③	1.5	1.75	-7.5 E-1	6.25 E-2	1.625
④	1.625	1.75	-3.594 E-1	6.25 E-2	1.688
⑤	1.688	1.75	-1.523 E-1	6.25 E-2	1.719
⑥	1.719	1.75	-4.504 E-2	6.25 E-2	1.735
⑦	1.719	1.735	-4.504 E-2	1.023 E-2	1.727

root is in $[1.719, 1.735]$

(d) Newtons method

$$x_n = x_{n-1} - \left(\frac{x_n^2 - 3}{2x_n}\right)$$

$x_0 = 1$

$x_1 = 2$

$x_3 = 1.750$

$x_4 = 1.732$

$x_n = x_4$ for $n \geq 4$.

(c) Secant method

$x_{-1} = 1 \qquad x_0 = 2$

$$x_{n+1} = \frac{f(x_n)\,x_{n-1} - f(x_{n-1})\,x_n}{f(x_n) - f(x_{n-1})}$$

$x_1 = 1.667$

$x_2 = 1.727$

$x_3 = 1.732$

$x_4 = 1.732$

proces stops.

(b) Regula-Falsi method

a_n	b_n	$f(a_n)$	$f(b_n)$	w_n
1	2	-2	1	1.667
1.667	2	-2.222E-1	1	1.728
1.728	2	-1.560E-2	1	1.732
1.732	2	-1.760E-4	1	1.732

proces terminates

(4) $g(x) = \pi + 0.5 \sin\left(\frac{x}{2}\right)$

~~$-1 \leq \sin(x/2) \leq 1$~~ $\quad 0 \leq x \leq 2\pi$

So $0 \leq x/2 \leq \pi$

$0 \leq \sin(x/2) \leq 1$

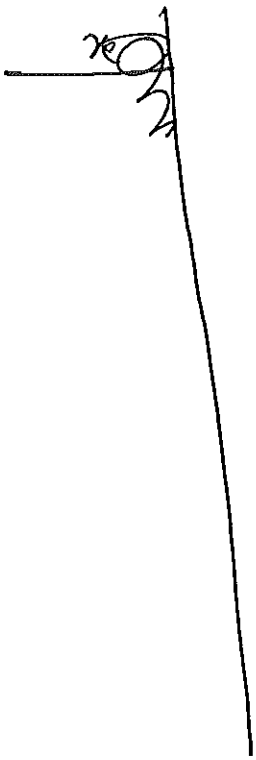
$g(x) \in [\pi, \pi + 0.5] \subseteq [0, 2\pi]$

So $g : [0, 2\pi] \to [0, 2\pi]$

$g'(x) = 0.5 \cos(x/2) \cdot \frac{1}{2}$

$|g'(x)| \leq 1/4$

So g has a unique fixed pt by theorem



$x_0 = 0$

$x_1 = 3.142$

$x_2 = 3.642$

$x_3 = 3.626$

$x_4 = 3.627$

$x_5 = 3.627$

fixed pt is 3.627 in 4 sig digits.