Expt-6
Resonance in Series RLC circuit

Aim:

To examine the behavior of current, voltage, and impedance of a series resonant circuit with respect to frequency. To determine Q-factor and bandwidth for various values of the damping resistor.

Introduction:

Resistor (R), Inductor (L) and Capacitor (C) are passive electric components, each having a typical I-V(Current-Voltage) characteristics. For an ideal resistor, the ratio of the applied voltage (V_R) and the resultant current (I_R) is a constant and equals the resistance offered by the resistor to the flow of electric current. The resistance of an ideal resistor is independent of the frequency. In other words an ideal resistor behaves identically in both DC and ac circuits. In ac circuits the voltage applied to an ideal resistor and the resultant current are in phase at all times.

However the same is not true for L and C. They offer impedance (or reactance) to the flow of electric current. Impedance may be understood as frequency dependent resistance.

The impedance (X_L) of an inductor of inductance 'L' is given as

$X_L = j\omega L$, where ω is the angular frequency and $j^2 = -1$,

while the impedance (X_C) of the capacitor of capacitance 'C is given as

$$X_C = -j/(\omega C)$$

Unlike the resistor, both the impedances X_L and X_C comprise of two components viz. the magnitude ($|X_L|$, $|X_C|$) and the phase angle (Φ_L, Φ_C). Here the phase angle refers to the angle between the voltage across and the current through inductor (or capacitor).

$$|X_L| = \omega L \text{ and } \Phi_L = \pi/2$$

$$|X_C| = 1/\omega C \text{ and } \Phi_C = -\pi/2$$

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Thus for a voltage (say *v*) across the inductor (or capacitor), the resultant current (*i*) will lag (or lead) by an angle of $\pi/2$. Inspection of the above equations reveals that the impedance of the inductor increases with the frequency and that of the capacitor decreases with the frequency. Also the phase angles are exactly opposite. Intuitively we can say that at some frequency the inductive and capacitive impedances will be equal and may cancel each other in some typical circuit configuration. Thus these two properties combined together can give rise to interesting frequency dependent phenomenon of electric resonance.

R, L and C can be connected in series or parallel to form a series resonant or a parallel resonant circuit.

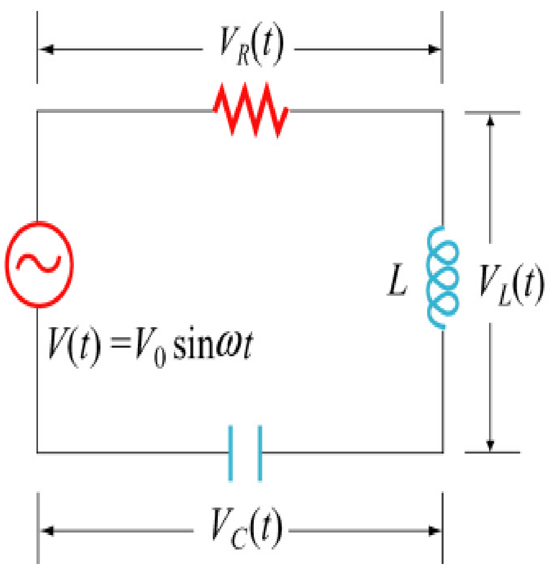

Figure 1 Circuit diagram of series RLC circuit [3]

Figure1 shows the circuit diagram of a series resonant circuit.

Applying Kirchhoff's Voltage law and solving the differential equation[3] we get

$$I_0 = \frac{V_0}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Being a series circuit, the current in the circuit (I_0) is ***same*** *through all the circuit elements.* The voltage across the individual elements will vary in magnitude and phase.

At a particular frequency (ω_0), $X_L = X_C$ and current becomes maximum. The expression for ω_0 is:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

ω_0 is called as resonant frequency and it depends only on the value of circuit elements.

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At resonance, the current in the circuit is limited by purely resistive components. A practical inductor may have a significant series resistance (r_L) and the voltage source will also exhibit some resistance (called as internal resistance of the source-R_{INT}).Thus the total resistive component (R_T) in the circuit will be summation of the above, i.e.

$$R_T = R + R_{INT} + r_L$$

Note: R_{INT} = 20 Ω for the function generator available in the laboratory.

