Expt-8
Centrifugal Force

Aim:

This experiment requires you to determine the centrifugal force experienced by a car which is at rest on a track rotating with a constant angular velocity (See Fig.1). The car comes to rest (after an initial transient motion) at a fixed radius from the center of the rotating track, since the centrifugal force is balanced by the tension in the string, to which the car is tied. This string is connected to a spring balance (dynamometer) on the other end, which allows us to read the tension (and therefore the centrifugal force) directly. **This experiment involves determination of the centrifugal force as a function of (a) the mass, (b) the angular velocity and (c) the distance of the car from rotation axis.**

Apparatus:

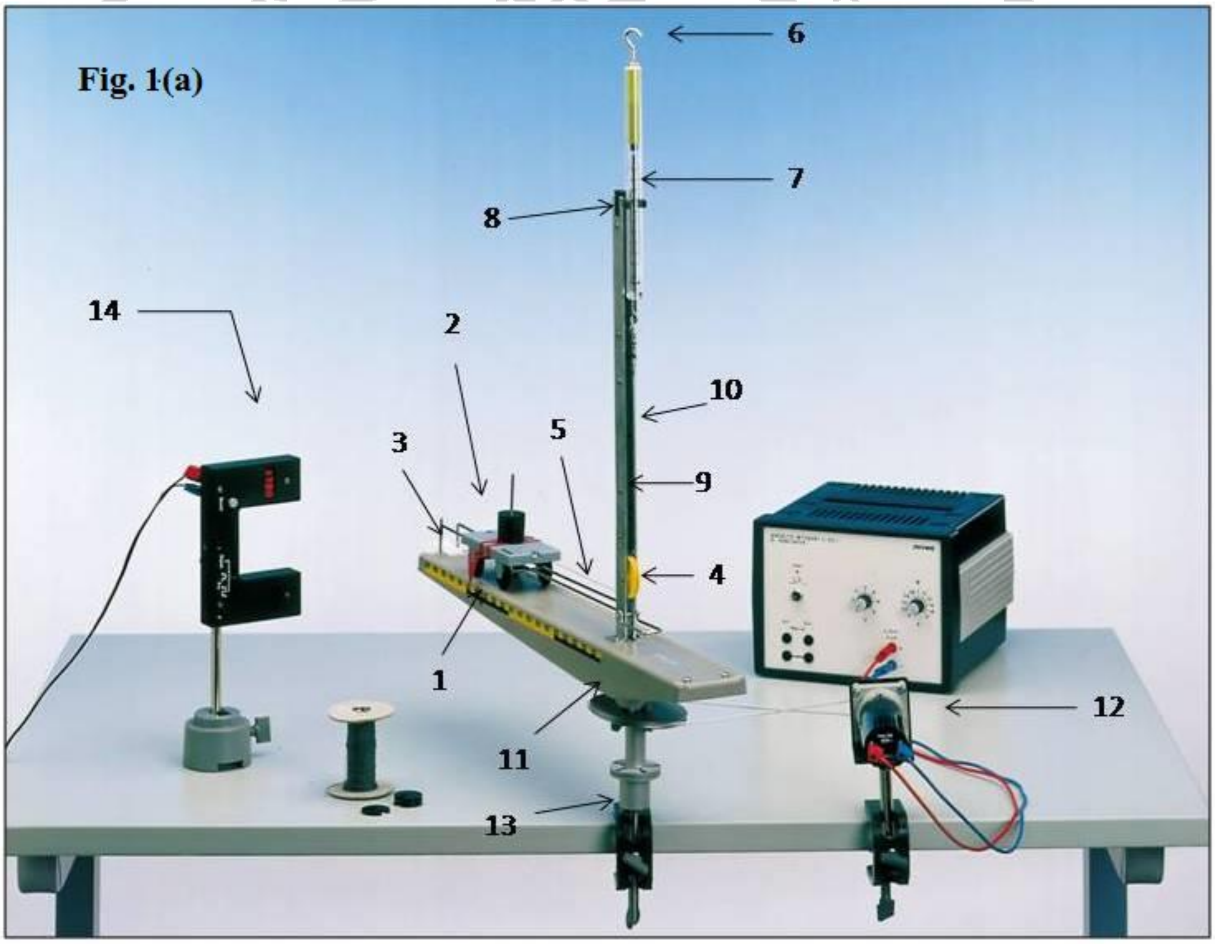

Fig.1 (a): *Experimental Setup*

1. Track
2. Car (shown with the holding pin and slotted weight, 50 g (each), positioned on it). **The mass of the car plus the holding pin is 50 g +/- 1 g**.
3. Holding bow to keep the car from flying off the track at high rotation speeds.
4. Guiding roller for the connecting thread (5).
5. Connecting thread for connecting the car (2) to a dynamometer (6).
6. Dynamometer (e.g. spring balance 1 N)
7. Dynamometer holder. One end is plugged into one of the holes (8) and the other end holds the dynamometer.
8. Holes into which the dynamometer holder can be plugged.