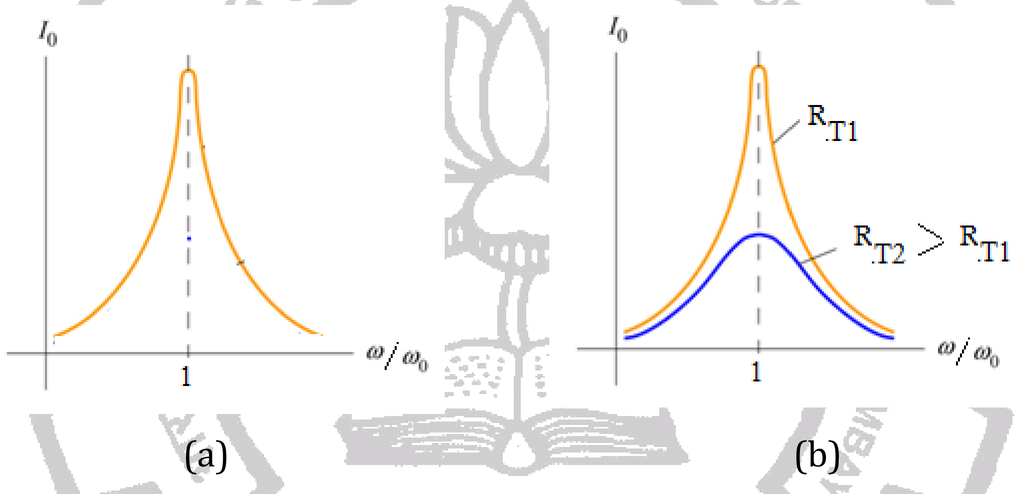

Figure 2 Frequency response of current in series RLC circuit [3]

Figure 2a shows frequency dependency of I_0 wherein ω/ω_0 is plotted on a logarithmic scale. We see that for frequencies well below and above the resonant frequency the current is minimum and reaches a maxima at ω_0.The value of maxima is determined by R_T. If we determine the frequency response with a higher value of R_T then the maxima will differ as shown in fig 2b. Also the curve becomes broader with ω_0 remaining constant in both the cases.

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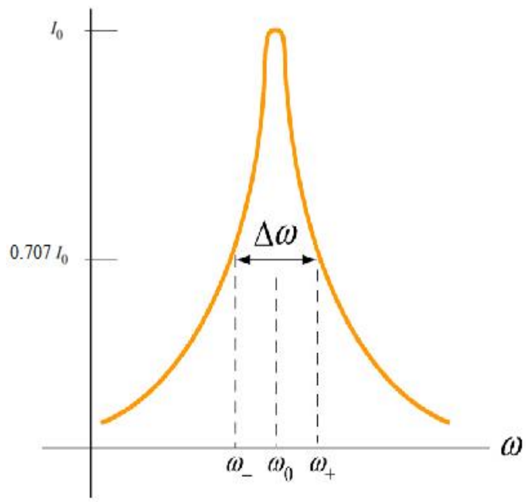

Figure 3 Bandwidth of a series RLC circuit [3]

Adjacent figure 3 shows the procedure to calculate bandwidth ($\Delta\omega$) of the resonant curve. The frequencies corresponding to $0.707I_0$, namely ω_+ and ω_- constitute the upper and lower cut-off frequencies of the response. At these frequencies, the power is half of the power at resonance frequency. Hence they are also known as half power frequencies. It is preferred to have a sharper response i.e. $\Delta\omega$ very small.

Q-factor and voltage magnification

Quality factor (Q) of a resonant circuit is defined as [4]

$$Q = 2\pi \frac{\text{maximum energy stored}}{\text{total energy lost per cycle at resonance}}$$

In terms of circuit components,

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}$$

Ideal inductor and capacitor do not dissipate energy. Energy is stored and released in alternate cycles. However resistor does not store but dissipates energy. Thus Q is a measure of energy storage relative to energy dissipation. Less energy dissipation (lower R), more sharper the resonance curve.

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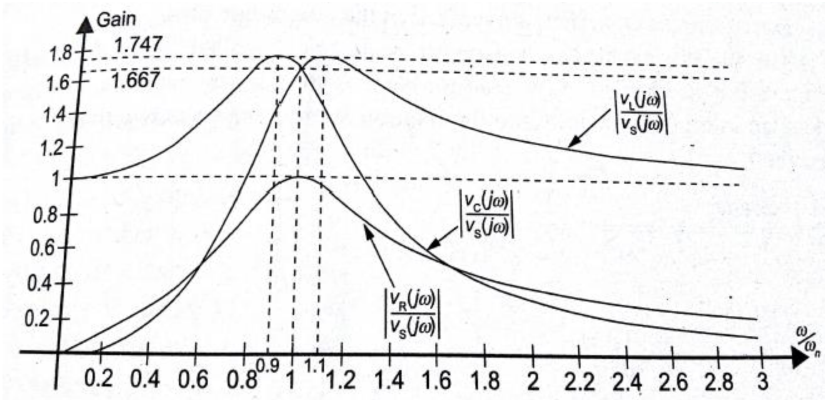


Figure 4 Frequency response of V_R, V_L and V_C [5]

Figure 4 shows frequency response plots of I_0, V_L and V_C in terms of the functions V_R/V, V_L/V and V_C/V respectively. At resonance ($\omega/\omega_0 = 1$),

$V_L = I_0\omega_0 L = (V/R).\omega_0 L = QV$, where V is input voltage

If Q is very large, then V_L will also be very large i.e. V_L will be magnified. However as seen from figure 4, the maximum value of V_L occurs beyond ω_0 and depends upon values of R, L and C. An exact analytical expression for the same can be found in [4].

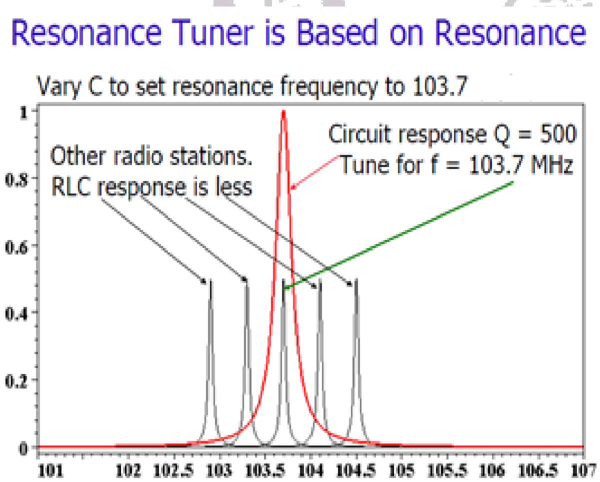


Figure 5 Resonance tuner

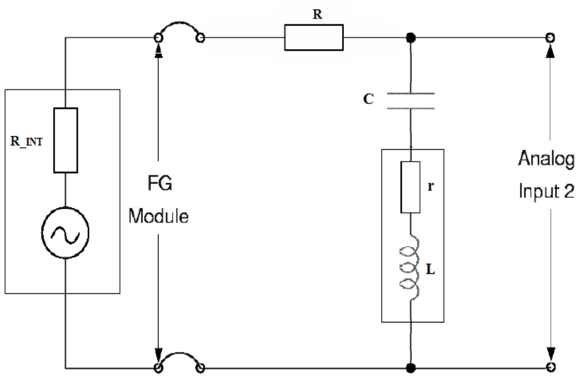
Applications of resonance in RLC circuits:

Series RLC circuit is used to tune in to specific radio stations by varying capacitor as shown in adjacent figure. Changing C changes ω_0 hence current will be maximum at a desired frequency whereas it will be negligible at other frequency bands.

Resonance in Series RLC circuit

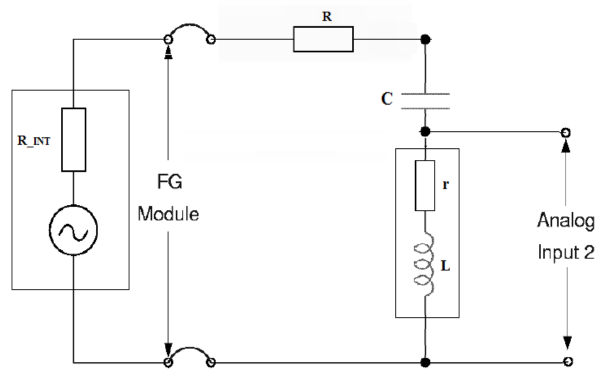
Practical circuit diagrams:

1. Resonance in series RLC circuit



(a) Resonance in series RLC

2. Voltage magnification in series RLC circuit



(b)Measurement of V_L in series RLC circuit

Figure 6 Practical circuit diagram for series RLC circuit [2]

A. Operational information:

a. Apparatus list

Cobra3 basic unit, Function generator (FG) module, 12V DC adaptors for basic unit and FG,PC loaded with appropriate software and USB cable (A-B), Digital multi-meter with L and C measuring ranges, Resistors, capacitors, inductors, shorting links and patch cords.