9. Locking pin for turning down the dynamometer holder arm (10). The hinge is freed by pulling the pin out as far as it will go and holding it out. The arm (10) can then be swung down by 90°. The locking pin re-engages in the new position.
10. Dynamometer holder arm, which can be turned down.
11. Grooves for power transmission from the motor and driving belt.
12. Motor.
13. Bearing unit (in the bench clamp) for putting the track.
14. Light Barrier with counter.

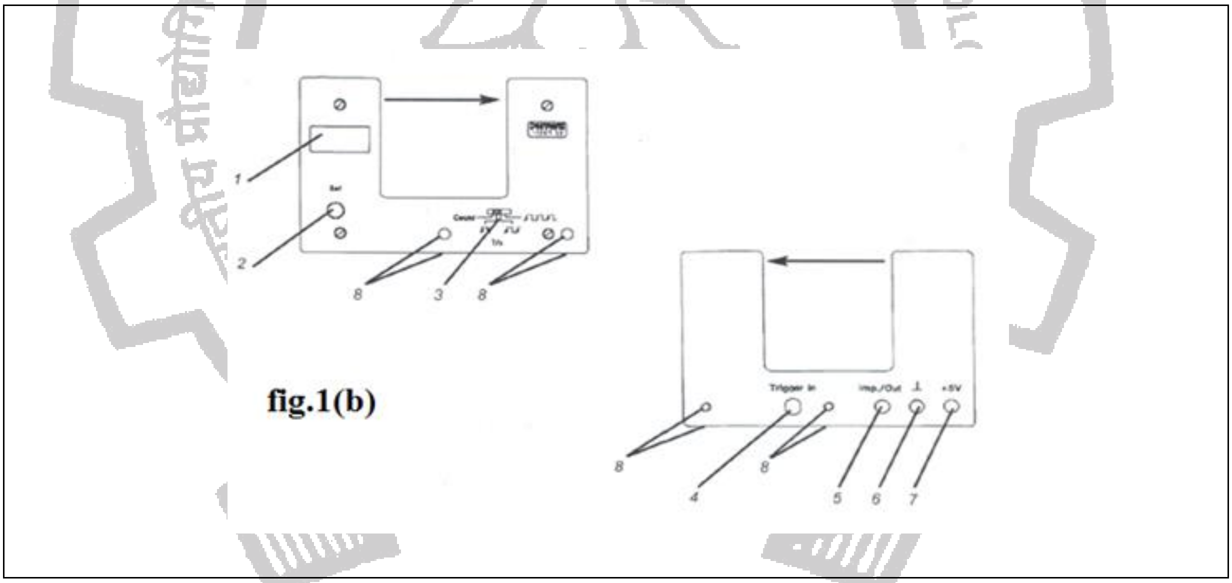

Fig.1 (b): *Light barrier with counter (14)*

The light barrier: Working principle

As the track passes through the light barrier, the light signal is blocked and a counter associated with the instrument measures the time interval between two consecutive blocking of the light barrier by the track. This time interval then corresponds to the time period of rotation of the track.

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To measure the time period, firstly, select the desired operation mode with the switch(In the position" ⌐⌐ "), and then press the SET button. Make sure that the light barrier is not blocked while you press the SET button. Only then is a previously selected operation mode concluded.

The duration between two light blockages is measured and displayed. Measurement range: 0 to 9.999 s. A new measurement is only possible after pressing "SET".