b. Familiarization with the experimental setup

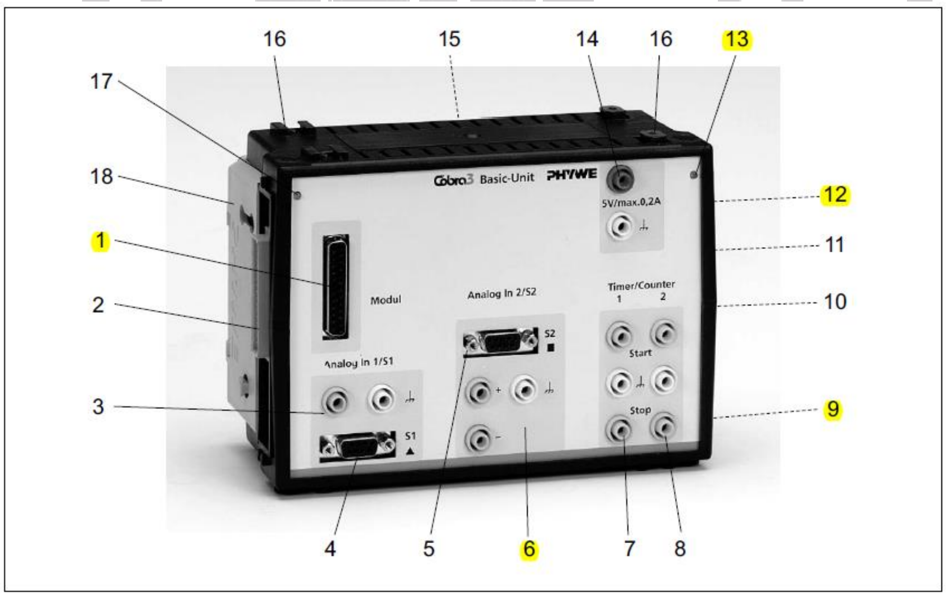


Figure 7 Cobra3 Basic unit [1]

Fig. 7 shows the prominent features of the Cobra3 basic unit that will be used as an interface between the PC and the various modules. The five features relevant to this experiment are highlighted in yellow. For information of the other features reader is directed to [1].

i. Power to the basic unit
The basic unit can be powered by a 12V/6W DC adaptor (Centre positive) through the low voltage socket (**no. 12**) provided on the right side wall.

ii. Power ON indication
A green LED (**no.13**) on top-right of the front panel acts as an indicator for power.

iii. Connection to the PC
The basic unit can be connected to the PC via an USB cable connected to the USB port (**no. 9**) on the right side wall and appropriate USB socket on the PC.

iv. Mounting for Function generator module
The function generator module can be mounted on the basic unit through the 25-pin D-type connector (**no. 1**).

v. Measurement port
The analog input voltage to be measured can be connected to port marked as "Analog In 2/S2" (**no. 6**) with two patch cords. This port is capable of measuring "**Earth free**" potential difference in 6-steps namely ± 30V, ± 10V, ±3V, ± 1V, ± 0.3V, ±0.1V

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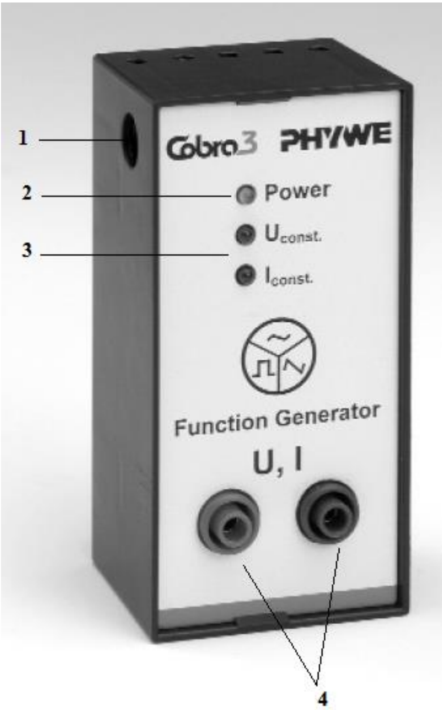

Figure 8 Cobra3 FG [2]

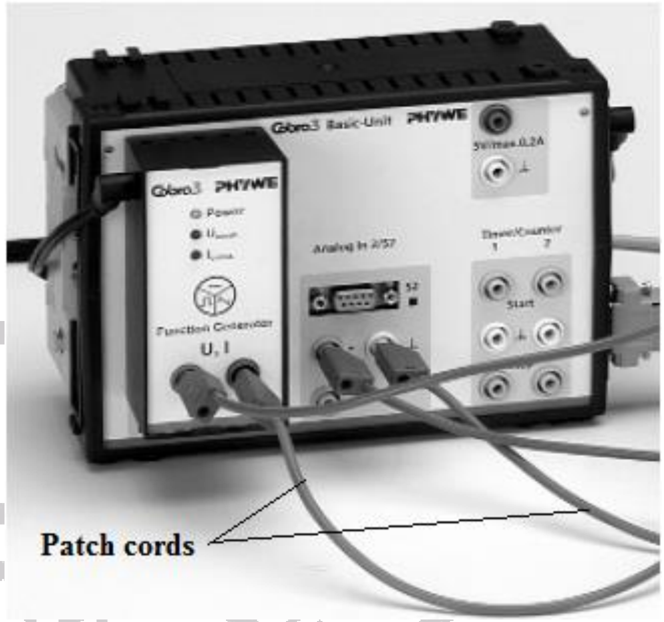

Figure 9 Cobra3 FG interconnected to basic unit [2]

The prominent features of FG as displayed in fig. 8 are:

i. Power to the FG

The FG can be powered by a 12V/6W DC adaptor (Centre positive) through the low voltage socket (**no. 1**) provided on the left side wall.

ii. Power ON indication

A green LED (**no.2**) on the front panel acts as an indicator for power.

iii. FG mode

The FG can be operated in the constant voltage mode ($U_{const.}$) or the constant current mode ($I_{const.}$). The corresponding LED on the front panel (**no. 3**) will glow as an indication of the particular mode.

iv. Output port

The output of the FG can be accessed through two connectors on the front panel (**no. 4**) through suitable patch cords.

Fig. 9 shows the interconnection of the FG with the Basic unit. The FG is mounted on the front panel of the basic unit. Electrical connectivity of the FG is automatically established via 25-pin D connector on the FG and the basic unit.

Resonance in Series RLC circuit

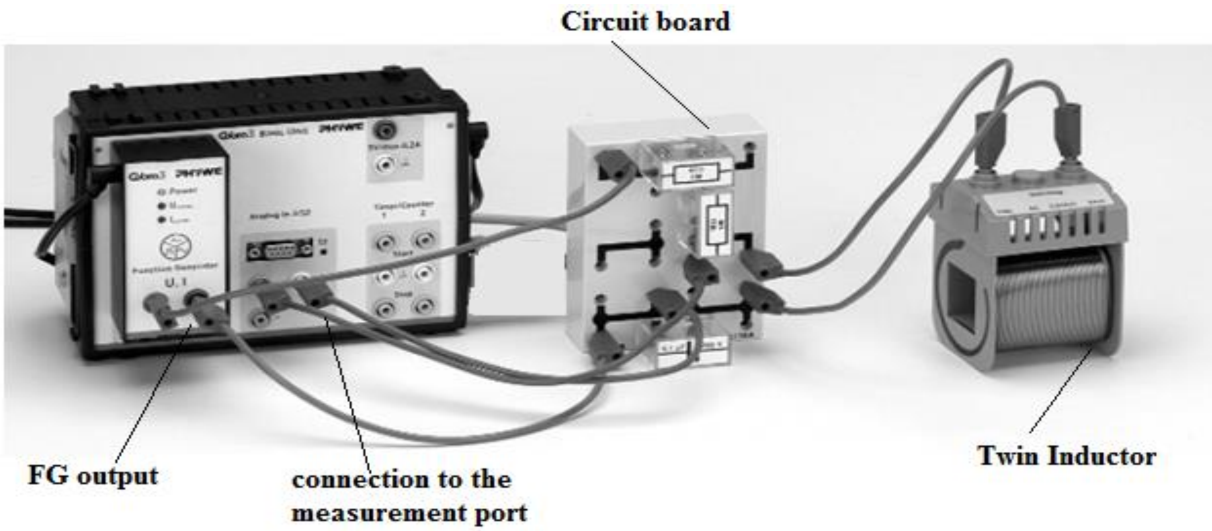

Figure 10 Circuit assembled and connected to FG and Basic unit

The FG is already mounted on the Basic unit. Follow the circuit diagram (Fig.6a) for assembling the circuit. Use the provided circuit board to connect relevant resistor(s) and capacitor(s). The Twin Inductor is actually two inductor coils connected in series. Connect any one of these or the series combination to the circuit using patch cords. Make connection to the measurement port and FG using suitable patch cords. Show the connection to your TA. Once the connections are verified you can power the FG and the Basic unit. Verify that the corresponding Power-ON LED's of both the units are lit. Now you are ready to interface the setup to the PC.