Theory and evaluation:

In the reference system which rotates with the angular velocity ω, the equation of motion of a mass point (mass *m*, position vector *r*) reads:

$$m\frac{d\boldsymbol{v}}{dt} = -\nabla U + m\boldsymbol{r} \times \frac{d\boldsymbol{\omega}}{dt} + 2m\boldsymbol{v} \times \boldsymbol{\omega} + m\boldsymbol{\omega} \times (r \times \boldsymbol{\omega}) - \boldsymbol{F}$$

Here, $\boldsymbol{F}$ is the centrifugal force we are interested in**.**The external force field U (gravitational field) is compensated by the track, the angular velocity ω is

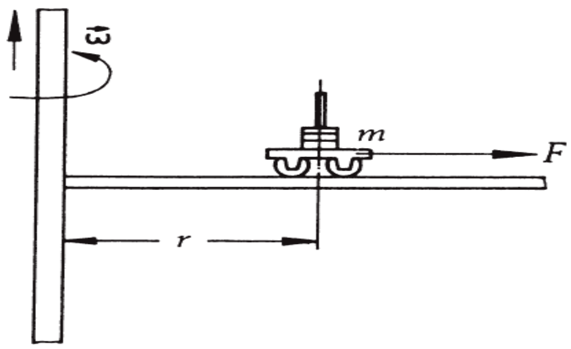

Fig.2: *Schematic showing the car of mass m, at rest in the reference frame of the rotating track (rotating at an angular velocity ofω)*

constant, and the car is at rest in the rotating reference system. Therefore U = 0, $\vec{v}$ = 0,$\vec{\omega}$= const.
Thus for the particular set up we have in this experiment,

$$\boldsymbol{F} = m\boldsymbol{\omega} \times (r \times \boldsymbol{\omega}) = m\omega^2 r$$

since $\boldsymbol{r} \perp \boldsymbol{\omega}$.

Note:

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The centrifugal force experienced by the car is balanced by the tension in the thread connecting the car to the spring balance, which in turn, is balanced by the elongation of the spring in the spring balance, following Hooke's law $F_Z = -kz$, where $k$is the spring constant and z is extension of spring. Thus, measuring off F_Z directly from the spring balance (reading in Newton) allows us to determine the centrifugal force F.

Set-up and Procedure:

The experimental set-up is arranged as shown in Figure 1(a) above. The experiment needs you to essentially vary three parameters; m, r, andω.

To vary m, you are provided with variable slotted weights, which can be put on top of the car. The r can be adjusted by pulling the dynamometer (spring balance) up or down (for a chosen m and ω). The value of ω can be varied by changing the power supplied to the motor rotating the track. You are required to adjust the voltage of the power supply.

F_Z is directly read off from the spring balance. For calculation of ω, you need to find the time period, T, of rotation of the track. This is determined using the light barrier with counter. In the "↑⌊↑" mode, the time between two consecutive blockages (by the rotating track) is measured, which gives T.

The red pointer fitted to the car indicates the distance of (the center of gravity of) the car from the axis of rotation. The distance can be read off directly from the scale shown on the track.

1. Determination of the centrifugal force as a function of mass.

The car is loaded progressively with additional external weights. The track is rotated at a constant ω by choosing a constant setting of power supply (e.g. the usual suggested range of variation 50-70V). Three measurements of T are taken and the average is taken for calculation ofω. Once ω is fixed, an adhesive tape is used to mark the position (r = 20 cm) where we want the car to stop when the track rotates. However, as the mass is varied, the car will tend to stop at different values of r. It can be brought back to the position of the adhesive tape, by adjusting the vertical position of the dynamometer. This way r remains constant during this experiment. Please remember to switch off the track rotation while pulling the dynamometer up or down.

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For each every chosen mass m, the force on the spring (F_Z), and (ideally) therefore, the centrifugal force F, is directly read off from the dynamometer, while the track is rotating. This $F_Z = F$ must be tabulated together with the mass m, for plotting. The plot of F_Z versus m must then be (least square) fit with a straight line (See the section on plot fitting and error analysis). Plot F_c vs m, on the same graph to compare the theoretical and experimental values.