Using the software:

You can launch the application on the PC through the shortcut on the desktop. The application lets you configure and control the various actual and virtual devices, set the display and use plotting and analysis tools.

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One needs to correctly configure the following before proceeding to start the experiment.

1. **FG**
2. **Analog In2/S2 settings**
3. Virtual device
4. **Settings chart of PowerGraph**
5. Displays chart of PowerGraph
6. Universal Writer settings
7. Channel Manager
8. **Analysis tools**

Majority of the settings common to all the experimental modules are already done and you need to understand and use some typical ones in-order to perform the experiment seamlessly.

If you wish to first get a feel of the system without bothering too much about the settings then:
From the opening screen, select **“Guage” → “Powergraph→**Click on the **FG icon → Continue** A screen with many windows each displaying some parameter say voltage etc. will be displayed. Once the system is ready, click “**start measurement**”. Data acquisition will be started and each parameter will be updated. After the designated number of data points are obtained the system will display the result of acquisition in new window. You may print the **graph** and **data points** using **Print** utility.

<u>Screenshots showing various configuration and settings</u>

1. To set the FG:

From the opening screen, select**“Guage” → “Powergraph”** to obtain the window of fig 11a→Click on the **FG icon** to obtain window of fig 7b. Do the necessary settings and click **“OK”**

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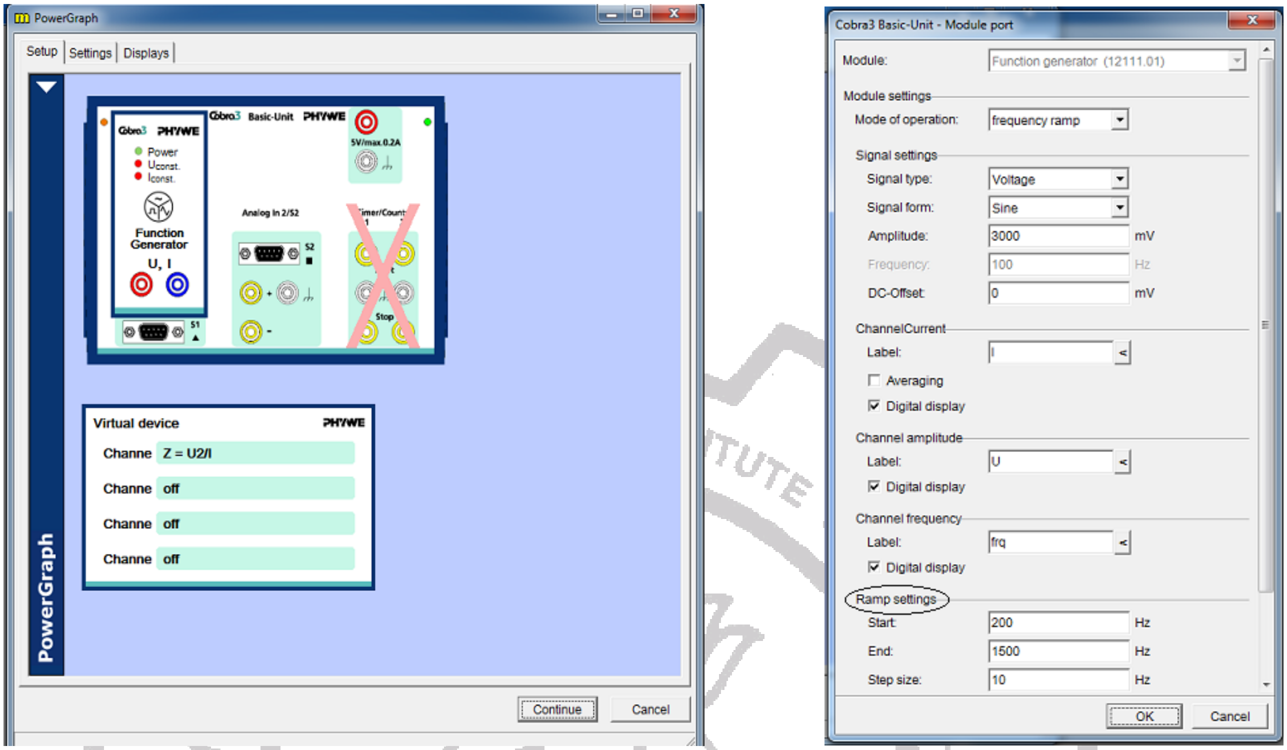

(a) (b)

Figure 11 Setting the FG

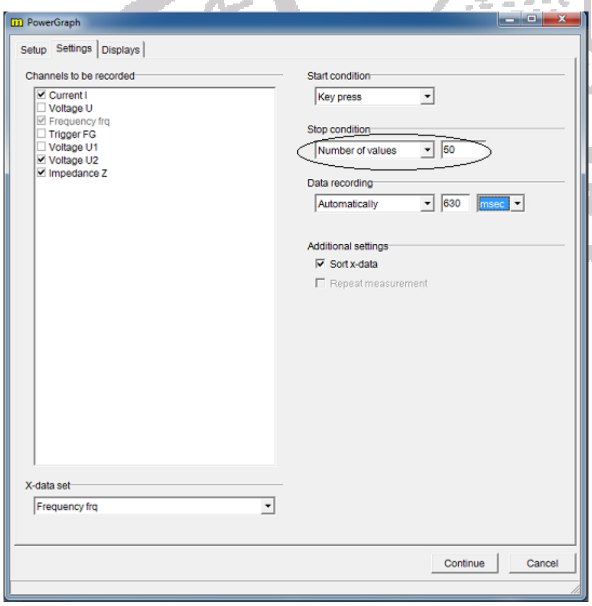

Figure 12 Setting the number of data points

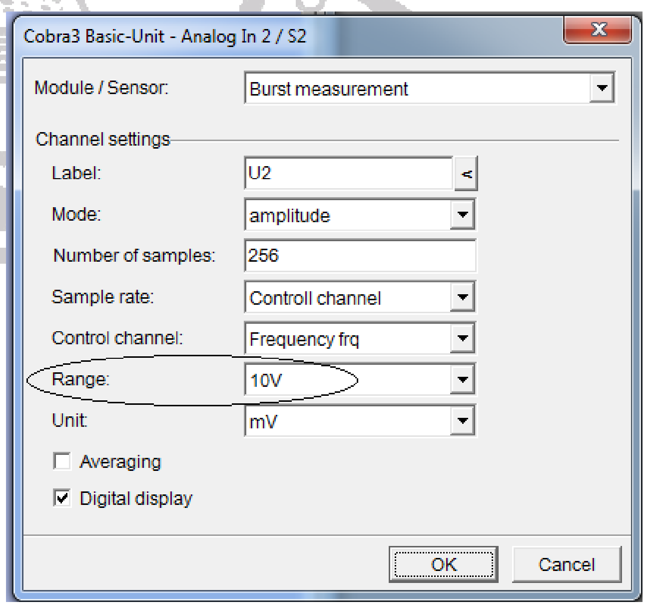

Figure 13 Configuring the measurement port Analog In2

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c. Choosing the optimum parameters

a. Limits of frequency sweep

Too little observations results in loss of data while too many data points add to the clutter without providing any additional insight. You can judiciously select the total number of data points using the appropriate settings.

Generally, one should take enough data points on either side of the resonant frequency. However, in a linear plot, the tail of the response looks too long. The minimum and maximum frequency of frequency sweep should be chosen based on these considerations.

b. Step size of freq. sweep

It depends upon expected bandwidth (Δf) that in turn depends upon the component values.($\Delta f = R/2\pi L$ for series resonant circuit). Typically step size should be at least ¼ of BW.

c. Amplitude of input sinusoidal voltage

It depends upon the minimum impedance of the circuit and maximum current supplying capability of the FG. For e.g. amplitude of the FG voltage should not be more than 3V for total circuit impedance of 50Ω.

d. Range of the measuring port Analog In2

It depends upon the amplitude of sinusoidal voltage, resolution required and available measurement ranges.

d. Pilot reading to *test that the circuit works functionally correct*

Obtain plots of I(f), V_{L-C} (f) and Z_{L-C} (f).

From the nature of the I(f) plot, quickly verify whether the circuit is working correctly, i.e. at resonance and note the f_0. If not, you may have to check your wiring once again!

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e. Precautions:

1. Power ports to both the FG module and the Basic Unit are on the sidewalls of the respective module/unit. Attempting to apply power to other ports can result in serious damage!