2. Determination of the centrifugal force as a function of angular velocity.

In this measurement, the mass of the experimental car stays constant (e.g. m=100 gm). Again, a predetermined radius (e.g. r = 20 cm) is marked with a piece of adhesive tape. At different angular velocities, the car is brought to position r, by adjusting the position of the dynamometer. To varyω, the voltage is varied at the power supply, and the average of two readings of the time period is taken using the light barrier. The force $F_Z = F$ at different ω are read off from the dynamometer and tabulated together with the values of the voltage (V), two readings of time period (T_1 and T_2), the average time period $\bar{T}$, and the calculated values of ω. Then F_Z and F_c must be plotted (on the same graph) as a function of ω^2, which is then (best fit) fit with a straight line using best to compare both the results (See the section on plot fitting and error analysis).

3. Determination of the centrifugal force as a function of distance from the axis of rotation:

Here, the mass of measuring car remains constant (e.g. m=100 gm). The motor is rotated at a constant angular velocity (e.g. V=60 V). Here you take 5 readings of T and use the average value ($\bar{T}$) to calculateω (See section on plot fitting and error analysis). The radius of the orbit r of the car is increased by means of displacement of the spring balance. The reading of the dynamometer F_Z and the radius r are measured. You will then tabulate the values of r, F_Z, and $F = m\omega^2 r$ and plot both F_Z and F on the same plot. On the plot of F, show error bars (See the section on error analysis).

Plot Fitting and Error Analysis:

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1) For all the three experiments mentioned above, F_c and F_Z should be plotted using best fit, except the analysis for F_Z vs m, plot should be performed using least square fit. To be able to fit a straight line, $y = mx + c$, the average values of $\bar{y}$ and $\bar{x}$ should be calculated from the data as

$$\bar{y} = \frac{1}{N}\sum_{i=1}^{N} y_i \text{ and } \bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

and

$$m = \frac{N\sum_{i=1}^{N} x_i y_i - \sum_{i=1}^{N} x_i \sum_{i=1}^{N} y_i}{N\sum_{i=1}^{N} x_i^2 - (\sum_{i=1}^{N} x_i)^2}$$

and

$$c = \bar{y} - m\bar{x}$$

The values of m and c should then be used to fit the straight line to the data.

2) For the third measurement ($r -$ dependence) , a plot of the function $F_C = \bar{m}\bar{\omega}^2 r$ as a function of r has to be plotted on the same plot as (measured) F_z versus$r$. For this plot, first the value of ω needs to be determined. To calculate, ω the time period of rotation of the track T, needs to be measured. Three readings of T should be taken at each set value of r. Average and standard deviation should be calculated for all five values of T. The average value can be calculated as

$$\bar{T} = \frac{1}{5}\sum_{i=1}^{5} T_i$$

Then calculate the standard deviation as

$$\delta T = \sqrt{\frac{1}{5}\sum_{i=1}^{5} (T_i - \bar{T})^2}$$

Calculate $\bar{\omega} = 2\pi/\bar{T}$and $\delta\omega$ from the relation

$$\frac{\delta\omega}{\overline{\omega}} = \frac{\delta T}{\overline{T}}$$

Mass of the car, $m = \overline{m} + \delta m = 50 \pm 1$ gm. Uncertainty in r is $\delta r = \pm 0.5$ cm. So you can calculate the centrifugal force as $F_C = m\overline{\omega}^2 r$ and the error in calculating F_C as

$$\frac{\delta F_C}{F_C} = \frac{\delta m}{m} + 2\frac{\delta\omega}{\overline{\omega}} + \frac{\delta r}{r}$$

In the plot of F_C versus r and, show the error bars. Provide explanations for discrepancies.

Precautions:

1. The current and voltage knobs should be turned down to zero before switching the power supply ON.
2. Take great care that no person comes so near the apparatus that he or she could be hit by the rotating track.
3. The track must turn truly horizontally; otherwise the dynamometer reading may fluctuate considerably. If necessary, correct the footing of the bench clamp by partly inserting a piece of cardboard beneath.
4. When motor is on, track will rotate and it will pass through light barrier. So adjust the height and distance of barrier so that the rotating track will safely pass through light barrier.
5. For keeping omega constant in first and third part of the experiment use toggle switch. Do not change the knob position.