2. Power the FG and Basic unit only after the TA has verified the assembled circuit personally.

f. Typical trouble scenes:

a. The frequency response is flat to a value >> 0 ----- (LC short circuited)
b. Very large current drawn ----- (R is shorted out)
c. No current at all ----- (power source not connected, R –open, one of the wires of LCR circuit not connected)
d. Resonance peak shifted ----- (Different value of L and/or C used)
e. Q factor different ----- (Different value of R, L or C used)

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D. Experimental modules

Module 1: Resonance in series RLC circuit

Part A: Basic measurements

1. Internal resistance (R_{INT}) of the FG________Ω(**from the literature,** IT CANNOT BE MEASURED DIRECTLY BY A MULTI-METER. ATTEMPTING TO DO SO MIGHT DAMAGE THE FG)

2. Using Digital multi-meter (DMM), measure and record the values of circuit components

Component	Run1	Run2
Inductor (L)	H	H
Resistance of inductor (r_L)	Ω	Ω
Capacitor (C)	μF	μF
Resistor (R)	Ω	Ω

3. Total resistance (R_T) of the series RLC circuit = ________Ω (Run1), ________Ω (Run2)

Part B: Frequency response

1. Connect the components as per circuit diagram (fig.6a).
(Turn on supply to FG and Basic unit only after verification of your circuit by TA)
2. Based on the expected resonant frequency (f_0) and bandwidth (Δf), set: minimum frequency = _________ Hz , maximum frequency = _________ Hz and step size of the frequency ramp= _________ Hz
3. *Take a pilot reading*
Obtain plots of I(f), V_{L-C} (f) and Z_{L-C} (f).
From the nature of the I(f) plot, quickly verify whether the circuit is working correctly, i.e. at resonance and note the f_0. If not, you may have to check your wiring once again!

4. Actual observation

If the pilot reading is satisfactory then determine the following parameters from the dataset of the plot. Compare the same with their theoretically expected values wherever applicable. Repeat the same with a new value of series resistor (Run2)

	Run1, R_T= Ω		**Run2, R_T = Ω**	
Parameter	Theoretical	Experimental	Theoretical	Experimental
I_0 (mA)				
0.707*I_0 (mA)				
f_0 (Hz)				
f_L (Hz)	**Can ignore at this stage**		**Can ignore at this stage**	
f_H (Hz)				
Δf (Hz)				
Q				

5. For both the runs, determine and hence mark whether the resonant frequency (f_o) equals

$(f_L+f_H)/2$ (i.e. arithmetic mean) or $(f_L*f_H)^{1/2}$ (i.e. geometric mean)

Plot both the resonance curves on the same graph and mark f_0 and Δf of both the responses. Do they show a trend with respect to the value of damping resistor? State and justify <u>analytically</u>.

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Module 2: Voltage magnification in series RLC circuit

1. Re-wire the connection of the measurement port to measure the voltage across the inductor (fig. 6b).
2. Obtain plots of I(f) and V_L(f).
3. From the dataset of the plot, determine the following and compare with their theoretically expected values

	Theoretical	**Experimental**
I_0 (mA)		
0.707*I_0 (mA)		
f_0 (Hz)		
f_L (Hz)	**Can ignore at this stage**	
F_H (Hz)		
Δf (Hz)		
Q		
V_L (V)		

What is the voltage expected across the capacitor (V_C) at resonance?
____________ V.

Is there a relation between V_L and V_C? If yes, then state the relation______________.

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E. Data analysis

	Average	Relative % error w.r.t. theoretical value $\frac{(Experimental - Theoretical) * 100}{Theoretical}$
I_0 --- Run1	NA	
I_0 --- Run2	NA	
f_0	**Hz**	
Δf --- Run1	NA	
Δf --- Run2	NA	
Q--- Run1	NA	
Q--- Run2	NA	
V_L	NA	

F. Results and discussion

Co-relate the experimental results for I_0, f_0, Δf, Q and V_L with their theoretical values and account for discrepancies if any.

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G. Fun questions for bonus marks/viva: *(TA's may expand the list)*

a. Why sinusoidal input only? What will happen if we use square, Triangular or DC voltage?
b. Should we use the FG as a voltage source or current source? Justify.
c. Use of a voltage ramp?
d. effect of offset voltage, if any?
e. need of R_C into the circuit?
f. Calculate the maximum power dissipated across R_C and R_D. Based on this calculation, suggest the power rating to be selected for safe operation.

References

[1]http://repository.phywe.de/files/bedanl.pdf/12150.50/e/1215050e.pdf accessed on June 13, 2015
[2]https://www.phywe.de/en/geraetehierarchie/physics/electricity/electromagnetic-oscillations-and-waves/12111-00#tabs3 accessed on June 13, 2015
[3] http://web.mit.edu/viz/EM/visualizations/coursenotes/modules/guide12.pdf as accessed on June 13, 2015
[4] http://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-071j-introduction-to-electronics-signals-and-measurement-spring-2006/lecture-notes/resonance_qfactr.pdf as accessed on June 13, 2015
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